DOCUMENT RESUME

ED 058 022	SE 002 115
TITLE	Novel Experiments in Physics. A Selection of Laboratory Notes Now Used in Colleges and Universities.
INSTITUTION	American Association of Physics Teachers, Washington, D.C.; American Inst. of Physics, New York, N.Y.
REPORT NO	Pub-R-165
PUB DATE	Apr 64
NOTE	424p.
EDRS PRICE	MF-$0.65 HC-$16.45
DESCRIPTORS	*College Science; *Experiments; Instructional Materials; Laboratory Experiments; *Laboratory Procedures; *Physics; Teaching Procedures
IDENTIFIERS	American Association of Physics Teachers

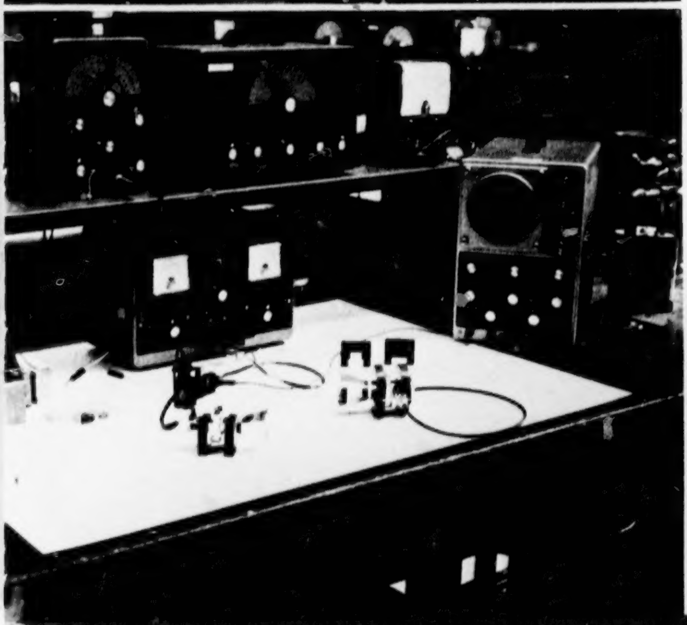
ABSTRACT

The present volume was compiled by the AAPT Committee on Apparatus for Educational Institutions and by the AAPT-AIP Center for Education Apparatus in Physics to assist physics departments in the never-ending task of keeping their undergraduate laboratories up to date. The purpose of the book is to put before physics teachers for their consideration new experiments or interesting ways of doing well-known experiments. It contains reprints of experiment notes in physics. The experiments were selected by members of the Apparatus Committee from laboratory manuals and sets of laboratory notes submitted by over one hundred and thirty colleges and universities in the United States. Eighty-one experiments are included from the various areas of experimental physics usually treated as a part of an undergraduate major in physics. No attempt has been made to separate experiments designed for advanced laboratories from those usually performed as a part of elementary courses. (Author/TS)

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
OFFICE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY

NOVEL EXPERIMENTS IN PHYSICS

a selection of laboratory notes now used in colleges and universities



Project sponsored by the
Committee on Apparatus for Educational Institutions
of the American Association of Physics Teachers

Published by the **American Institute of Physics**

1

Pub. R-165

NOVEL EXPERIMENTS IN PHYSICS

A selection of laboratory notes now used in colleges and universities

Project sponsored by the
Committee on Apparatus for Educational Institutions
of the American Association of Physics Teachers
Published by the
American Institute of Physics
335 East 45 Street, New York 10017

Committee on Apparatus for Educational Institutions of the American Association of Physics Teachers

Walter R. French, Jr.
Nebraska Wesleyan University

John G. King
Massachusetts Institute of Technology

Harry F. Meiners
Rensselaer Polytechnic Institute

W. C. Kelly, *Secretary*
American Institute of Physics

H. V. Neher
California Institute of Technology

Alan M. Portis
University of California, Berkeley

Howard P. Stabler
Williams College

Allan M. Sachs, *Chairman*
Columbia University

April, 1964

Printed in U.S.A.

PREFACE

The present volume was compiled by the AAPT Committee on Apparatus for Educational Institutions and by the AAPT-AIP Center for Educational Apparatus in Physics to assist physics departments in the never-ending task of keeping their undergraduate laboratories up to date. It contains reprints of experiment notes in physics. The experiments were selected by members of the Apparatus Committee from laboratory manuals and sets of laboratory notes submitted by over one hundred and thirty colleges and universities in the United States. Eighty-one experiments are included from the various areas of experimental physics usually treated as a part of an undergraduate major in physics. No attempt has been made to separate experiments designed for advanced laboratories from those usually performed as a part of elementary courses.

The purpose of the book is to put before physics teachers for their consideration new experiments or interesting ways of doing well-known experiments. The Committee decided to do this in the most detailed and convenient way possible - by photo-offsetting the actual experiment sheets that are given to students. The only modification in these materials has been the addition of a few notes to refer the reader to other sources of information about apparatus or experimental procedures.

This book is for the teacher and not the student. It is in no sense an "official" or comprehensive manual of experiments and will have missed its mark if it is so treated. Many good experiments - ones that should be a part of the laboratory work of the physics student - are not included since many are already well known. The goal of the Committee was to select experiments that seemed interesting in themselves, instead of attempting to determine the best experiment in any group or covering any given topic. Rather than treat this collection as a description of a complete laboratory program, the physics teacher should regard it as a source of ideas for the possible modification of existing programs, selecting ideas that seem interesting and applicable. The experiments cover a wide range, both in the difficulty of the theoretical topics treated and in the sophistication of the equipment used. Some experiments could be easily adopted by any college; others involve almost "irreplaceable" or highly elaborate equipment and have been included as an indication of what is possible rather than as a model for duplication. If the present book proves useful to teachers, the Committee will attempt to compile similar volumes from time to time as new experiments are devised in physics departments. Comments from users of this book will be welcome.

The Committee thanks the physics departments that provided copies of their laboratory manuals, especially those that kindly consented to having some of their experiments reprinted here. We wish that it were possible to give the names of the many physicists who contributed to the development of the eighty-one experiments included here. We have had to be content with giving the name of the institution at which the experiment was developed. All of the members of the Committee on Apparatus in 1963-64 helped with this project, but special acknowledgment is due Walter French, John King, H. Victor Neher, and Howard Stabler who, with the undersigned, carried out the initial review of the experiments being considered for this book, and W. C. Kelly, who supervised all phases of the preparation. The project was coordinated by F. E. Christensen, Director of the AAPT-AIP Apparatus Center, which is supported by a grant from the National Science Foundation. Mrs. Margaret T. Llano of the AIP Department of Education and Manpower saw the book through the press.

Publication of this book is a part of the overall program of the AAPT Committee on Apparatus to promote the development of new equipment and to assist physics teachers in obtaining new instructional apparatus. Among other projects sponsored by the Committee are the AAPT-AIP Apparatus Drawings Project, Apparatus Notes and Apparatus Reviews in the AMERICAN JOURNAL OF PHYSICS, national surveys of apparatus needs, meetings with manufacturers to encourage the production of new educational equipment in physics, and Apparatus Competitions at New York meetings of the AAPT. The Committee serves as the advisory committee for the Center for Educational Apparatus in Physics, which provides staff support and implementation for the work of the Committee. Further information about these activities can be obtained from the Center at 335 East 45 Street, New York, N.Y. 10017.

Allan M. Sachs, Chairman

AAPT Committee on Apparatus
for Educational Institutions

April 10, 1964

CONTENTS

MECHANICS

MEASUREMENT AND EXPERIMENTAL ERRORS 1
Columbia University

TIME INTERVAL MEASUREMENT 7
University of California, Berkeley

VELOCITY WHEEL - THE STUDY OF INSTANTANEOUS VELOCITY. 15
University of Maryland

VELOCITY WHEEL - THE DETERMINATION OF THE VELOCITY. 19
AS A FUNCTION OF TIME, AND THE ACCELERATION
University of Maryland

CONSERVATION OF MOMENTUM. 23
Cornell University

CONSERVATION OF MOMENTUM. 29
Massachusetts Institute of Technology

LINEAR AIR TROUGH 33
California Institute of Technology

TRAJECTORIES. 51
Harvard University

THE VELOCITY OF A BULLET. 57
California Institute of Technology

A. CENTRIPETAL FORCE, B. CONSERVATION OF ANGULAR MOMENTUM. . . 59
Cornell University

WORK AND ENERGY 65
Massachusetts Institute of Technology

VELOCITY WHEEL - DYNAMICAL ANALYSIS 73
University of Maryland

ROTATIONAL DYNAMICS 77
The Florida State University

SPHERICAL PENDULUM: ANGULAR MOMENTUM 81
University of Minnesota

MECHANICAL RESONANCE. 87
Stanford University

COUPLED LINEAR OSCILLATOR 89
University of Minnesota, Duluth Branch

MOTOR DRIVEN GYROSCOPE 91
Massachusetts Institute of Technology

THE MAXWELL TOP. 99
California Institute of Technology

THE SYMMETRICAL TOP. 103
Dartmouth College

MOLECULAR PHYSICS, HEAT, SOUND

SPECIFIC HEAT OF GRAPHITE. 105
University of Colorado

VISCOSITY OF GASES 107
California Institute of Technology

ULTRASONICS. 111
Michigan State University

ELECTRICITY and MAGNETISM

ELECTROSTATIC FORCES AND THE ABSOLUTE MEASUREMENT OF VOLTAGE 117
Massachusetts Institute of Technology

MAGNETIC FIELDS. 125
Columbia University

EDDY CURRENTS. 129
Harvard University

RESONANCE PHENOMENA IN ALTERNATING CURRENT CIRCUITS. 135
Columbia University

ELECTRONICS

THERMIONIC EMISSION. 143
Massachusetts Institute of Technology

TRANSISTOR CHARACTERISTICS 147
University of Delaware

AMPLIFICATION. 151
University of California, Berkeley

FREQUENCY RESPONSE AND NEGATIVE FEEDBACK 161
University of California, Berkeley

ELECTROMAGNETIC WAVES

VELOCITY OF LIGHT. 169
Harvard University

TO DETERMINE THE REFRACTIVE INDEX OF A. GLASS, B. WATER, BY BREWSTER'S LAW. 177
Iona College

CONSTRUCTION AND USE OF A FABRY-PEROT INTERFEROMETER 179
William Jewell College

POLARIZED LIGHT AND THE FARADAY EFFECT 183
Massachusetts Institute of Technology

DISPERSION AND DOUBLE REFRACTION OF QUARTZ 189
University of Minnesota

STUDY OF MICROWAVES AS AN EXAMPLE OF WAVE PHENOMENA. 191
University of Colorado

MICROWAVES 195
Massachusetts Institute of Technology

MICROWAVE PROPAGATION. 201
University of California, Berkeley

MICROWAVE DIFFRACTION. 213
University of California, Berkeley

ATOMIC PHYSICS

THE MILLIKAN OIL DROP EXPERIMENT 223
California Institute of Technology

ELECTRONIC CHARGE. 227
University of California, Berkeley

CHARGE-MASS RATIO OF ELECTRONS 231
Oberlin College

MEASUREMENT OF e/m FOR THE ELECTRON. 237
Dartmouth College

THE ZEEMAN EFFECT (OPTICAL DETERMINATION OF e/m) 243
Stanford University

THE RATIO OF e/m FOR THE ELECTRON. 245
Massachusetts Institute of Technology

PLANCK'S CONSTANT. 261
Newark College of Engineering

MASS SPECTROMETER. 265
University of California, Berkeley

THE SINGLE CRYSTAL X-RAY SPECTROMETER. 275
University of Pennsylvania

CHARACTERISTIC X-RAY SPECTRA 279
University of California, Berkeley

ABSORPTION OF X-RAYS 283
University of Pennsylvania

COMPTON SCATTERING OF X-RAYS 287
Massachusetts Institute of Technology

THE COMPTON EFFECT 291
University of Colorado

ENERGY STATES OF SODIUM. 297
Lehigh University

FRANCK-HERTZ EXPERIMENT. 305
University of California, Berkeley

OPTICAL PUMPING. 311
University of California, Berkeley

ZEEMAN EFFECT. 313
Purdue University

SOLID STATE PHYSICS

ELECTRICAL CONDUCTION BY SOLIDS. 321
Brown University

VELOCITY DISTRIBUTION OF THERMOELECTRONS 327
Stevens Institute of Technology

SEMICONDUCTOR TEMPERATURE COEFFICIENT OF RESISTANCE. 331
University of Minnesota, Duluth Branch

DRIFT MOBILITY OF CARRIERS IN GERMANIUM. 333
Purdue University

THE USE OF THE HALL EFFECT TO STUDY THE FORCE
ON A CHARGE MOVING IN A MAGNETIC FIELD 339
University of Colorado

RESISTIVITY AND HALL EFFECT. 343
Purdue University

THERMOELECTRIC POWER OF GERMANIUM AGAINST COPPER 355
Purdue University

COLOR CENTERS. 359
Dartmouth College

NUCLEAR and HIGH ENERGY PHYSICS

GEIGER COUNTERS AND SCALING SYSTEMS. 361
Oak Ridge Institute of Nuclear Studies

SCINTILLATION COUNTER. 371
Massachusetts Institute of Technology

RANGE OF ALPHA PARTICLES 379
North Carolina State College

THE ABSORPTION OF GAMMA RAYS IN LEAD 383
University of Colorado

ARTIFICIAL RADIOACTIVITY 391
University of Pennsylvania

THE ACTIVATION OF COPPER 395
North Carolina State College

COINCIDENCE MEASUREMENTS 399
North Carolina State College

NUCLEAR INDUCTION. 403
University of California, Berkeley

MÖSSBAUER EFFECT 407
California Institute of Technology

BETA RAY SPECTROMETRY. 411
Oak Ridge Institute of Nuclear Studies

THE RUTHERFORD SCATTERING EXPERIMENT 415
Massachusetts Institute of Technology

SCATTERING OF ALPHA PARTICLES. 425
Stanford University

COLLISION OF PARTICLES 427
Dartmouth College

LIFETIME OF THE MU MESON 433
Massachusetts Institute of Technology

MESON DECAY: ANALYSIS OF BUBBLE CHAMBER EVENTS. 437
Columbia University

ANALYSIS OF A K+ MESON DECAY IN NUCLEAR EMULSION 447
University of California, Berkeley

COVER PHOTOS...courtesy of California Institute of Technology
and the University of California, Berkeley

Mechanics

11

Columbia University

References: (1) Statistical Treatment of Experimental Data by Hugh D. Young, McGraw-Hill (paperback); (2) Probability and Experiment Errors in Science by Lyman G. Parratt, Wiley.

GENERAL DISCUSSION

The Laws of Physics are based on measurements obtained in experimental observations. The results must include not only numerical values (expressed in appropriate units) but also estimates of the reliability of the measurements. Strictly speaking, we can never measure any quantity "exactly"; we obtain only an approximate value. The degree of uncertainty, however, can be indicated by assessing that there is a certain probability that the true magnitude will lie within definite limits. The separation of these limits from the measured value is referred to as the "uncertainty" or "error" of the measurement.

Errors may be classified into two general categories, systematic and random.

1. Systematic Errors. These include prejudice on the part of the observer, improper use of or adjustment of equipment, inherent defects in the equipment, or neglect of such effects as temperature, pressure, humidity, etc. Careful attention to and elimination of major sources of systematic errors are, of course, basic to good experimentation. After minimizing the systematic errors, an estimate of the residual uncertainties should be included in the result of each experiment. Note that for a series of similar measurements, the net systematic errors will tend to be of one sign; that is, the main result will not be a spread in values obtained but a shift of the mean value in a given direction.

2. Random Errors. A series of measurements in which the systematic errors have been minimized will still contain variations due to causes that lie beyond the control of the observer. The presence of such errors is made evident by discrepancies among individual observations made under apparently identical conditions. Into this category fall:

(a) Variations in instrumental readings in measuring a well defined quantity. For example, variations due to parallax and human judgment in interpolating between marks on a ruler used to measure the length of a rod.

(b) Variations known as statistical fluctuations, which arise not from errors in observation but from the element of chance associated with the sampling of a population or of any random distribution. For example, variations in recording counts from a random source of radioactive particles over equal lengths of time.

Both types of random errors are subject to the same kind of analysis, and both will be studied in this experiment.

Illustrative Example I. (Example of Random Error, type a)

The table lists $n = 16$ measurements of the length of a single brass rod. Each measurement is made by interpolating by eye between 0.5 mm divisions on a ruler.

Measurement	Length mm	Deviation $d = L - \bar{L}$	d^2
1.	22.1	+ .17	.062
2.	22.0	+ .07	.005
3.	21.9	- .03	.001
4.	21.8	- .13	.017
5.	21.8	- .13	.017
6.	21.7	- .23	.053
7.	21.9	- .03	.001
8.	22.0	+ .07	.005
9.	21.9	- .03	.001
10.	22.3	+ .37	.137
11.	21.9	- .03	.001
12.	22.1	+ .17	.062
13.	21.9	- .03	.001
14.	21.8	- .13	.017
15.	22.0	+ .07	.005
16.	21.8	- .13	.017
	$\Sigma L = 350.9$	$\Sigma\lvert d\rvert = 1.82$	$\Sigma\lvert d\rvert^2 = .402$

$$\bar{L} = \frac{\Sigma L}{16} = 21.93 \quad \text{a.d.} = \frac{\Sigma|d|}{16} = 0.11 \quad \text{s.d.} = \left[\frac{\Sigma|d|^2}{16}\right]^{1/2} \quad \text{or}$$

$$\text{preferably} \quad \left[\frac{\Sigma|d|^2}{15}\right]^{1/2} = 0.16$$

Average deviation of mean:

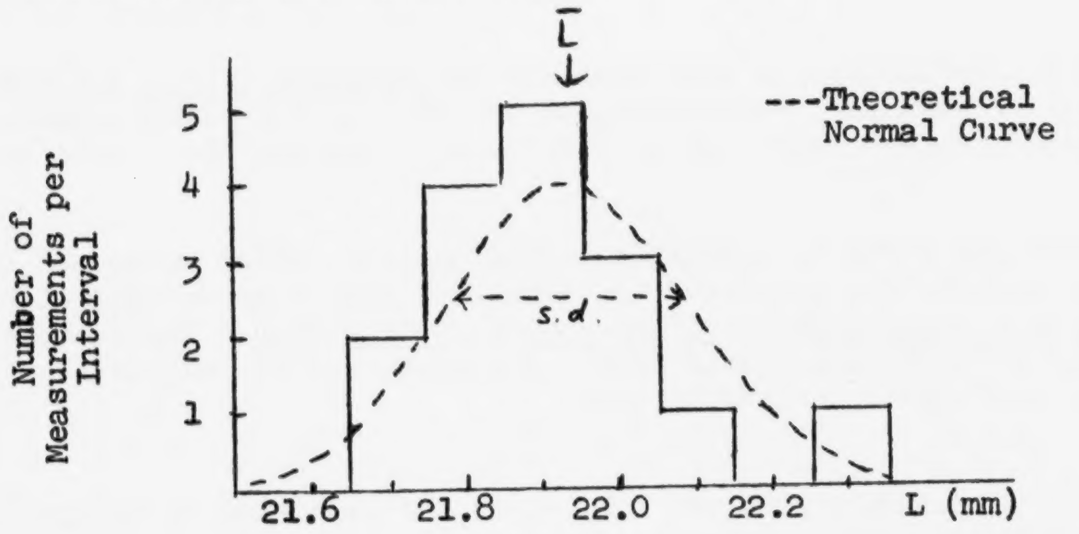
$$\text{A.D.} = \frac{1}{\sqrt{16}}\ \text{a.d.} = 0.03$$

Standard deviation of mean:

$$\text{S.D.} = \frac{\text{s.d.}}{\sqrt{16}} = 0.04$$

Final Result: L = 21.93 ± 0.04 mm

The data may be represented by the histogram, indicated by the solid lines in the figure below.



STATISTICAL ANALYSIS

Distribution.

It is obvious that the histogram in Example I can not be represented accurately by a simple mathematical curve. However, as n, the number of measurements, is increased, the histogram will approach the shape of a symmetrical bell-shaped curve (assuming that the measurement error is in fact random). An example of such a curve, the so-called Gaussian or Normal Error distribution, is indicated by the dashed curve in Fig. 1. A detailed statistical analysis makes use of the properties of this curve, or of a similar "Poisson" distribution. (e.g. see reference sections 8 and 9.) In this experiment, we will omit the details of the statistical distribution, limiting the discussion to apparently arbitrary measures of the deviations of the data about the mean.

Mean. For a symmetrical distribution, the "most probable" result is identical with the mean, i.e. the arithmetical average of the n individual measurements, ($\overline{L}$ in the table.) The best single value from the set of measurements is thus the mean; what remains is to estimate the error.

Average and Standard Deviation. The dispersion of an actual set of n measurements about their mean value is an experimental indication of the random errors involved in the measurement. The simplest index of this dispersion is the "average deviation", (a.d.) which is defined as the average of the absolute values of the deviations of the measurements about the mean. (See column 3 of the table.)

$$d_i = L_i - \overline{L} \qquad\qquad \text{a.d.} = (1/n) \sum_{i=1}^{n} |d_i|.$$

A more widely used index of dispersion is the root-mean-square or "standard deviation", (s.d.), calculated in column 4 of the table:

$$(\text{s.d.})^2 = (1/n) \sum_{i=1}^{n} (d_i)^2.$$

In order to account for the inaccuracy of locating the "true" mean from a single set of measurements, this measure of precision is usually increased by replacing n by n - 1. The conventional definition is thus:

$$(\text{s.d.})^2 = \left[1/(n-1)\right] \sum_{i=1}^{n} (d_i)^2.$$

For large values of n, of course, the difference is negligible. The standard deviation assigns more weight to large deviations, since the deviations are squared before averaging.

The standard (or average) deviation of a set of measurements provides a numerical guess as to the likely range of values into which the next measurement may fall. For a Gaussian distribution, 68% of the measurements fall within a region of (+ or -) one standard deviation from the mean. (For a Gaussian distribution, a.d. = 0.8 s.d., and 58% of the measurements lie within one average deviation of the mean.)

Standard Deviation for a Distribution of Random Events.
For random errors of type a, as in the example of measuring a length the s.d. of the distribution of a set of measurements depends on experimental details, such as the inaccuracy of the instrument, etc. For random errors of type b, as in the measuring of the number of random counts recorded in a time interval, the s.d. of the distribution of a large set of such measurements can be predicted on a statistical basis solely from m, the mean number of counts in each interval. The distribution will approach a "Poisson" distribution function, which has the property that:

$$\text{s.d.} = \sqrt{m}$$

(See reference 1, section 8.)

Standard Deviation of the Mean.
The precision of the mean, which depends on a set of n measurements, is, of course, greater than the s.d., which indicates a probable range for a single measurement. If we were to record a second set of n measurements, the value of the second mean would in general differ from the value of the first; but the difference would be expected to be less than the s.d. of either set. In principle, we could make N sets of n measurements, find the distribution of the N means, and compute the standard deviation of the means (which we will call, S.D.) about the overall mean of all of the measurements. The above procedure is fortunately not necessary, since from statistical theory we can actually predict a value for S.D. from the s.d. of a single set of n measurements, namely:

$$\text{S.D.} = \text{s.d.} / \sqrt{n} \, . \qquad \text{(See Reference 1, section 12.)}$$

We then use this value of S.D. as the best estimate of the precision (or "error") of the measured mean - e.g. see the final result of the illustrative example.

Propagation of Errors.
When a result of interest involves some combination of measured quantities, the precision or s.d., of the result can be computed in terms of the measured values of the S.D. for each quantity.

Addition and Subtraction. When the means of the measurements of two different quantities are added, the S.D. of the sum may be expected to be less than the sum of the two S.D.'s, since there is as much chance of two random deviations being of opposite sign as of the same sign. It can in fact be shown from statistical theory (see reference 1, section 14) that for addition or subtraction:

$$\text{mean of } (x \pm y) = \text{mean of } x \pm \text{mean of } y,$$

and

$$\left[\text{S.D. of } (x \pm y)\right]^2 = \left[\text{S.D. of } x\right]^2 + \left[\text{S.D. of } y\right]^2 .$$

i.e. S.D.'s are added "in quadrature".

Multiplication and Division. In multiplying two quantities, the precision of the product depends on the fractional (or percentage) uncertainty in each factor rather than the absolute uncertainties. More precisely, again it can be shown that:

$$\text{mean of } xy = (\text{mean of } x)\ (\text{mean of } y)$$

$$\left(\frac{\text{S.D. of } xy}{\text{mean of } xy}\right)^2 = \left(\frac{\text{S.D. of } x}{\text{mean of } x}\right)^2 + \left(\frac{\text{S.D. of } y}{\text{mean of } y}\right)^2 .$$

Similarly,

$$\text{mean of } x/y = (\text{mean of } x)\ /\ (\text{mean of } y)$$

$$\left(\frac{\text{S.D. of } x/y}{\text{mean of } x/y}\right)^2 = \left(\frac{\text{S.D. of } x}{\text{mean of } x}\right)^2 + \left(\frac{\text{S.D. of } y}{\text{mean of } y}\right)^2$$

i.e. fractional S.D.'s are added "in quadrature". More generally, it can be shown that:

$$\left(\frac{\text{S.D. of } x^{\ell}y^{m}}{\text{mean of } x^{\ell}y^{m}}\right)^2 = \left[\ell\,\frac{(\text{S.D. of } x)}{(\text{mean of } x)}\right]^2 + \left[m\,\frac{(\text{S.D. of } y)}{(\text{mean of } y)}\right]^2.$$

PROCEDURE

Part (a): Measurement of the Density of a Brass Cylinder.

Apparatus: Ruler, micrometer, vernier calipers, balance, and a machined cylinder of brass.

Manipulations: Using the ruler, make a series of independent measurements of the length and diameter of the cylinder. To insure that these are independent, i.e. to minimize the influence of each reading on the following one, the ruler should be shifted after each reading and the positions of both ends should then be recorded.

Use the balance to measure the mass of the cylinder.

Use the micrometer and vernier calipers to make a single, more accurate measurement of the diameter and length of the cylinder.

Computations: Compute the mean, s.d., and S.D. for each set of measurements. From these, find the mean and S.D. for the volume and density of the cylinder.

Compute the volume and density of the cylinder for the more accurate measurements and compare.

Part (b) Measurement of the Counting Rate of a Geiger Counter.

Apparatus: Geiger counter with high voltage supply and scaling circuit, stop watch, and radioactive source with long half-life.

Manipulations: The instructor will turn on the counter and make the necessary adjustments. Do not change the setting on the voltage control. Record the number of counts registered by the geiger counter in a 15-second interval . Repeat this measurement as many as 50 times.

Computations: Compute the mean a.d., s.d., and S.D. from the measurements.

Plot a histogram of the measurements, indicating on it the computed values.

Compare the results with those predicted by a statistical theory for random events.

QUESTIONS

1. How does the computed precision of the measurements in part (a) compare with a reasonable estimate of the error in reading the ruler?
2. How does the s.d. of the distribution of measurements in part (b) compare with the what would be expected statistically in a large set of measurements of the mean number of random counts?
3. List possible systematic errors in this experiment.
4. If you wish to measure the flux of particles (i.e. counts /cm^2/sec) in part (b), how carefully would you have to measure the area of the counter in order not to limit the accuracy of the result?
5. How many counts are required in a single measurement to give a statistical precision of 1% ?

TIME INTERVAL MEASUREMENT

University of California, Berkeley

Purpose: To measure a small time interval by electronic counting.

Principle of operation: This experiment consists of a precision measurement of "g". The measurement is effected in the following way. A metal sphere falls in vacuum past three pairs of slits. (see fig. 1). The interruption of the light beam to photomultiplier tube #1 causes the gate-forming circuit to produce a positive voltage on its channel A output. The interruption of the light to P.M. #2 causes the gate-forming circuit to terminate the channel A output pulse and to start a similar pulse on output B. PM #3 terminates the channel B pulse. Output A of the gate-forming circuit is fed into the channel A gating tube on the oscillator chassis. This gates the oscillator into the channel A scaler. Channel B operates identically.

Description of components:

a. Slit system and tower

The tower can be evacuated. There is in the top, an electromagnet to hold the sphere until dropped. The three forward slits are fixed in position. The three after slits may be moved vertically. The slit widths may be adjusted. The PM chassis may be removed by removing the retaining screws. The PM tube is a type 931A. The circuit diagram of the PM chassis is shown in fig. 2. The circuit diagram for the lamps and magnet are shown in fig. 3.

b. Oscillator chassis

The circuit diagram of the 100 KC crystal oscillator and the gate tube (with associated amplifier) are shown in fig. 4.

c. Gate forming circuit

A block diagram of the gate forming circuit is shown in fig. 5, and a detail diagram of each block in figs. 6, 7, and 8. The non-overloading amplifier ("long-tail pair") is essentially a cathode follower driving a grounded grid amplifier. The Schmitt trigger circuit is used as a discriminator. It produces an output pulse whose height is independent of the input pulse height, provided the latter is sufficiently great. The scaling pair (also called "flip-flop") is a bi-stable multivibrator. The Schmitt circuit and the scaling pair are described in Elmore and Sands, "Electronics: Experimental Techniques."

d. Scaling circuit

The block diagram of the scaler is shown in fig. 9. The scaling pairs are triggered only by negative pulses. Thus they pass on one pulse for every two put in. The scaler records the number of pulses put into it and presents this number in the binary system.

Procedure: Remove the PM chassis. With the cathetometer align each slit pair and its light source to lie in the same horizontal plane. Measure the vertical distance between slit pairs with the cathetometer. The current through the lamps should not exceed 3.5 amps. Replace the PM chassis, and turn the knobs (PM voltage controls) counterclockwise to their limit. This places the smallest voltage across the tube. Turn on the PM high voltage. Adjust the slit widths and voltage controls until the attached meter shows less than ten micro-amps across the tip jacks on the PM chassis (The meter reads a fraction of the PM anode current). The slits should be as narrow as possible. Check that bioses on the gate tubes, the gate forming circuit and scale are correct. Interrupt the light with a card to determine that the apparatus is working properly. Evacuate the tower. Lift the ball with the magnetron magnet. Release the ball and take the count. Repeat at least twenty times.

Report:

1. Very brief abstract
2. Very brief description of what you did.
3. Tabulation of results.
4. Statistical analysis of error including the computation of the standard deviation of your value of g. (See Michels, "Adv. Elect. Meas." or Beers, "Theory of Error".)
5. Complete analysis of systematic errors. Use this to assign an estimated uncertainty to your results.
6. Derive the formula which gives g as a function of the two measured distanees and times.
7. Explain the operation of:
    a. PM circuit
    b. Long-tailed pair
    c. Gate tube
    d. Scaling pair. What limits the speed at which it can count? Approximately how fast can the pair in fig. 8 count?
    e.* Schmitt trigger circuit. For the circuit in fig. 7, at what grid #1 voltage will the regenerative action occur? What is the expected magnitude of the output pulse? Explain clearly how the circuit acts as a discriminator (i.e. how it is used to trigger at a given voltage.)

* Elmore and Sand's circuit uses different parameters. Therefore a passage copied from them is not considered a satisfactory answer.

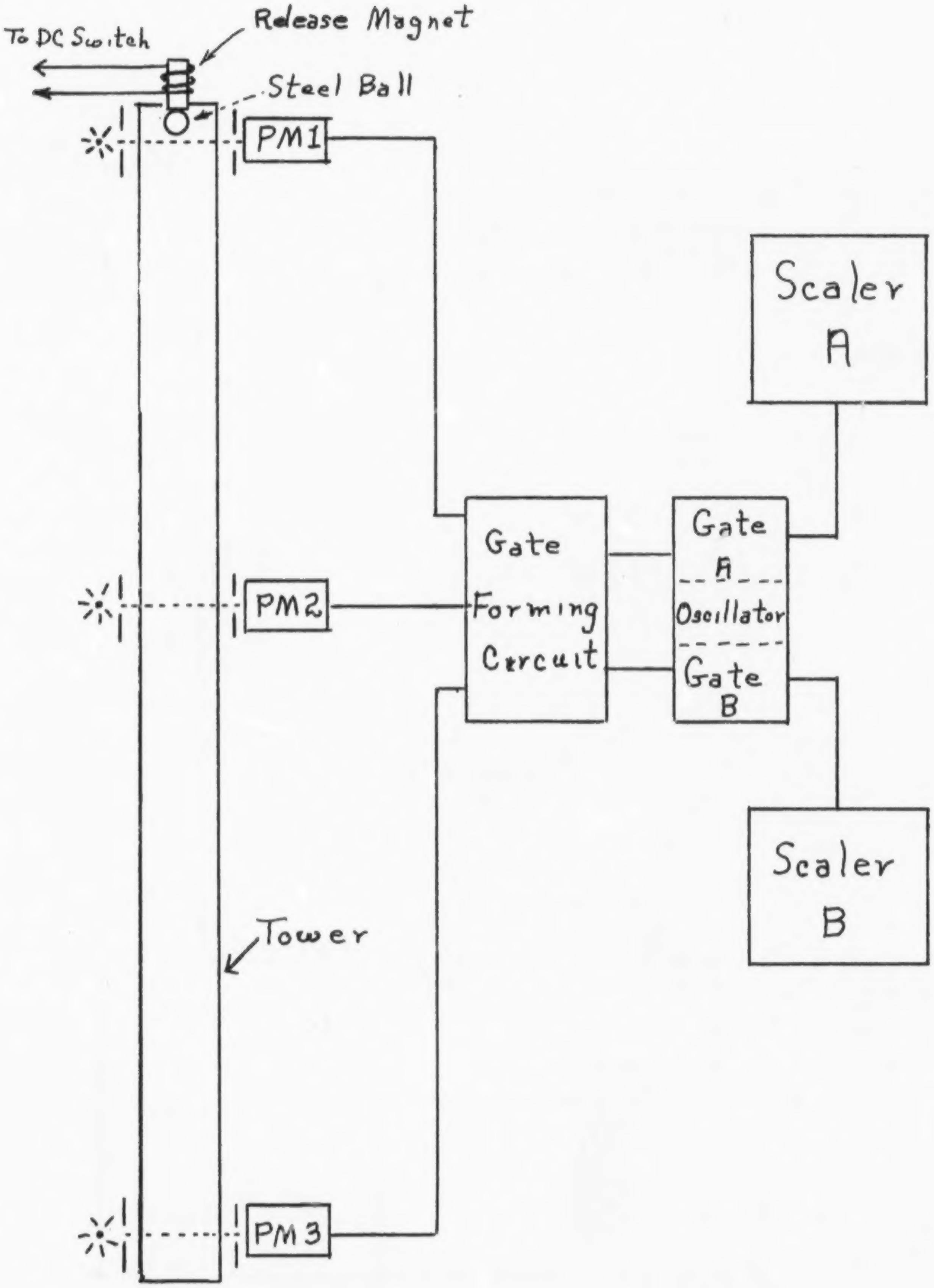

To DC Switch
Release Magnet
Steel Ball
PM1
PM2
PM3
Tower
Gate Forming Circuit
Gate A
Oscillator
Gate B
Scaler A
Scaler B

Figure 1

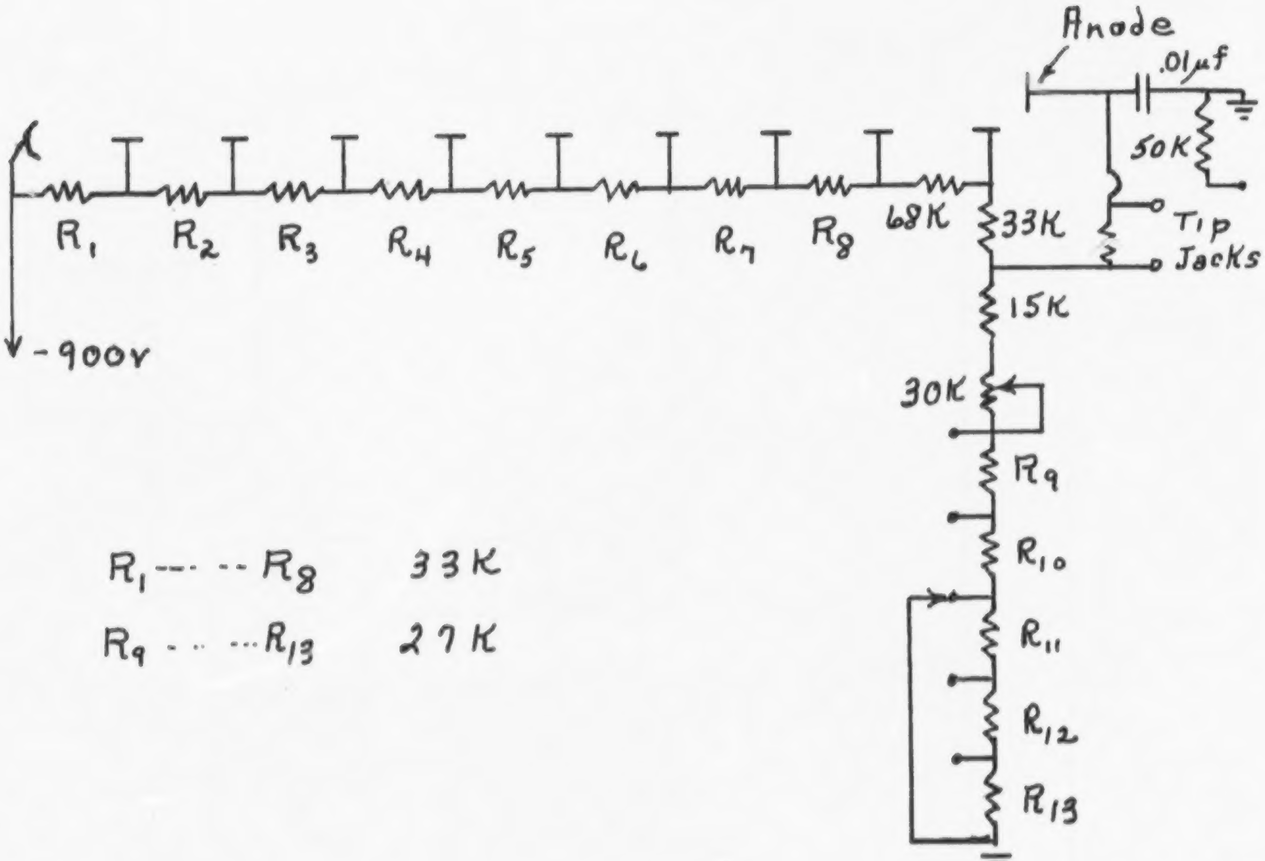

Anode
.01µf
50K
Tip
Jacks
R₁
R₂
R₃
R₄
R₅
R₆
R₇
R₈
68K
33K
15K
30K
R₉
R₁₀
R₁₁
R₁₂
R₁₃
-900v
R₁ ---- -- R₈ 33K
R₉ - - - ... R₁₃ 27K

Figure 2

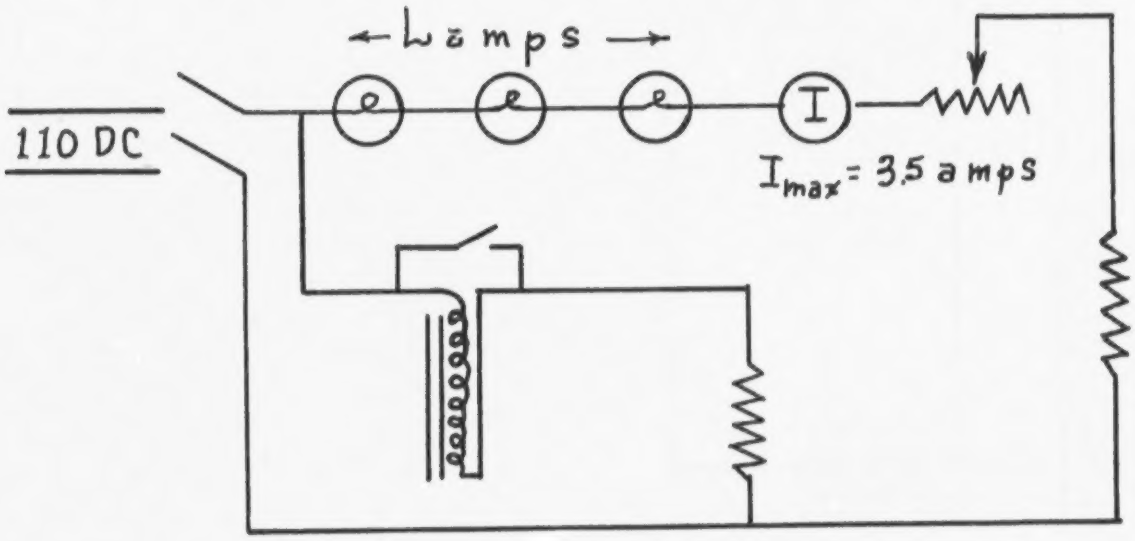

← Lamps →
110 DC
I
Iₘₐₓ = 3.5 amps

Figure 3

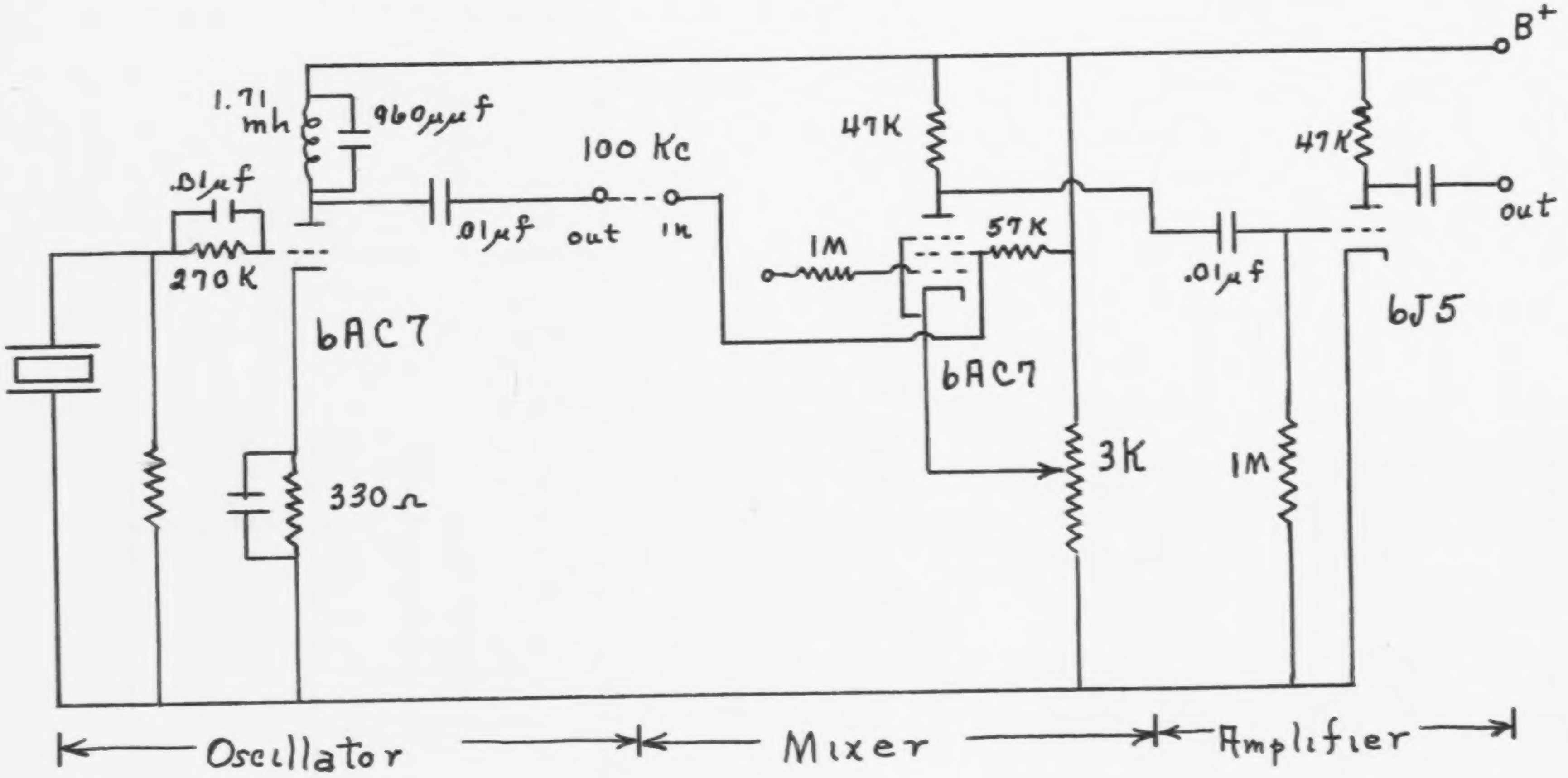

B+
1.71 mh
960μμf
.01μf
270K
.01μf
100 Kc
out
in
6AC7
330Ω
47K
1M
57K
6AC7
3K
47K
out
.01μf
6J5
1M
Oscillator
Mixer
Amplifier

Figure 4

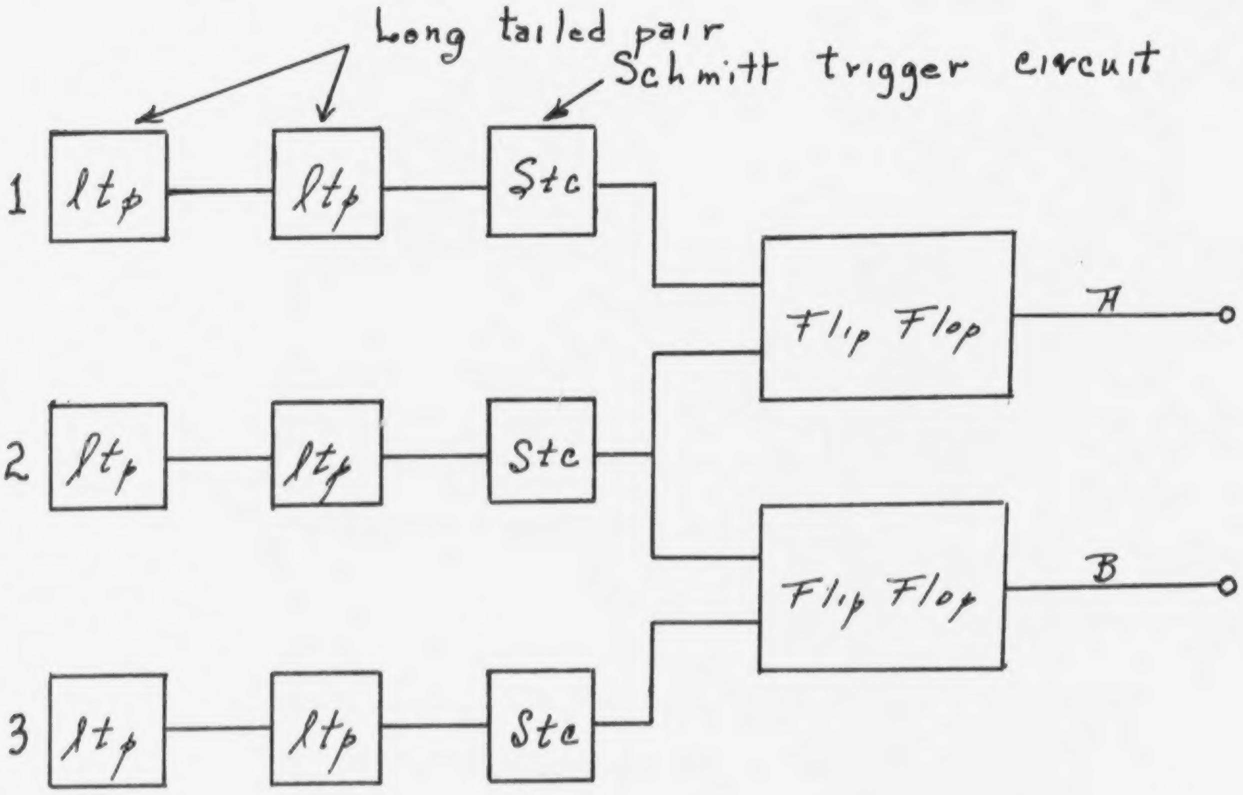

Long tailed pair
Schmitt trigger circuit
1
2
3
ltp
ltp
ltp
ltp
ltp
ltp
Stc
Stc
Stc
Flip Flop
Flip Flop
A
B

Figure 5

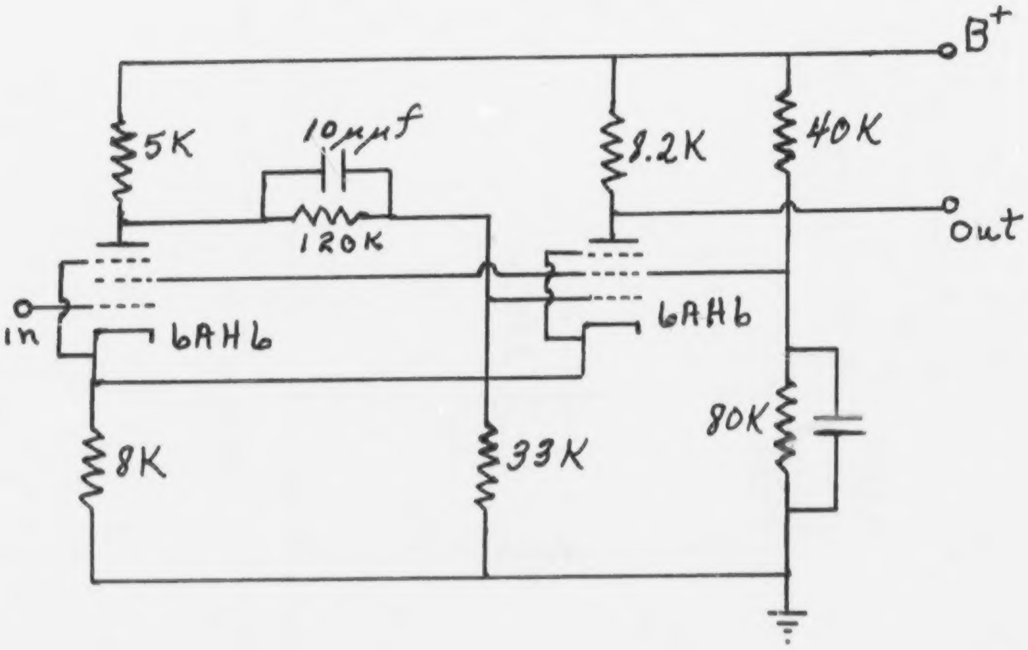

B+
5K
10μμf
120K
8.2K
40K
out
in
6AH6
6AH6
80K
8K
33K

Figure 6

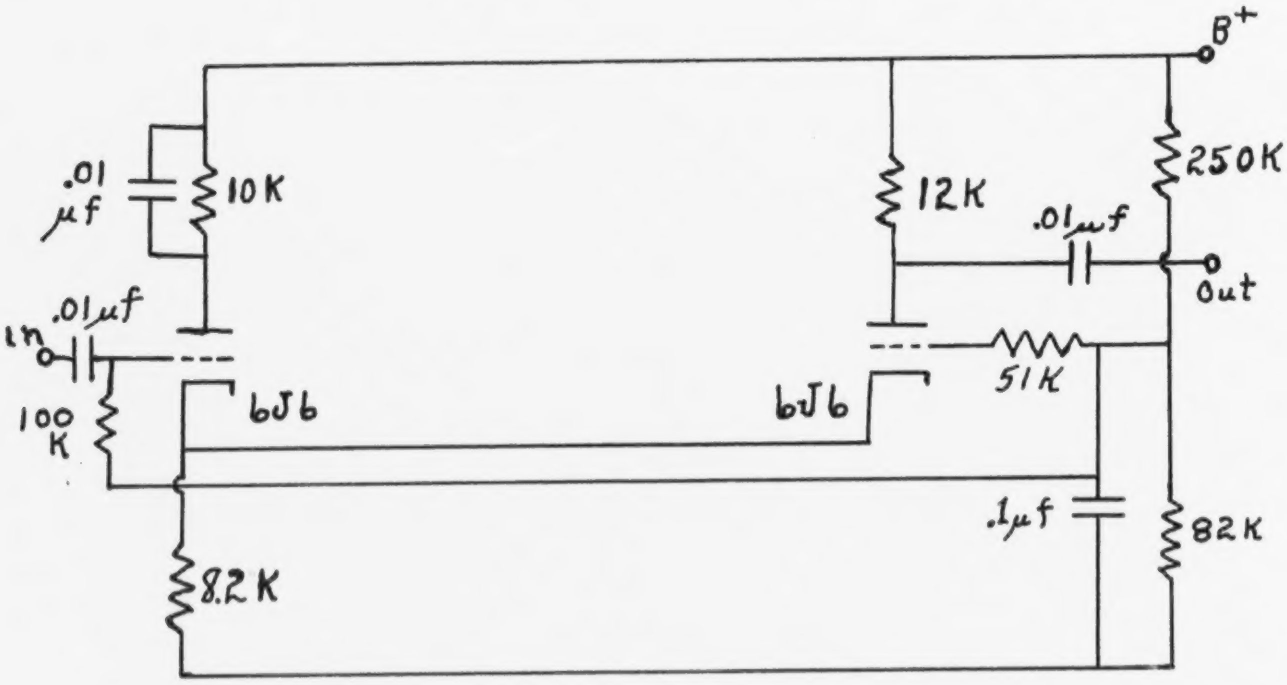

B+
.01 µf
10K
12K
250K
.01µf
Out
in
.01µf
100 K
6J6
6J6
51K
.1µf
82K
8.2K

Figure 7

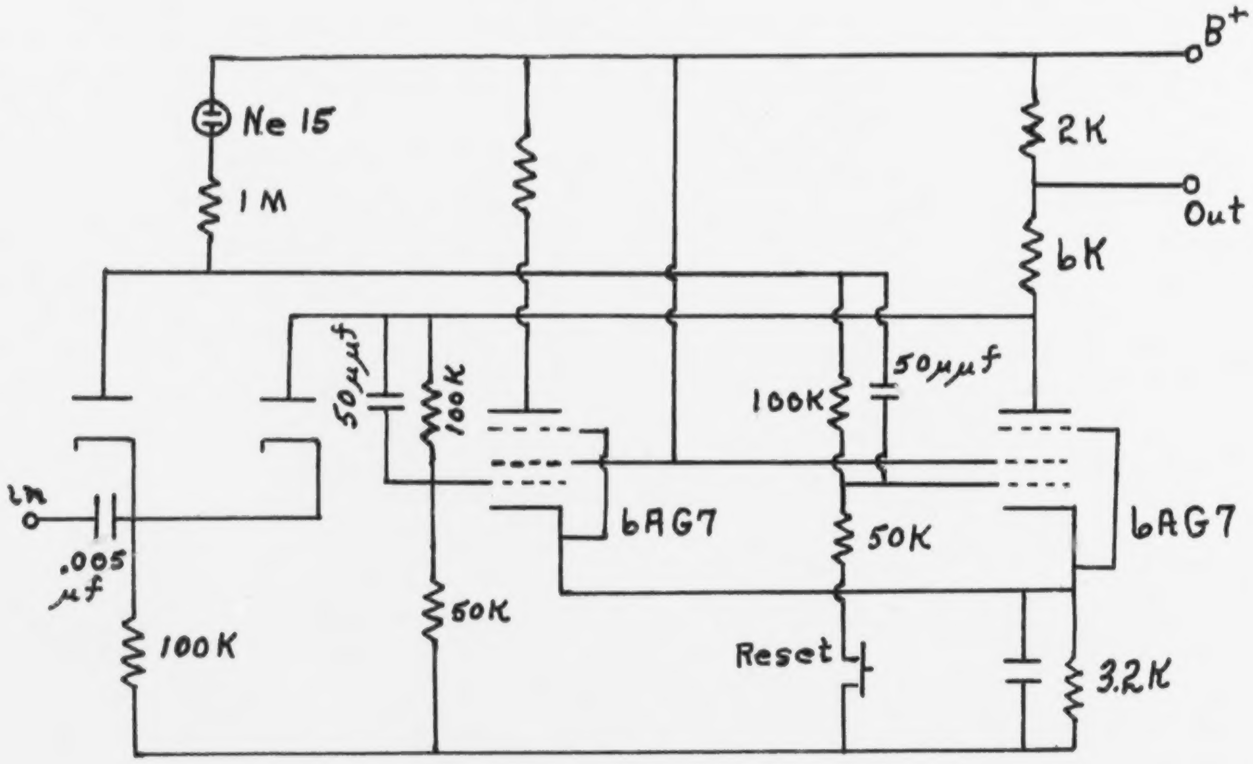

B+
Ne 15
1M
2K
Out
6K
50µµf
100K
100K
50µµf
in
.005 µf
6AG7
50K
6AG7
100K
50K
Reset
3.2K

Figure 8

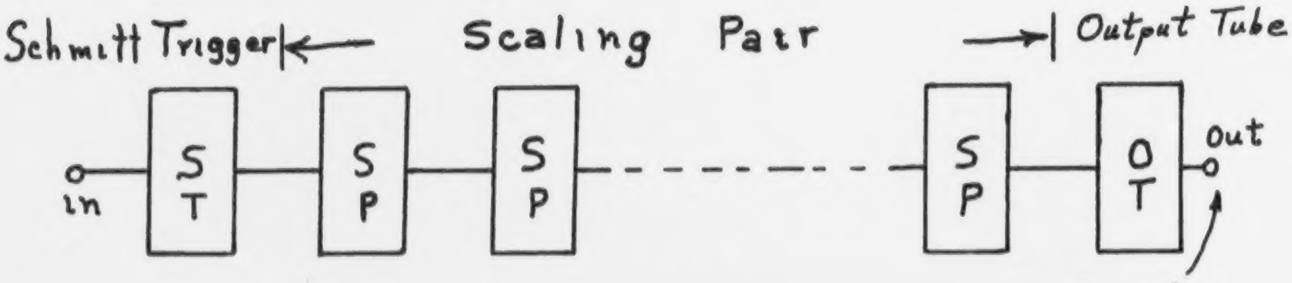

Schmitt Trigger
Scaling Pair
Output Tube
in
S T
S P
S P
S P
O T
out
To Register

Figure 9

VELOCITY WHEEL-THE STUDY OF INSTANTANEOUS VELOCITY

University of Maryland

- - - - - - - - - - -

Special Note

This experiment, the one following, and the experiment on rotational dynamics beginning on page 73 are part of a sequence of four interrelated and interdependent laboratory experiments at the University of Maryland. The purpose of these experiments is the demonstration of most of the principles of mechanics covered in an introductory course. All of the experiments are carried out with the same apparatus. The student develops a sense of continuity and also is not troubled by having to face unfamiliar equipment at the beginning of each laboratory session.

The apparatus consists of an inclined pair of rails and a wheel. The axle of the wheel is of small diameter while the wheel itself has a large moment of inertia. The result is a rather small acceleration and velocities which are small enough to be accurately measured with a stop watch.

The first three of the four experiments deal with such kinematic concepts as the position vs. time function, instantaneous velocity and the time derivative, and acceleration. The final experiment deals with the dynamical analysis of the motion and the calculation of acceleration. In the final experiment the moment of inertia is determined from its oscillation frequency in a torsion pendulum.

The importance of error analysis is stressed throughout. Errors are determined for the directly measured acceleration and for the acceleration calculated from the laws of motion. The comparison between these acceleration values is made in the light of their errors.

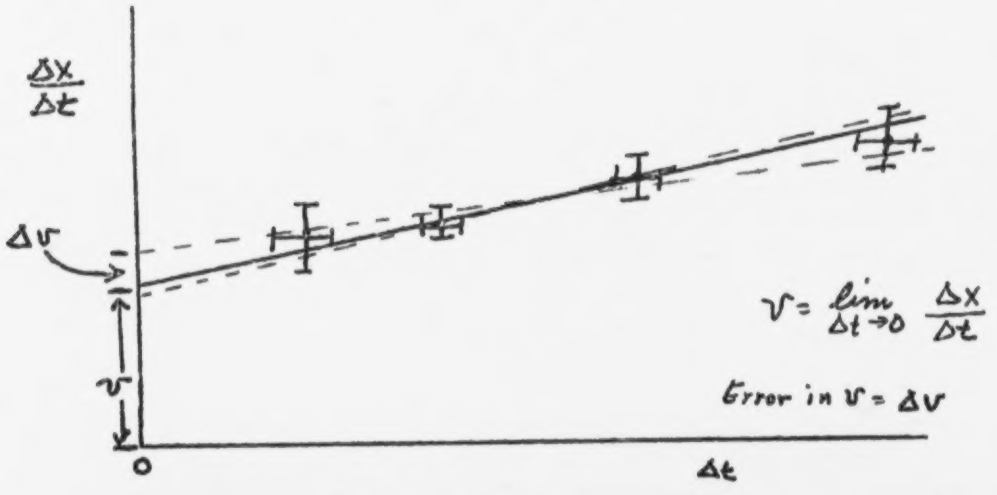
- - - - - - - - - - -

In the first velocity wheel experiment the position vs. time function, x(t), was determined. In the present experiment, we will measure the velocity of the wheel at a single point x along the incline by a process analogous to the mathematical operation of differentiation.

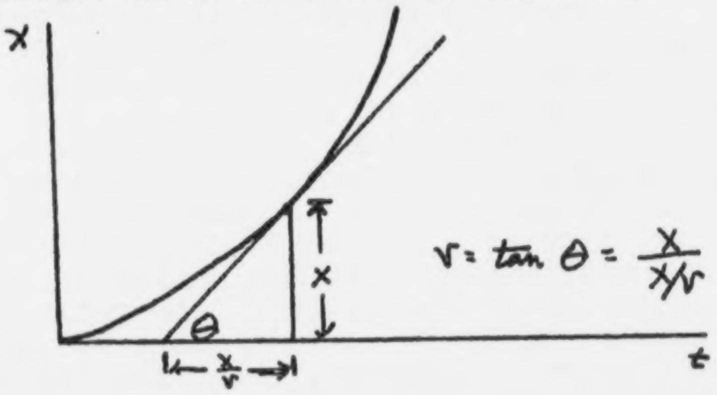
When we speak of the velocity at a particular instant of time we are of course referring to the instantaneous velocity at that instant of time. On the other hand the conventional measurement of velocity involves the determination of the finite time Δt required for the wheel to traverse a given distance Δx. The ratio of these two numbers, namely $\Delta x/\Delta t$ is the average velocity of the wheel in the time interval Δt. One must bear in mind the fact that the velocity is continuously increasing even diring the time interval Δt. In arriving at the instantaneous velocity at a time t (or more precisely for this experiment, at a particular position x) one must employ a method analogous to taking the derivative. That is, to consider the limit of the ratio $\Delta x/\Delta t$ as Δt shrinks to zero. Experimentally it is impossible to measure an infinitesimally small Δx, or Δt. However, the following procedure can be used to deduce ratio $\Delta x/\Delta t$ in the limit of $\Delta t \to 0$ from its values for measurably large Δt's.

We will attempt to determine the velocity of the wheel at a particular position x, corresponding to a certain time t. The time corresponding to the choice of x can be quickly read off of the x vs. t plot determined in Experiment II. Indicators are set at the position x and at a second position a distance Δx further down the incline. The time Δt, required for passage between these two points can be determined with a stop whatch as an average of at least five trials. The ratio $\Delta x/\Delta t$ is the average velocity in the time interval Δt. The process in repeated only now for smaller intervals Δx. These $\Delta x/\Delta t$ values can be plotted vs. their corresponding Δt and a smooth curve (straight line) fitted to the plotted points. The instantaneous velocity at x is taken to be the intercept of this curve with the $\Delta x/\Delta t$ axis.

A typical plot of this sort is illustrated in figure 1. The error in the velocity is obtained from the intercepts of the "extreme" curves (dashed) drawn so as to have intercepts of the maximun, and minimum possible values and yet be contained within the error bars of the plotted points.



The velocity determined in this fashion should be equal to the slope of the tangent line to the x(t) curve at the corresponding time t. To test this, construct a line of slope = v and passing through corresponding x and t point on the x(t) plot (Experiment 2). This procedure is illustrated in figure 2.



Procedure: Make sure you have the same equipment that you used for Experiment 2. First level the incline in the usual way and then set it up on the second step of the supporting block.

1. Set the 1st marker at point about 1/3 to 2/3 down the incline. This is the point at which the instantaneous velocity will be determined. Record the meter reading x of this point and determine the corresponding time t from the x(t) curve obtained in Experiment 2.

2. Set the 2nd marker a distance $\Delta x = 16$ cm. further down the incline. Starting the wheel at rest at the top of the incline and determine the time Δt required to traverse the distance between the markers averaged over at least five trials. Compute the standard deviation and error in the mean for these data.

3. Repeat the above procedure for $\Delta x = 12, 8, 4$ cm.

4. Compute the ratios $\Delta x/\Delta t$ for each setting and plot $\Delta x/\Delta t$ vs. Δt as in figure 1. The error bars for Δt are the errors in the mean. The error in $\Delta x/\Delta t$ due to the Δt error is $\Delta x/\Delta t \; \frac{\sigma/\sqrt{5}}{\Delta t}$ where $\sigma/\sqrt{5}/\Delta t$ is the fractional error in Δt.

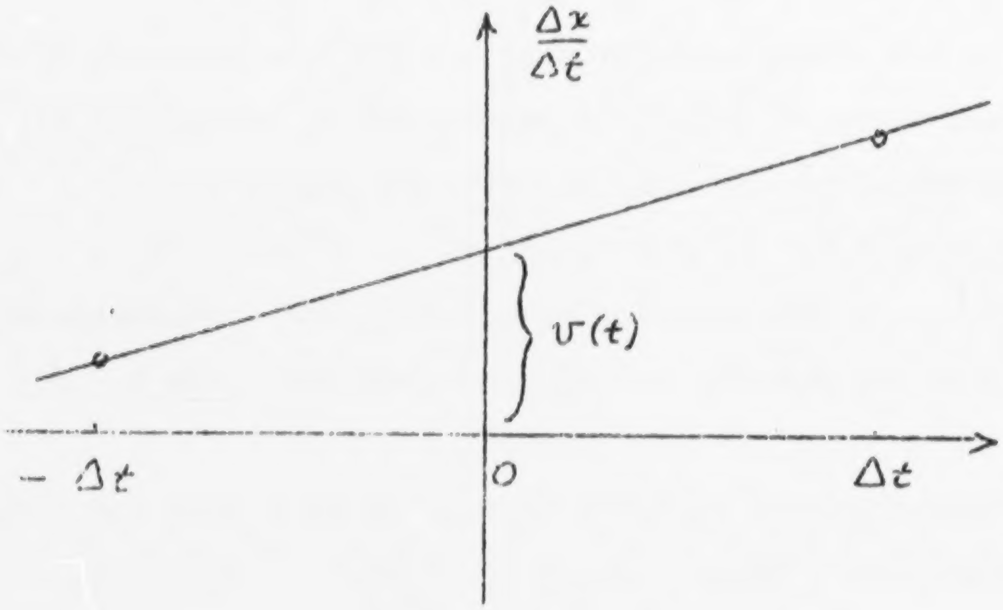
5. Fit the best straight line to these points. The instantaneous velocity v is the intercept of this line with the axis.

6. Construct the tangent line to the x(t) curve from Experiment 2 in the manner descrived in figure 2.

VELOCITY WHEEL-THE DETERMINATION OF THE VELOCITY AS A FUNCTION OF TIME, AND THE ACCELERATION

University of Maryland

In the second velocity wheel experiment, (EXPERIMENT III), the instantaneous velocity was determined at a particular time t by extrapolating the function $\frac{\Delta x}{\Delta t}$ of Δt to $\Delta t \rightarrow 0$. It turned out that this was a linear function. That is, the $\frac{\Delta x}{\Delta t}$ vs. Δt points could be fitted by a straight line. This is by no means true for all motions. This feature of the wheel's motion enables us to determine instantaneous velocities with relative ease. Thus, the instantaneous velocity at a particular time t is the same as the average velocity between the times t - Δt and t + Δt, independent of the choice of Δt. In order to show that this is true, we imagine that the average velocity $\frac{\Delta x}{\Delta t}$ is measured between the times t - Δt and t, and t + Δt and t. The resulting $\frac{\Delta x}{\Delta t}$ vs. Δt plot is illustrated in figure 1.



The two plotted points are connected by a straight line in accordance with the results of EXPERIMENT III. Furthermore, the intersection of this line with the $\frac{\Delta x}{\Delta t}$ axis gives the instantaneous velocity v(t) at t. But this intersection also corresponds to the average of the two $\frac{\Delta x}{\Delta t}$ values which in turn is the average velocity in the interval t − Δt and t + Δt. We will come back to this point later.

The purpose of this experiment is to determine the function v(t) and from it, the acceleration. One must measure the instantaneous velocities at various times and plot v vs. t. You will observe that these plotted points can be fitted by a straight line, the slope of which is the acceleration.

The function v(t) can, therefore, be represented as:

$$v(t) = at + v_0$$

where v_0 is the wheel's velocity at t = 0. We have, therefore, a case of unidirectional or rectilinear motion with constant acceleration. The function representing the wheel's position as a function of time is:

$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

where x_0 is the wheel's position at t = 0. The position of the wheel corresponding to t = 0, as determined in EXPERIMENT II, is x_0 = 0 and therefore the function x(t) is given by:

$$x(t) = \frac{1}{2}at^2 + v_0 t$$

We return to the question of equating the instantaneous velocity at a time t to the average velocity between the times t − Δt and t + Δt.

The instantaneous velocity at t is derived from the function x(t) by differentiation. Thus, $v(t) = \frac{dx(t)}{dt} = at + v_0$

The average velocity between t $-\Delta t$ and t$+\Delta t$ is by definition

$$v_{ave.} = \frac{x(t+\Delta t) - x(t-\Delta t)}{(t+\Delta t) - (t-\Delta t)} = \frac{x(t+\Delta t) - x(t-\Delta t)}{2\Delta t}$$

The values of x(t) at t $+ \Delta t$ and t $- \Delta t$ are:

$$x(t+\Delta t) = \tfrac{1}{2} a \cdot (t+\Delta t)^2 + v_0 (t+\Delta t)$$
$$= \tfrac{1}{2} at^2 + v_0 t + (at+v_0)\Delta t + \Delta t^2$$

$$x(t-\Delta t) = \tfrac{1}{2} a \cdot (t-\Delta t)^2 + v_0 \cdot (t-\Delta t)$$
$$= \tfrac{1}{2} at^2 + v_0 t - (at+v_0) \cdot \Delta t + \Delta t^2$$

Substituting these values into the expression for $v_{ave.}$, one obtains

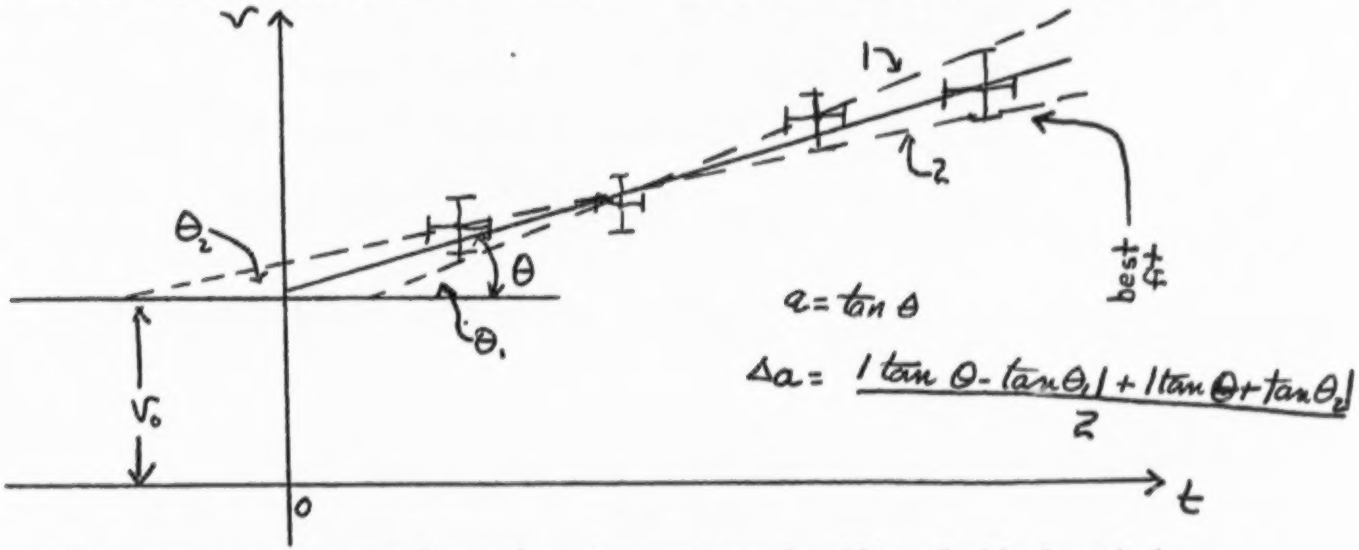
$$v_{ave.} = \frac{2(at+v_0) \cdot \Delta t}{2\Delta t} = at + v_0$$

This is the same as the derivative of x(t) at the time t.

Procedure

1. Determine the instantaneous velocities at four positions along the incline. For the first measurement, set markers at 10 cm. and at 20 cm. Measure the time Δt for traversing the interval Δx = 10 cm., five times. Calculate the mean andthe error in the mean. The ratio $\frac{\Delta x}{\Delta t}$ is the instantaneous velocity at the time $t_{10} + \frac{\Delta t}{2}$ where t_{10} is the time required to move from x = 0 to x = 10 cm. The time t_{10} can be read off the x(t) curve obtained in Experiment II, or can be quickly remeasured during the present experiment. Calculate the error in $\frac{\Delta x}{\Delta t}$ (see Experiment III).

Repeat the above procedure for settings 25 cm. - 35 cm., 50 cm. - 60 cm., 75 cm. - 85cm. Calculate the velocities v and their corresponding times t and plot these points (and error bars) on millimeter graph paper. Plot t along the abscissa and v along the ordinate. Fit a straight line to the plotted points and determine the acceleration (slope) and the initial velocity (v intercept).


v
1
2
θ_2
θ
θ_1
best fit
$a = \tan\theta$
$\Delta a = \frac{|\tan\theta - \tan\theta_1| + |\tan\theta + \tan\theta_2|}{2}$
v_0
0
t

The errors in v and a, due to errors in the plotted points, can be estimated by drawing "extreme" curves 1 and 2 through the plotted points as illustrated in figure 2.

The acceleration a is taken to be the slope of the "best" fitted line. The error in the acceleration may be taken as the average of the absolute deviations between a and the slopes of the two extreme curves. Thus, $\Delta a = \frac{|\tan\theta - \tan\theta_1| + |\tan\theta - \tan\theta_2|}{2}$

The error in v_0 can be likewise taken as the average of the absolute deviations between v_0 and the intercepts of lines 1 and 2.

2) Plot the function $x(t) = \frac{1}{2}at^2 + v_0 t$ using values for a and v_0 as determined in part 1. Superimpose on this plot points for the x vs. t values as determined in Experiment II.

For an isolated system of particles the total momentum of all the particles is a constant. We will study this law in a fairly well isolated system of steel spheres. We can observe elastic and inelastic collisions.

Discussion:

We will observe collisions between spheres. The velocities of the spheres before and after the collisions can be measured. In all of the collisions observed one particle will be initially at rest and the other will be moving with a well determined velocity. The apparatus is shown in the figure.

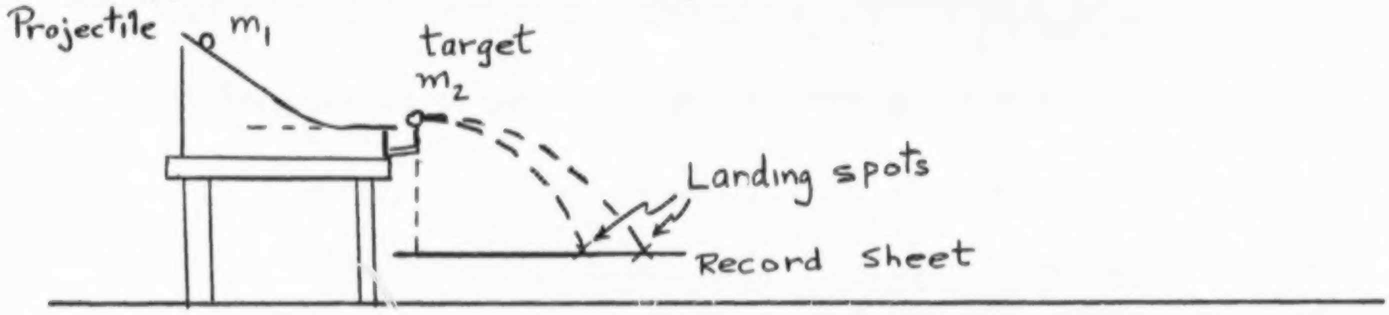
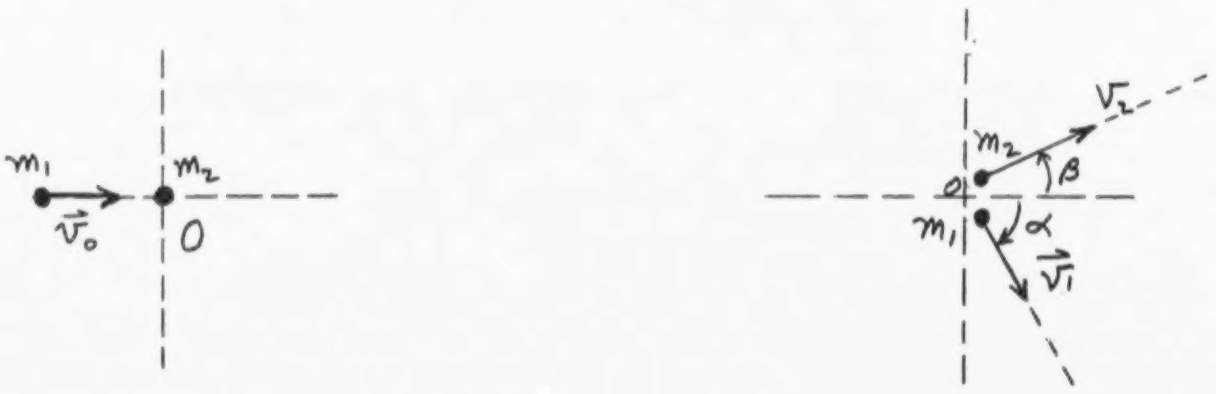


Figure 1.

The particle m_1 rolls down the track and collides with m_2 which is on an adjustable pedestal. The trajectories of the two particles after the collision are shown in the Figure 1. The spots where each particle lands on the horizontal record sheet are determined with the aid of a carbon paper on top of the record sheet. The point 0 is directly below the pedestal on which m_2 rests before the collision. Point 0 can be determined with the aid of a plumb bob. Prove to yourself that the distance between point 0 and the landing spot for one of the particles is proportional to the velocity of that particle just after the collision if the collision occurs in a horizontal plane. Figure 2 is a diagram of the collision as seen from above.



Before

After

Figure 2.

The law of conservation of momentum states that the following vector sum must hold.

$$m_1\vec{v}_o = m_1\vec{v}_1 + m_2\vec{v}_2 \qquad (1)$$

Equation (1) can be expressed in terms of components of momenta parallel and perpendicular to v_o.

$$m_1v_o = m_1v_1 \cos\alpha + m_2v_2 \cos\beta \qquad (2)$$

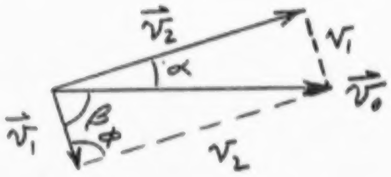
$$m_1v_1 \sin\alpha = m_2v_2 \sin\beta$$

The angles α and β are treated as positive angles as defined in Figure 2.

Consider three types of collisions.

Case A. "Perfectly elastic" collision between spheres of equal mass, $m_1 = m_2 = m$.

From equation (1) it follows that $\vec{v}_o = \vec{v}_1 + \vec{v}_2$. The relationship between the magnitude of v_o and the magnitudes of v_1 and v_2 is given by the law of cosines.



$$v_o^2 = v_1^2 + v_2^2 - 2\, v_1v_2 \cos\phi$$

For a perfectly elastic collision, the mechanical energy is conserved.

$$\tfrac{1}{2}mv_o^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}mv_2^2$$

$$v_o^2 = v_1^2 + v_2^2$$

Compare this with the law of cosines equation and you conclude that

$$\cos\phi = 0$$

$$\therefore \phi = 90$$

We can then conclude that the locus of a landing spot, for collisions of different α and β, will be a circle of diameter v_o. This is shown in Figure 3.

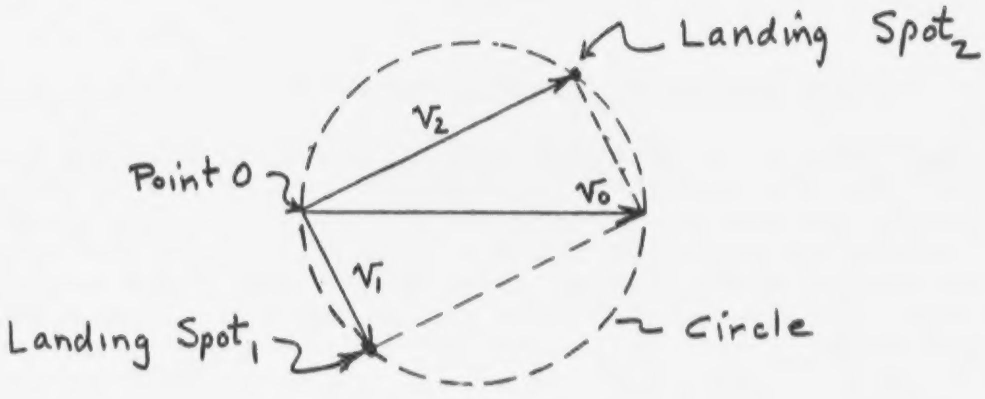

Landing Spot$_2$
V_2
Point O
V_0
V_1
Landing Spot$_1$
Circle

Figure 3.

Case B. "Perfectly elastic" collisions between particles of unequal mass.

For this case, it can be shown that all landing spots of the target (m_2) are on a circle of diameter = $\frac{2m_1 v_o}{m_1+m_2}$ and that all landing spots of the projectile, (m_1) are on a circle of diameter = $\frac{2m_2v_o}{(m_1+m_2)}$. These circles are concentric and the target circle passes through point 0. See Figure 4.

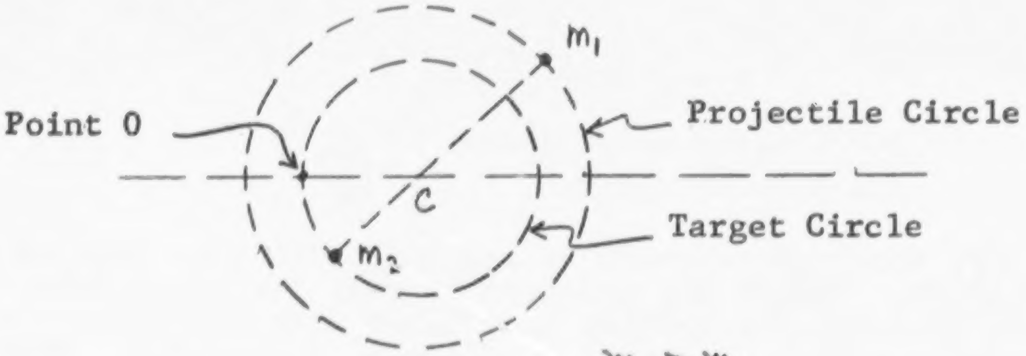

m_1
Point 0
Projectile Circle
C
Target Circle
m_2

Figure 4.

$m_2 > m_1$

The center of mass of m_1 and m_2 is at the center of both circles at the instant the particles hit the record sheet.

Case C. Completely inelastic collision. Projectile sticks to target after impact.

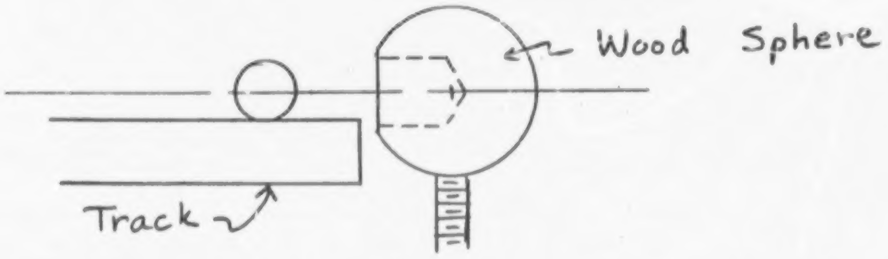
Study of this event should further emphasize the importance of the conservation of momentum law in predicting the motion after impact.

Procedure: Record vertical distance between top of pedestal and box floor.

Case A. Make sure that the end of the accelerating track is horizontal. Set the pedestal so that the target is at the same vertical height as the projectile as it comes off the track. Try a few collisions to position the box that will hold the record sheet. The record sheet can be held to the box floor with tape or thumbtacks. Locate point 0 with the plumb bob. Cover the record sheet with carbon paper and locate the landing spot for the projectile with the target removed. Make records of landing spots for 20-30 collisions for $m_1 = m_2$. After each collision identify each pair of spots with a symbol different from that used for other pairs.

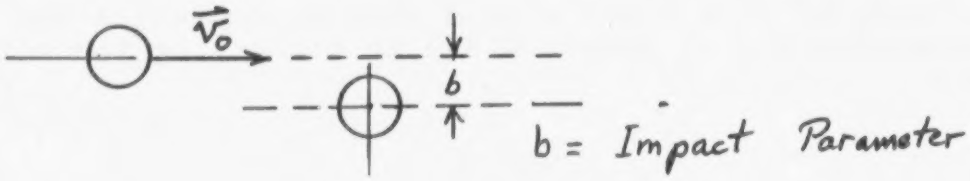
Case B. Choose spheres of unequal mass using the smaller mass as the projectile. The level of the pedestal will have to be carefully re-adjusted. Use a new record sheet. Establish point0 and the "velocity" of the projectile on the record sheet. Make a record of collisions on the record sheet. Identify each pair of collisions. Record the diameter and mass of each sphere.

Case C.



Use the wooden sphere as the target in this part of the experiment. Adjust the pedestal so that when this sphere is placed upon it, the opening is aligned with the anticipated projectile path. Use the 3/8" diameter projectile. When the projectile hits the sphere, it should stay in it until the two hit the record sheet. Locate point 0 on the record sheet. With the balance determine the mass of the projectile and the mass of the sphere.

(Definition: The Impact Parameter is the perpendicular distance between the velocity vector of the projectile and a line parallel to v_o passing through the center of the target sphere.)



Report:

1. What is the momentum of the projectile just before the collision in each of the above cases, A, B, C?

2. From positions of four different collisions (differing impact parameters) of A determine whether momentum and mechanical energy (KE) are conserved. Compare magnitude and direction of the total momentum of the system before and after the collision. If mechanical energy is not conserved, what fraction of the initial KE of the projectile was lost to heat, etc. in each collision under consideration. (Clearly indicate which collisions are used.)

3. Repeat 2. for case B.

4. Compare final total momentum of both particles of the inelastic collision (part C.) to initial momentum of projectile. Compare initial and final kinetic energies. From the known masses of target and projectile in C. calculate from theory the fraction of the initial KE of the projectile that is converted into heat in this collision. Compare with experimental results.

5. Optional: Starting from laws of conservation of momentum and KE (for a perfectly elastic collision) prove the statement made under case B. above concerning the diameters of circles of target and projectile of unequal mass.

Massachusetts Institute of Technology

In the lectures it was demonstrated that when two objects moving along the same straight line collide, their total momentum is the same before and after the collision, provided no external forces act on either of the objects. In other words, in this one-dimensional experiment the total momentum of the two objects is conserved. In this experiment we will try to find out if momentum is also conserved for collisions which take place in two dimensions, i.e., in a plane. The main difficulty associated with such an investigation is the isolation of the system from external effects, particularly friction. In this experiment we attempt to reduce the effects of friction by floating the colliding bodies on a thin cushion of air.

I. Description of the Apparatus: The apparatus consists of two heavy circular pucks and a flat glass plate covered with a sheet of gray Teledeltos paper and a sheet of plain white paper. Compressed air is supplied through plastic hoses suspended from the laboratory ceiling and extending to the bottoms of the pucks. It escapes by flowing between the underside of the puck and the supporting surface, providing a thin film of air upon which the puck can slide with negligible friction. Also available are metal disks for changing the mass of one of the pucks.

The data is recorded using a sparking mechanism. A spark from the steel point at the center of one of the pucks jumps across the air gap and through the white paper, flows along the conducting surface of the Teledeltos paper to the other puck, where it jumps the air gap to the second spark point. The current density is high enough to char the white paper at the first gap but too low to do so at the second. Thus the white paper will be marked at the position of the center of mass of the first puck. After an interval of $\frac{1}{120}$ of a second a spark flows in the opposite direction marking the position of the second puck. In this manner the position of each of the pucks is recorded sixty times a second, and the velocity of the pucks is proportional to the displacement between successive spark marks.

II. Experimental procedure: Level off the supporting surface so that undesirable gravitational forces are negligible. Adjust the air hoses and the rate of air flow so that friction and the effects of the air hoses are at a minimum. Perform and record a collision between two pucks of different masses. Construct x-and y - axes on the white paper at right angles to each other. Find the x-and y - components of the velocities of each of the pucks before and after the collision.

Before the collision: v_{1x} = ____ cm/sec ± ____ cm/sec

v_{1y} = ____ cm/sec ± ____ cm/sec

v_{2x} = ____ cm/sec ± ____ cm/sec

v_{2y} = ____ cm/sec ± ____ cm/sec

After the collision: v'_{1x} = ____ cm/sec ± ____ cm/sec

v'_{1y} = ____ cm/sec ± ____ cm/sec

v'_{2x} = ____ cm/sec ± ____ cm/sec

v'_{2y} = ____ cm/sec ± ____ cm/sec

Measure the masses of the two pucks.

m_1 = ____ gm ± ____ gm ; m_2 = ____ gm ± ____ gm.

Compute the x- and y-components of the momenta of each of the pucks. before and after the collision.

Before the collision: p_{1x} = _____ gm-cm/sec ± _____ gm-cm/sec

p_{1y} = _____ gm-cm/sec ± _____ gm-cm/sec

p_{2x} = _____ gm-cm/sec ± _____ gm-cm/sec

p_{2y} = _____ gm-cm/sec ± _____ gm-cm/sec

After the collision: p'_{1x} = _____ gm-cm/sec ± _____ gm-cm/sec

p'_{1y} = _____ gm-cm/sec ± _____ gm-cm/sec

p'_{2x} = _____ gm-cm/sec ± _____ gm-cm/sec

p'_{2y} = _____ gm-cm/sec ± _____ gm-cm/sec

Be sure to include the accuracy or experimental uncertainty in each of your measurements. Remember that when you perform a calculation the uncertainty usually increases; the experimental uncertainties of two numbers add when the numbers are added or subtracted, and the relative or percentage uncertainties add when the numbers are multiplied or divided. Now, to the experimental accuracy of your measurements - was the total momentum conserved during the collision? On a sheet of graph paper draw the vector momentum before and after collision and attempt to explain differences.

III. Motion of the Center of Mass. Physicists sometimes prefer to describe the motion of a system of particles, not by specifying the motion of each of the individual particles, but by specifying the relative motions of the individual particles and the motion of the center of mass. Thus we may describe our two-puck system not only by giving the velocities of both of the pucks at all instants of time, but also by giving the velocity of the center of mass of the two pucks and their relative velocity. From Newton's laws of motion one obtains that, for a system acted on by no external forces, the velocity of the center of mass is constant.

On your data sheet, find the center of mass of your two-puck system at several instants of time. Do these points lie on a straight line? Find the velocity of the center of mass before and after the collision. Are they equal?

The relative velocities play an important role in classifying a collision as elastic or inelastic. If the relative velocity of the two bodies after a collision is equal but opposite to what it was before the collision, the collision is said to be perfectly elastic. If the relative velocity after the collision is zero, the collision is said to be perfectly inelastic. From your data find the relative velocities before and after the collision.

You may think of many other interesting experiments which may be performed with these pucks. If you desire to try one, discuss it with your instructor. He will help you obtain the necessary equipment.

- - - - - -

References: Ingard and Kraushaar, Introduction to Mechanics, Matter, and Waves, pp. 88-96.

Marcley, R.G., "Air-Suspended Pucks for Momentum Experiments" American Journal of Physics, 28, 1960, pp. 670-674.

LINEAR AIR TROUGH

California Institute of Technology

I. Introduction: The eight-foot air troughs in the Physics 1 laboratory can be used for a multitude of interesting and important experiments. The nearly complete absence of friction combined with the nearly complete elasticity of the bumpers, makes it possible to attain considerable precision in dynamical measurements even if the friction and loss of velocity at collision is ignored altogether. Further, the losses due to these two effects may be experimentally determined and corrections thus made for any particular experiment, thus permitting even greater precision to be attained.

The following list of experiments is by no means exhaustive. It should serve as a point of departure for original experiments of the students' own invention. The experiments are listed approximately in the order of increasing sophistication, in terms both of the measurements that must be made and the theory that is needed to analyze the motions. This is not necessarily the same, of course, as the order of interest or importance. Indeed, some of the simplest experiments are the most interesting.

A WORD OF CAUTION: THESE AIR TROUGHS AND GLIDERS WILL OPERATE WELL ONLY IF THEY ARE CLEAN AND SMOOTH. USE CARE IN HANDLING THE GLIDERS AND IN PLACING THEM IN THE TROUGH. THEY WILL DENT EASILY. TREAT THEM AS YOU WOULD YOUR WATCH. AVOID HIGH VELOCITY IMPACTS, ESPECIALLY BETWEEN THE HEAVIER GLIDERS.

Some suggestions on the use of the air troughs:

1. Before using, wipe the groove of the air trough with a tissue paper wiper wet with xylene. Also wipe the surfaces of the gliders.
2. There is a small effect of the air jets at the ends of the gliders. To minimize this, use a minimum of air pressure.

3. To get a feeling for the way in which the gliders behave, you should try a number of simple experiments such as: (a) Note the effects of the jets on the ends of both the heavy and light gliders. (b) Find the approximate variation in slope of the trough that produces the least detectable acceleration in each direction. Do this for both the aluminum and steel gliders. (c) Note the relative slowing down of the two kinds of gliders when the trough is horizontal. (d) Vary the air pressure and repeat the above observations.

II. Experiments Not Requiring a Time Measurement

If one ignores the loss of speed through frictional effects, a number of important experiments can be performed which do not require the use of a stop watch or timer. One assumes that the relative speeds of two objects are proportional to the distances traversed in the same (unknown) time interval. Thus, if two gliders start simultaneously from known points along a level trough and also reach the two ends simultaneously, their relative speeds can easily be found.

A. Action-reaction experiments

1. Gliders are available which repel each other by spring pressure and expansion of gas. Using equal gliders, one can establish that the acquired velocities are always equal and opposite.
2. By tying together various numbers of equal gliders, and by using the above interactions, one can establish that the acquired velocities are inversely proportional to the respective numbers of masses tied together; e.g. if two masses are repelled by three masses, the acquired velocities are in the ratio 3:2.

3. By adopting one glider as a "standard" and proceeding as above, one can define and measure relative *inertial mass* in terms of the relative velocities acquired in mutual repulsion interactions. One can also check the consistency of such measurements: If the ratio of the mass of one body to that of the standard is $m_1 : m_2$, and a second body to the standard is $m_2 : m_s$, do the two bodies have a mass ratio (*measured directly* in the same way) of $m_1 : m_2$? Does this depend on the material?

B. *Impact experiments*

The steel bumper springs on the gliders and at each end of the trough are almost perfectly elastic, so that the properties of elastic as well as inelastic impacts may be studied. A typical impact will involve the collision of two gliders of masses m and M, moving with initial velocities v_1 and V_1, while after impact their velocities will be v_2 and V_2 respectively.

If one assumes that the bumpers are perfectly elastic, one can measure the ratio of mass of the colliding gliders by allowing the lighter mass to hit the heavier one, with the latter initially at rest. The student should derive the following formula for the relative velocities of the two masses after such a collision.

$$\frac{v_2}{V_2} = \frac{1}{2}\left(1-\frac{M}{m}\right) . \quad \text{----------------} \quad (1)$$

Can one also perform this experiment with the lighter mass initially at rest?

In such a collision as above, you will be able to find a position along the trough where, after collision, the two gliders strike the ends at the same time. The ratio of these two distances traveled is, of course, also given by Eq. (1) above.

Suppose that instead of a steel bumper at the end of the trough, a jet of air were used to retard, stop and reverse the direction of the glider. Would this make a satisfactory "bumper" at the end?

III. Experiments Using a Stop-timer

A wide variety of experiments may be performed using a stop-clock or a stop-watch. Some of these possibilities are as follows:

A. The acceleration of gravity can be determined by tilting the trough a known amount and measuring the acceleration of the glider. Two procedures are suggested:

1. Release a glider from rest at known distances along the track and measure the times required to reach the end.
    a. How and why does the calculated value of g depend upon the tilt of the trough? The distance of travel? The mass of the glider?
    b. Is there an optimum value of the slope of the track which gives maximum accuracy? If so, what is it?
    c. Can one avoid having to level the track very precisely initially?
    d. What factors contribute to the uncertainty in value you find for g and how was their influence minimized in your experiment?

2. Release a glider from any point along the trough and let it bounce at the lower end. Measure the elapsed time between successive bounces and the maximum distance traveled up the trough. Vary the glider, slope, air pressure and the initial
   a. Consider the same questions as in 1 above.
   b. Which method should give the more reliable value of g and why?

B. The effects which cause the glider to slow down can be determined quantitatively. These effects are two in number; (a) the slowing down due to viscous drag between the glider and the trough surfaces and (b) velocity lost at the end bumpers. One way of separating the two effects is as follows: Measure the velocity as a function of the distance traveled as the glider bounces back and forth between the end bumpers on a level trough. In this kind of measurement, it is desirable to use two stop-timers, starting one as you stop the other.

You should then plot a curve of the average velocity for a trough length vs distance, or trough lengths traveled. It is suggested that you go from say, 30 cm sec^{-1} down to a few cm sec^{-1}. You can study the slope of such a curve as a function of air pressure and kind of glider.

If you now clamp another glider rigidly in the trough so that the test glider travels only 1/4 of a trough length between end bumpers the effect of the bumpers will be 4 times as important as before for the same distance traveled. Another curve may then be plotted of velocity vs distance. Be sure to label each curve you plot.

The distance may again be reduced by a factor of 4 so that now the effects of the bumps becomes 16 times as important as for a full trough length.

Proceed in this manner as far as you can, plotting appropriate curves.

From these data you should be able to draw a curve of velocity vs distance for no bumps.

If it is assumed that the viscous drag along the trough is due to the viscosity of the air film, the differential equation that needs to be solved may be written as follows:

$$\frac{dv}{ds} = \frac{-A\eta}{md} \text{ ----------------------- } (1)$$

where s is the distance, A the area of contact of glider and trough, m the mass of the glider, d the thickness of air film and n the viscosity of the air. The solution of Eq. (1), putting in the condition that the initial velocity at s = o is v_o, one finds,

$$v = v_o - \frac{s}{\tau} \text{ ------------------- } (2)$$

where $\tau = \frac{md}{A\eta}$ and is called the "characteristic" time of the motion. From the slope of your curves, find the characteristic time for your particular conditions.

Eq. (2) predicts that the velocity should decrease linearly with distance. Do you find this to be true?

If one desires to have the least slowing down of the glider due to viscous forces, what conditions would need to be satisfied?

What can you say about the change of velocity per bump? Does it depend on the velocity? Is the percentage change of energy per bump independent on the velocity?

If you wish the two effects of slowing down of the glider to be small, would you use high or low velocities? It may be helpful to find the percentage change of velocity, using the above data, due to each cause for a trough length, assuming certain velocities such as 30 cm $\sec^{-1}$ and 5 cm $\sec^{-1}$.

C. Another method of determining the characteristic time is as follows: With a tilted trough, you will find that as the glider rebounds, it stops and reverses direction of motion at continually decreasing distances up the trough. Obviously, for no losses, this reversal point would remain constant. Hence this Δs from one reversal point to the next is a measure of the viscous drag, (neglecting losses at the bumpers. Is this justified from your previous results?)

The solution of the appropriate differential equation shows that,

$$\Delta s = \frac{8}{3} \; \frac{1}{\sqrt{2g\theta}} \; \frac{s^{1.5}}{\tau} \quad \text{-----------} \quad (3)$$

where τ is the characteristic time as defined previously, θ is the tilt of the trough and the other quantities have their usual meanings. Experimentally then, one determines the reversal points for a given slope of the trough. Eq. (3) suggests that the Δs's be plotted vs s on log-log coordinate paper. Draw the best straight line through your points. Eq. (3) predicts that the slope should be 1.5. Is your line consistent with this value? From your plot, you should be able to write down an equation like (3) with an appropriate constant. Assuming this constant and that in Eq. (3) to be equal, you can solve for the characteristic time Compare this with your previously determined value.

D. The mass of the air trough itself can be found by suspending it from the ceiling in a slightly tilted position, and letting a glider, which starts from rest, collide with the end bumper. The trough will then swing like a pendulum. Using the known time and distance of travel of the glider, its velocity, as it strikes the bumper, may be found. Measuring the amplitude and period of the swing of the trough immediately after impact permits the velocity acquired by the trough to be calculated. Conservation of momentum and energy then give the mass of the trough in terms of that of the glider.

IV. Experiments Using a Spark-timer

The motion of gliders can be analyzed in greater detail by using a spark timer and waxed paper tape. Collisions, the acceleration of gravity and action-reaction experiments can all be performed using this method of time vs distance measurement. (Note: to minimize the randomness of just where the spark goes through the paper, the pointed electrode should be as close to the paper as possible).

In addition to the above experiments, others, which are most conveniently carried out using a spark timer are as follows:

A. The effects of track irregularities upon the motion can be analyzed by letting a glider move very slowly along a level or slightly inclined track.

B. Motion with a strong damping force may be analyzed. A bar magnet, embedded in a glider, generates electrical "eddy currents" in the aluminum track. These eddy currents produce a drag force which is precisely proportional to the velocity. This is just the same kind of damping that results from the air film between glider and trough, but in the case of these eddy currents, is much more pronounced.

Suppose the glider starts from rest on an inclined plane. A solution of the appropriate differential equation shows that, at any time t, the velocity is given by,

$$v = v_o\,(1 - e^{-t/\tau}), \text{ ----------- } \quad (4)$$

where v_o is the so-called terminal velocity of the glider, and τ is the characteristic time, which in this case is very small compared with the case with no magnet in the glider.

Eq. (4) suggests that you use semi-log paper when you plot your data. From your curve you should be able to find both τ and v_o.

C. The sparker and waxed paper may also be used to find a value of the characteristic time with the usual glider with no magnet. In this case you would use Eq. (2) and find the velocity as a function of distance. The characteristic time you find can be compared with other values you have found provided you have used the same conditions.

D. Experiments to find the acceleration produced on a given mass by a definite force may be performed with the spark timer and waxed paper tape. A light spring and scale may be fastened to the heavier steel gliders. A calibration of the spring may be made either by using known masses or by knowing the mass of the glider and finding the extension of the spring for various tilts of the trough. On a level trough you may then determine whether a known force produces the acceleration required by Newton's laws of motion. (With a little practice you will find that you can keep the spring extended to a given point by pulling with your hand and walking along the trough).

Reprinted from AMERICAN JOURNAL OF PHYSICS, Vol. 31, No. 4, 255-262, April, 1963
Printed in U. S. A.

Linear Air Trough

H. V. NEHER AND R. B. LEIGHTON
California Institute of Technology, Pasadena, California

(Received 13 November 1962)

The construction and operation of a linear air trough, a device for floating small rectangular blocks (called gliders) on an air film, is described. The apparatus is useful for demonstrations and laboratory study of one-dimensional particle mechanics. The very small friction that is present is due to the viscosity of the air film, and causes the speed of a glider to decay with a time constant of the order of a few hundred seconds. A type of bumper has been designed which yields a coefficient of restitution greater than 0.99. Ten of these air troughs have been used in a student laboratory for a year, and have proved quite successful in experiments involving Newton's laws, collisions, damped harmonic motion, and motion on an incline.

I. INTRODUCTION

PUCKS that float on a film of gas have been described in the literature,[1] and have been widely used to demonstrate and study motion in two dimensions. For qualitative measurements, the gas-supported puck is a very suitable and, indeed, a very striking device. Two methods for admitting the gas to the base of the puck have been used. One of these uses a small tube from an overhead supply of gas. The other uses a supply of gas carried along by the puck itself. In the latter case, the gas supply may consist of a compressed gas like a small carbon dioxide capsule, or a supply of solid carbon dioxide may be carried in a container and the gradual subliming of the solid furnishes the required gas supply. Rubber balloons have also been used.

There are obvious limitations imposed by the use of all of the above systems of supplying the necessary gas. A much better arrangement would be to inject the gas through numerous holes in the plate on which the pucks are to float. However, this system would become involved, from a mechanical point of view, especially if the flat surface were very large.

Injecting the gas through holes in the surface on which the puck glides does become practical for a linear device, and such a device is herein described. We shall adopt the name *air trough* for such a piece of apparatus, and shall call the moving blocks *gliders*. While it is true that some features of collisions of objects are lost if only the one-dimensional case is studied, many phenomena may be studied with precision and ease with the linear system. It seems unlikely that, in an introductory laboratory, very much time would need to be spent on collisions in two dimensions if the characteristics of collisions in one dimension had been studied quantitatively with an air trough.

II. DESCRIPTION OF THE APPARATUS

The trough is conveniently made with plane sides with a 90° angle between them. To supply the gas (e.g., air) at regular intervals along the trough, two rows of equally spaced inserts with small holes are placed along each side. These tap the air supply in the two manifolds that run the full length of the trough.

While the original models were fabricated by welding together commercially available "U" and right-angle aluminum extrusions, later models were made with a special aluminum extrusion. To maintain the necessary straightness and rigidity, the extrusion is fastened to an aluminum I-beam by means of screws and epoxy resin. For an 8-ft-long trough, a 4-in. I-beam provides sufficient rigidity, while a 12-ft trough requires a 6-in. I-beam. A cross section of the extrusion and I-beam is shown in Fig. 1.

Also shown in Fig. 1 is a cross section through four of the numerous inserts which feed air from the common manifold to the surface on which the glider moves. These inserts are 0.063-in.-o.d. stainless steel tubing and have a 0.006-in.-diam coaxial hole. Such tubing is commercially available. These inserts are spaced along the trough at intervals of one inch. They are staggered in the four rows so that, with the proper length of

---

[1] Robert G. Marcley, Am. J. Phys. **28**, 670 (1960).

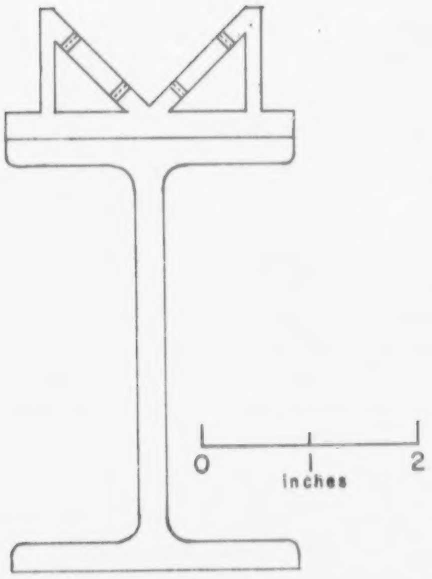


FIG. 1. A cross section of the special aluminum extrusion and the I–beam to which it is fastened. Also shown are cross sections through the stainless-steel inserts with their small holes through which the air flows from the two manifolds.

glider, only one jet of air at a time is uncovered as the glider moves along the trough. This is desirable because of a small Bernoulli action on the end surface of the glider.

Because of the rather large number of inserts that are required, the following method was used to make them in quantity:

(1) A suitable length of steel tubing, e.g., $\frac{5}{8}$ in. i.d., $\frac{3}{4}$ in. o.d., is plugged at one end and partly filled with zinc. While the zinc is molten, suitable lengths of the stainless steel tubing are forced down into the liquid. If the whole is kept hot, the molten zinc will rise up around the small tubing. Some 70 pieces of the stainless steel tubing may be so inserted into a $\frac{5}{8}$-in.-i.d. steel tube.

(It has been reported to the authors by Professor Malcolm Correll of the University of Colorado that epoxy resin may be substituted for zinc for holding the inserts in place. The epoxy may be removed with acetone. This procedure avoids the possibility of the inserts being attacked by the acid when the zinc is removed.)

(2) The steel tubing, filled with zinc and small tubing, is next cut into wafers about $\frac{3}{16}$ in. long.

(3) A surface grinder or abrasive wheel is then used to smooth the wafers. A grinding wheel with loose grit should not be used as the small holes are apt to become closed. Most of the 0.006-in. holes, at this stage, should be open at both ends.

(4) To clear out the burrs from the remaining holes a 0.005-in.-diam tungsten wire may be used, or, if a miniature sand blast is available, this may be effectively employed.

(5) The zinc is now etched out of the wafers to free the stainless-steel tubing, using hydrochloric acid. Care should be taken to be sure that the inserts are not also attacked by the acid.

(6) To round the edges on the inserts and thus make it easier to force them into holes in the aluminum, they may be tumbled. One may place a large number in a jar lined with carborundum paper and rotate this slowly about a horizontal axis. Tumbling for a period of 12 to 24 h at 1 or 2 rps should be sufficient.

Before the inserts are put into the holes, the trough should be well fastened to a suitable I-beam, after which the sides of the trough are planed. It is found that the gliders, when floating on an air film and nearly stationary, are sensitive to slopes of 1 part in 20 000. Thus, the requirements on the variations of the planed surface are ± a few thousandths of an inch in the length of the trough.

The holes for the inserts are conveniently drilled with a jig either before or after the trough is planed. These holes should be slightly smaller than the inserts so that the latter will be held firmly by the aluminum.

The manifolds in the extrusion are closed with suitable end plates. Air is admitted to these manifolds by connections to an air line furnishing a pressure of 5 to 40 psi. The air should be filtered to avoid stoppage of the small holes.

It is convenient for many experiments to have stops with suitable bumpers for the gliders at each end of the trough. If the I-beam is made several inches longer than the trough, space will be available at each end to fasten an aluminum block in which a spring bumper, shortly to be described, may be mounted.

It is desirable to have a means of changing the inclination of the trough by a definite amount. This may be done by equipping one end of the trough with a screw on which is mounted a dial with a scale. A stationary scale indicates the integral number of turns. Near the other end of the trough a cross piece is fastened which in turn can rotate in two metal blocks that may be rigidly clamped to a bench or table. A suitable choice of pitch of screw and length of lever arm gives a change of slope of 1 in 1000 for one rotation of the screw.

III. AUXILIARY EQUIPMENT

(1) The gliders may be made of materials such as aluminum, brass, and steel. The two faces

that fit in the trough should be ground straight and have the proper angle with respect to each other. The top corner of the gliders is best truncated by milling a flat surface. This not only serves the purpose of easily identifying the proper orientation of the glider but, also, serves as a convenient surface on which to fasten various auxiliary devices. It is desirable also to provide a means of lifting the heavier gliders to make it easier to place them in the trough without causing damage. A baîl type of handle is easily mounted at each end.

Suitable bumpers on the ends of the gliders as well as at the ends of the trough are very desirable. A satisfactory design is shown in Fig. 2. The spring material used in the bumper is available commercially as clock spring stock. Plug C (Fig. 2) has two flat places to take the double leaf springs D, when C is forced into B. The two springs thus take on the shape of a "U" with the thinner material (0.006×0.375 in.) being backed up by the thicker spring (0.016×0.437 in.) which in turn is backed up by a screw. The spacings should be such that the thin spring can distort by about 2 mm before touching the thick spring, and this can distort about 1½ mm before bottoming on the screw. This combination avoids distorting the springs beyond their elastic limit at hard impacts.

The energy loss at the ends is very much less for the thin spring material than when the thicker spring is used. With the steel glider (2.1 kg), and using 0.016-in.-thick springs on the glider and stops at the ends of the trough, the loss of energy on impact with a velocity of about 30 cm sec^{-1} was about 12%. Changing all bumpers to the 0.006-in.-spring material reduced the loss at the ends to less than 1%.

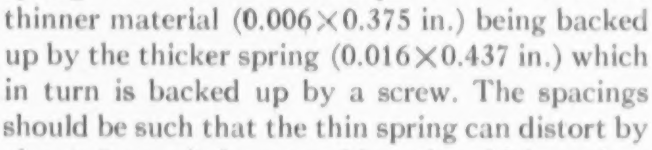


FIG. 2. Design of the bumper used in each end of most of the gliders as well as in the stops at the ends of the trough. Using these bumpers, collisions are more than 99% elastic.

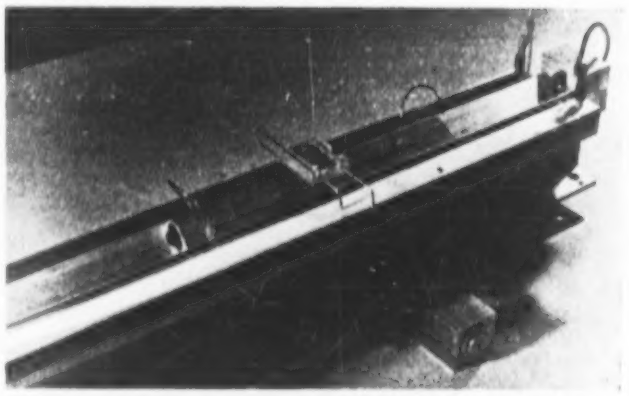


FIG. 3. The arrangement for using the insulated rails for making a spark record on waxed paper, is here shown. Also attached to the gliders is a pointer for visual determination of the position of the glider. Note also the double-leaf spring used as a bumper.

(2) For studying the motion of a glider, it is often desirable to have a spark record. A surface for holding the paper, that runs the full length of the trough, as well as an electrically insulated rail are easily mounted on the base of the extrusion as shown in Fig. 3. An insulated metal probe, mounted on the glider, carries the high potential from the spark rail to its pointed end which moves along just above the waxed paper.

A convenient rate of sparking is 1 per half-second. This may be achieved with a cam, operating a microswitch, and driven by a small synchronous motor. An automobile high tension coil is a convenient source of high voltage.

(3) On the side of the angle piece on which the waxed paper is mounted, a suitable metric scale may be fastened. A pointer mounted on the glider provides a means of determining its position: (See Fig. 3).

(4) For studying damped harmonic motion, long helical springs may be used to fasten to each end of a glider, the other end of the springs being fastened to the ends of the trough. With a period of about 5 sec, the logarithmic decrement is about 0.02 for a steel glider. These springs may be wound on a lathe from piano wire.

(5) To study forced, damped simple harmonic motion, a variable speed, geared motor may be used. An amplitude of about 1 cm is convenient. A crank on the motor is attached to one of the gliders. Wires from this glider connect to a similar glider near the other end of the trough. This second glider is fastened by means of a spring to the end of the trough. Long helical springs then connect from these two driven gliders to the third glider at the center of the trough. Thus the end gliders are driven with a definite amplitude

at a variable frequency. The motion of the driven glider may then be studied.

(6) The driven glider, in 5 above, may be provided with a damping proportional to its velocity by mounting on a glider, made from a piece of aluminum angle, a horse-shoe shaped permanent magnet. It is desirable to place soft iron inserts into the aluminum to produce a strong magnetic field that links the aluminum of the extrusion. Means may also be provided for raising and lowering the magnet on the glider, thus changing the damping. By such means, the Q of the system, with suitable springs, may be changed from about 2 to 120.

(7) To study action and reaction, gliders may be provided with a mechanism to blow them apart with powder caps used in some toys. One may obtain such caps with sticky adhesive on the back at toy stores. Phenolic insulated pieces, in the form of piston and cylinder, are substituted for the bumpers in the gliders. An O-ring, mounted in the piston provides a convenient seal for the firing chamber. The piston contains a pointed metal rod with a suitable connection so that a high potential may be applied externally which will pass through and thus ignite the powder of the cap. The cap is stuck over the end of the pointed rod which in turn is flush with the end of the phenolic. The cylinder contains a metal screw in its base to which the spark jumps after passing through the cap. Suitable adjustments may be made so that after the cap is exploded and the gliders start off in opposite directions and return after bumping off the ends of the trough, they stick to each other when they come together.

(8) To study random motion of particles, a variable speed, sinusoidal drive may be provided by a crank-and-flywheel driven piston mounted at one end of the trough. A number of short gliders are placed in an inclined trough and they are kept in motion by this source of "heat" at the lower end. The frequency of drive should be a few strokes a second, and a large moment of inertia should be provided to minimize loss due to collision with the gliders. This piece of auxiliary apparatus could be combined with that in (5) above to provide a source for forced simple harmonic motion.

(9) To study normal modes and other coupled systems, it is desirable to have a spring that may be extended some distance and yet behave properly when compressed. Such a spring may be made from the same clock spring material as is used to make the bumpers. A section of the 0.006×0.375-in. leaf spring, cut about 24-in. long, bent into the shape of a semicircle gives a period of a few seconds when attached to the 3-in. long aluminum gliders. To attach this spring to the gliders, holes are drilled, say, a quarter inch from each end, then the last half inch of the spring is bent at right angles. The spring is then screwed down to the tops of the gliders. Two equal gliders so attached may be made to proceed along the trough in a "measuring worm" kind of motion.

(10) To study damped motion as such, the usual glider is not satisfactory because of the low damping. While the glider described in 6 above may be used, it may be desirable, especially for demonstration purposes, to embed a magnet in a glider. This may be done by using square aluminum tubing, $1\frac{1}{4}$ in. on the outside, with a $\frac{1}{8}$-in. wall, and placing a bar magnet inside, say $\frac{3}{4}$-in. in diameter and 8 in. long. On a slightly tilted trough, such a glider soon reaches its terminal velocity.

(11) Accelerated motion may be studied when the trough is level by pulling a glider, say the 2.1-kg mass, with a calibrated spring. A spring of this sort is described by one of us (H. V. N.)[2] as used with the Maxwell top. Such a device may be fastened to the top of the 12-in. steel glider. The student may either pull the spring to a certain point, or it may be pulled by a Hero type engine working with air pressure.

IV. USE AND CARE OF THE APPARATUS

Experience in using 10 of these troughs, almost every day for a period of six weeks, has shown:

(1) With common air filters in the building compressed air line, no trouble was found in the small holes becoming stopped.

(2) Because of the small clearances (approximately 0.003 in. between trough and glider) it is necessary to have the surfaces clean. It was found sufficient to clean both gliders and trough at the

---

[2] H. V. Neher, Am. J. Phys. **30**, 503 (1962).

beginning of a laboratory period with paper tissue and xylene.

(3) If the smooth surface of a glider is damaged, it may be readily ground flat again by rubbing it on fine carborundum paper backed by a flat surface. (We have had our aluminum gliders anodized, and those made of steel, plated with chromium. This treatment not only preserves the surface, but probably results in a surface less likely to be damaged by scratching.)

V. SOME GENERAL PROPERTIES OF THE AIR TROUGH

(1) The frictional forces retarding the motion of the gliders in the trough seem to be entirely due to viscous damping of the thin air film between the glider and the trough surfaces. As such, the frictional drag goes to zero at zero velocity. Further, the forces required to give the gliders a small velocity are extremely minute. As a consequence, the light aluminum gliders wander slightly due to disturbing influences, such as the Bernoulli forces arising from the jets of air blowing by the end faces. For this reason, brass or steel are more satisfactory materials from which to make most of the gliders. As is shown later, however, it is best in all cases to use reasonable velocities in experiments—up to 30 cm sec^{-1} for steel gliders, and up to 60 cm sec^{-1} for those made of aluminum.

(2) The amount by which the gliders are lifted by the air film, as a function of air pressure, may be determined by means of a dial indicator. In the case of the steel glider the results in Table I were found.

The figures in the last column were calculated from the data in the second column. The mass of the steel glider was 2.131 kg, and it was supported by 47 jets.

(3) With the help of the above data, the damping of the glider due to viscosity may be calculated. For a damping proportional to the velocity, the equation of motion, when the trough is horizontal, is

$$m\ddot{x}+(\eta A/d)\dot{x}=0, \tag{1}$$

where η is the coefficient of viscosity of the air, A the area of the surfaces, and d the spacing. The solution of Eq. (1) in terms of x and t is

$$v=v_0e^{-t/\tau},$$

where $\tau=md/\eta A$ is the characteristic time of the motion, or the time for the velocity to drop to $1/e$ of its initial value.

TABLE I.

Pressure	Vertical distance	Surface clearance
5 psi	0.0010 cm	0.0007 cm
10	0.0025	0.0018
20	0.0048	0.0034
30	0.0066	0.0047
40	0.0079	0.0056

Data were taken with waxed paper and sparker for the steel glider going in both directions, with an air pressure of 23 psi. Using the data in Table I, the characteristic time was calculated to be 233 sec. The measured τ from the tape, averaging the two directions, was found to be 275 sec.

The characteristic time for the 3-in. aluminum glider is about 60 sec.

(4) Equation (1) may also be solved for v in terms of distance traveled. Thus,

$$mv(dv/dx)+(\eta A/d)v=0,$$

or

$$v=v_0-1/\tau x. \tag{2}$$

This predicts that the velocity should decrease linearly with the distance traveled.

To test this, and also to find the losses due to the collision of gliders with the stationary ends, a second glider was clamped rigidly in the trough to vary the distance traveled between bounces. The velocities given in Fig. 4 are those found from stopwatch measurements between collisions, or for a number of collisions where the distance traveled between bounces was small.

It is seen that the velocity does really decrease linearly with distance traveled, even when many bounces or collisions occur. Thus in curve H where over 300 collisions took place, the velocity changed linearly with the distance. The behavior of curve A, taken with H indicates not only that the change of velocity is due to a damping proportional to the velocity, but the change of velocity is also proportional to the number of bounces. Thus the change of velocity at a bounce is independent of the velocity, and has a constant value. This change in velocity per bounce is found to be approximately -0.04 cm sec^{-1}. It is thus possible to construct a new curve (A′) which

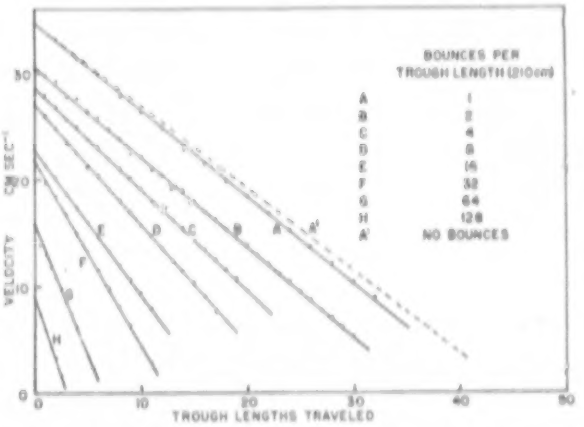


FIG. 4. Shown above is the velocity of the glider vs distance traveled for different numbers of collisions of the bumpers per unit distance traveled. The change of velocity with distance, due to the viscosity of the air film, is constant. Also, the change of velocity per bounce is constant, i.e., independent of velocity. The dotted line, A′, gives the expected velocity of the glider vs distance traveled in a very long, horizontal trough.

would represent the velocity vs distance for no bounces.

This curve A′ has a slope of 0.0036 sec^{-1} or $\tau = 280$ sec. This value should be compared with the value of $\tau = 275$ sec found from measurements made on a wax paper, using one trough length.

(5) As pointed out above, the change of velocity per bounce is independent of the velocity and has a constant value of about -0.04 cm sec^{-1} bounce^{-1}. Now the relative change of kinetic energy is $\Delta E/E = 2\Delta v/v$. Hence $\Delta E/E$ per bounce is not constant, but has a value of about -0.31% per bounce near the beginning of curve A and about -1.6% per bounce at the lower velocities of about 5 cm sec^{-1}.

Thus, where experiments are being performed in which losses due to damping or collisions are to be minimized, the larger velocities should be used. For example, at 30 cm sec^{-1}, it requires a glider about 7 seconds to travel the length of the trough. Yet, the change of velocity in going this distance is only 0.83 cm sec^{-1}, and when the glider collides with the end, it changes its velocity only 0.04 cm sec^{-1}. On the other hand, if a velocity of 5 cm sec^{-1} were used on the full length trough, the change of velocity would be the same due to damping but would amount to a decrease of 16 to 17% of the initial velocity. The velocity change due to the bounce at the end would, however, still be less than 1%.

VI. SOME EXPERIMENTS

The number of experiments that may be performed with the device, herein described is limited only by the ingenuity of those who have worked with such a piece of apparatus. It has its fascinations for faculty as well as students.

Some of the experiments that have been found instructive are as follows:

1. One may equip the gliders with various means of repelling each other. The use of leaf spring bumpers has already been described. One may also use horseshoe magnets mounted in the ends. A bar magnet should not be used because of the eddy currents induced in the aluminum extrusions. One may also use the expansion of gas as described earlier in connection with the exploding gun powder in toy caps. Thus one may establish experimentally that, with equal gliders, the acquired velocities are always equal and opposite, and independent of the kind of force involved.

2. Inertial Mass vs *Gravitational Mass.* Let one mass m_2 strike a stationary mass $m_1 (m_1 > m_2)$ somewhere along the trough, which is horizontal. There is one location of m_1 such that, m_2 and m_1, after collision, strike the ends at the same time. If the collisions are all nearly elastic, the returning masses will again collide close to the location of the original collision. Furthermore, m_2 will stop while m_1 will rebound with nearly the same speed it had originally. The process will repeat itself a number of times, if the system is not disturbed.

Assuming no loss of energy, it may be shown that the ratio of masses in the above experiments is

$$m_1/m_2 = (2l_2/l_1) + 1, \qquad (3)$$

where l_1 and l_2 are the distances traveled by m_1 and m_2, respectively, before colliding with the ends. Using steel and brass gliders whose respective gravitational mass ratio was 3.905, the inertial mass ratio found from Eq. (3) was 3.84. This difference is in the direction to be expected from the losses along the track suffered by the two masses.

3. The acceleration of gravity. If the trough is tilted at a small angle θ to the horizontal, there are two obvious methods for determining the acceleration due to gravity. One, is to time the glider in going down the inclined plane, starting from rest. The other, is to allow the glider to rebound at the end bumper and measure the time for the glider to go back up the trough, reverse its direction, and strike the bumper a second time. This latter method is to be preferred since, to a first order of approximation, the effect of losses on the trough cancel out. Thus the time required to go to its turning point is less than it would be without losses. On the other hand, the time to return to the bumper is greater than it would have been without losses.

The following data were found with a tilt of the trough of $\theta = 0.00200$, using a steel glider:

Distance of rebound	Time up and down	Calculated g
1.950 m	19.82 sec	9.93 msec^{-2}
1.495	17.43	9.84
1.004	14.30	9.83
0.790	12.75	9.72
0.474	9.88	9.71

4. Further measurements to determine the characteristic time. With an inclined trough, one may observe the successive points of reversal of the motion of the glider as it continues to rebound from the lower bumper. Were it not for losses, these points would, of course, all be identical. Hence the differences must be a measure of the losses. By solving the appropriate differential equation one finds that this Δx, to a first order of approximation, may be expressed as

$$\Delta x = [8/3(2g\theta)^{\frac{1}{2}}](x^{\frac{3}{2}}/\tau), \tag{4}$$

where x is the distance the glider travels up the trough, τ the characteristic time, while g and θ have their usual meaning.

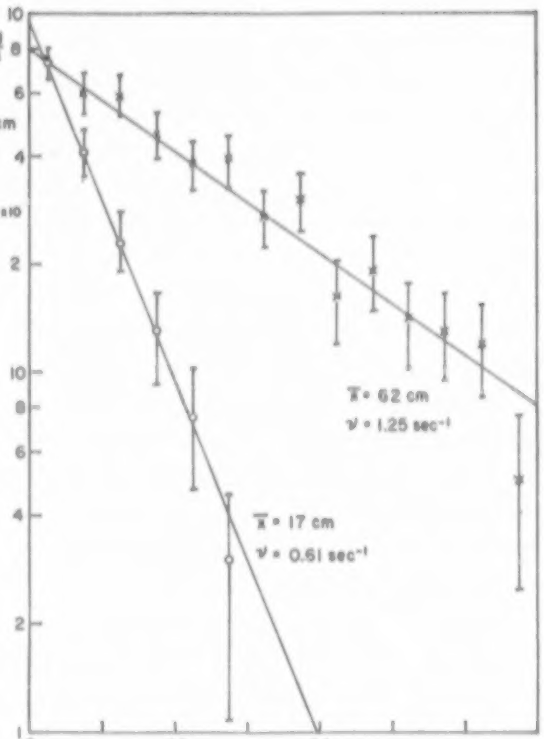


FIG. 6. On an inclined trough, a motor driven bumper, having a repetitious motion in a longitudinal direction at the lower end would be expected to give a random motion to the glider as it is repeatedly struck. The data in this figure were taken by dividing the length into uniform intervals and counting the number of times the glider reversed its motion in each interval. Randomness is indicated by the fact that the semilog plot gives a straight line.

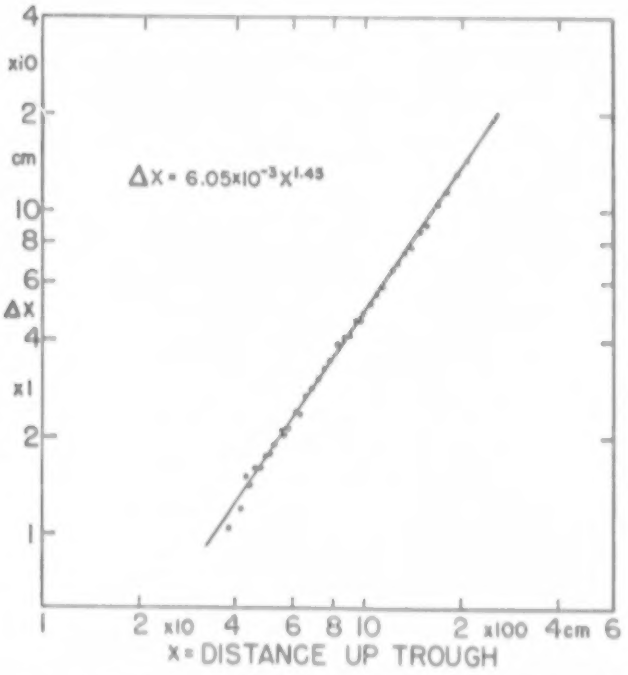


FIG. 5. For repeated bouncing-off the end bumper, with an inclined trough, the distance of reversal of motion of the glider decreases with each bounce. This constitutes another method of finding the characteristic decay time of the glider due to the air film.

The experimental values of Δx vs x are plotted in Fig. 5 in log–log coordinates. Here the tilt was $\theta = 0.00200$, the glider was of steel, and an air pressure of 18 psi was used. The equation of the curve is

$$\Delta x = 6.05 \times 10^{-3} \times 1.45. \tag{5}$$

Taking the two constants of (4) and (5) to be equal, the value found for the characteristic time is

$$\tau = 224 \text{ sec}.$$

This should be compared with the two values of 275 and 280 sec found previously. The difference may be accounted for from the difference in the air pressure used. The larger previous values of characteristic time were obtained with an air pressure of 23 psi. Using the data in Table I, the air film for this case was 3.84×10^{-3} cm thick. For the 18 psi used in the present experiment, the air film was 3.15 cm thick. Assuming a damping inversely proportional to the air film thickness, the value of 224 sec reduced to the same thickness of air, would become 274 sec. A high precision in the determination of this characteristic time should not be expected as differences in gliders, air pressure, foreign matter, etc., can all change the experimental value slightly.

5. If the variable-speed-motor-driven end-bumper described in Sec. III (8), is available, it is instructive to examine the energy distribution law for gliders which are in statistical equilibrium with this "heat source." A single glider is placed in the trough and the trough is tilted steeply enough to prevent the glider from reaching the high end, except rarely. Then, with the drive motor going at some convenient speed, the "heights" of successive bounces of the glider are measured, and tallied in, say, 10 cm intervals. A semi-log plot of this distribution should exhibit a straight line relation. Such distributions for two motor speeds, in approximately a 2:1 ratio, are shown in Fig. 6.

ACKNOWLEDGMENTS

We would like to take this opportunity to express our appreciation to the many individuals who have helped, in one way or another, to carry through this project; in particular, the personnel in the machine shop for their skillful work. We also wish to acknowledge the financial assistance of the Ford Foundation in supporting the development of such instructional laboratory equipment.

Harvard University

[*] Part I -- Theory

1. Derivation of Basic Equations: The apparatus shown in Fig. 1 consists of a long tube which has a fairly long downward incline, then curves and ends in a short, straight section. The tube is pivoted at point P and the inclination of the tube may be set by adjusting the turnbuckle. A small steel ball, released at the high end, rolls down the tube and emerges from the open end traveling as a free projectile. Once the ball has left the tube, its motion is determined only by V_o, the velocity with which it left the tube, and θ, the angle of elevation of the exit section of the tube.

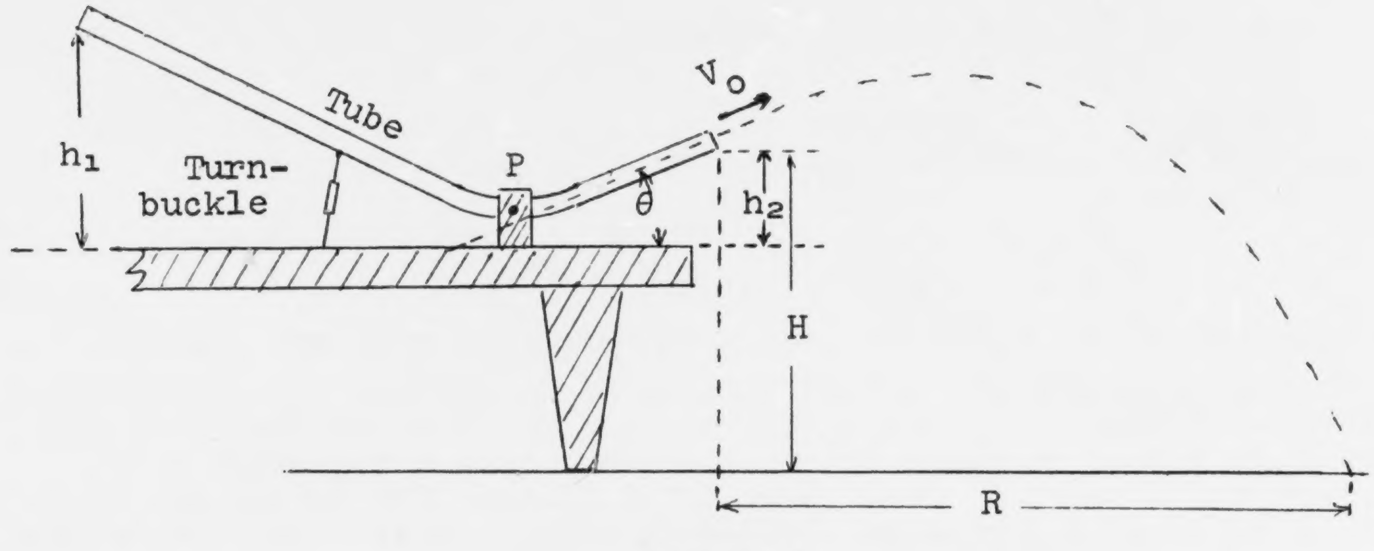


Figure 1.

Suppose that the ball slides through the tube and that the energy loss due to friction is negligible. Then, by conservation of energy,

$$\left[\text{Potential Energy at } h_1\right] = \left[\text{Potential Energy at } h_2\right] + \left[\text{Kinetic Energy at } h_2\right]$$

or

$$mgh_1 = mgh_2 + \tfrac{1}{2}mV_o^2. \qquad \text{Eq. (1)}$$

---

[*] Reprinted from Experimental Physics by D. Miller, G. Holton, and M. Hendersholt, Harvard University, Copyright 1962.

or $$V_o = \sqrt{2g(h_1-h_2)}. \qquad \text{Eq. (1a)}$$

The components of velocity at the end of the tube are therefore

$$V_{horiz} = \cos\theta \sqrt{2g(h_1-h_2)} \qquad V_{vert} = \sin\theta \sqrt{2g(h_1-h_2)}. \qquad \text{Eq.(2)}$$

If the ball is in flight for a time t, then

$$R = (V_{horiz})(t) \qquad \text{Eq.(3)}$$

and

$$H = \tfrac{1}{2}gt^2 - (V_{vert})(t). \qquad \text{Eq.(4)}$$

Solve Eq. (4) for t;

$$t = \frac{1}{g}\left[V_{vert} + \left\{(V_{vert})^2 + 2gH\right\}^{\frac{1}{2}}\right] \qquad \text{Eq.(4a)}$$

and substitute into Eq. (3), also making use of Eqs. (2), to find R:

$$R = 2\cos\theta\left[\sin\theta(h_1-h_2) + \left\{(\sin^2\theta)(h_1-h_2)^2 + H(h_1-h_2)\right\}^{\frac{1}{2}}\right] \qquad \text{Eq.(5)}$$

Question 1: (a) Does the value of g have any effect on R? (b) By the quadratic formula, Eq. (4) could have been written as

$$t = \frac{1}{g}\left[V_{vert} \pm \left\{(V_{vert})^2 + 2gH\right\}^{\frac{1}{2}}\right]$$

Explain, in 25 words or less, why the plus sign was chosen.

The two simplifying assumptions made in writing Eq. (1) are not realistic and Eq. (5) consequently cannot be expected to yield useful predictions of R, even though the remaining steps leading to Eq. (5) are all valid. It is much more appropriate to assume that the ball rolls down the tube without sliding. This, however, means that the ball not only moves along its path once it has left the tube, but it also rotates as it travels. The total kinetic energy of the ball at exit must therefore be written as $\frac{1}{2}mV_o^2 + \frac{1}{2}I\omega_o^2$ where I is the moment of inertia of the spherical ball about its center and ω_o is the angular velocity of the ball.

Question 2: (a) Look up the moment of inertia of a homogeneous sphere of mass m and radius r about its center.
(b) The angular velocity ω_o of the rolling sphere about its center is simply related to the velocity V_o with which the center of mass moves: $$V_o = \omega_o r.$$

Write a concise and convincing explanation of this relationship, aiming your presentation at someone who does not believe the equation.
(c) Now show that the final kinetic energy of the rolling ball is $(7/10)mV_o^2$.

Question 3: If the ball rolls down the tube, will it emerge from the exit end with higher or lower velocity than if it had slid down the tube? Answer briefly but sketch your reasoning.

Question 4: The ball also looses an appreciable amount of energy to friction and air resistance while rolling through the tube. Designate this energy loss as E_f and rewrite Eq.(1) to take into account both this frictional energy loss and the non zero component of rotational kinetic energy.

E_f may be measured experimentally. If the inclination of the tube so adjusted that the ball, released from the high end (at height h_{1f}), just comes to rest at the low end (at height h_{2f}) of the tube with zero velocity, it will be found that the difference in height between starting point and exit end is not zero. Indeed, by conservation of energy, the potential energy difference corresponding to this difference in height must just equal the loss to friction E_f. Therefore

$$E_f = mg\,(h_{1f} - h_{2f}). \qquad \text{Eq. (6)}$$

Question 5: Why is no mention made of kinetic energy in the above discussion of the determination of E_f?

Question 6: Substitute Eq. (6) into your result from Question 4 and thus derive an expression for V_o analagous to Eq. (1a). Employ the abbreviations

$$\Delta h = h_1 - h_2 \quad \text{and} \quad \Delta h_f = h_{1f} - h_{2f}.$$

h_1 and h_2 will not appear separately in this result.

Question 7: Use the results of Question 6 to write new expressions for V_{vert} and V_{horiz} corresponding to Eqs.(2).

Question 8: Use the results of Question 7 to derive a new formula for the range R, analagous to Eq.(5). Are you going to be able to use Eqs. (3), (4), and (4a) unaltered?

The correct answer to Question 8 may be written as

$$R = \left[\frac{10}{7}(\Delta h - \Delta h_f)\right](\sin\theta\cos\theta) + \cos\theta\left\{\left[\frac{10}{7}(\Delta h - \Delta h_f)\right]^2 \sin^2\theta + 2H\left[\frac{10}{7}(\Delta h - \Delta h_f)\right]\right\}^{\frac{1}{2}} \qquad \text{Eq. (7)}$$

Chances are that your answer to Question 8 did not come out in exactly this form, However, the terms in Eq. (7) are arranged in a form which is particularly suited for numerical computation.

2. Organization of Calculations: Calculations involving a formula as complicated as Eq. (7) must be carried out very systematically if confusion and wasted time are not to be the main results. First, use your raw data to calculate the quantities

$A = 10/7\ (\Delta h - \Delta h_f)$ $B = \sin\theta$

$C = \cos\theta$ $H = H$

together with percentage uncertainties (Percentage because these quantities appear multiplicatively in Eq. (7).)

Now A, B, C and H may be substituted into Eq. (7) to yield

$$R = ABC + C\sqrt{A^2B^2 + 2HA} \qquad \text{Eq. (7a)}$$

The necessary error computations may be done mentally, since the final quotation of error is invariably rounded off to the nearest percent or tenth percent in our work here.

Part II -- Experiment

a.) Clean the tube as well as you can by drawing the swab through the tube several times. You may want to repeat this during the experiment to insure smooth rolling. Listen to the sound which the ball makes as it rolls through the tube; jarring noises may indicate the presence of dirt obstructions.

b.) Devise an experimental procedure for measuring Δh_f and outline it briefly. The apparatus incorporates some aids; Pins have been located on the sides of the tube and these mark the position of the center of mass of the ball at their levels. There are three pins: one at the point from which the ball is released, another at the exit end of the tube, and a third one at an intermediate point on the final straight section of the tube. The actual adjustment of height differences is made by a turnbuckle. This feature permits very small differences to be set with ease.

After deciding on your procedure for measuring Δh_f, decide also how the uncertainty in Δh_f is to be determined and explain the method briefly.

c.) Take all data necessary to find Δh_f, together with uncertainty.

d.) Set the inclination of the tube so that the starting point is about 40 cm above the baseboard. Do not readjust the tube for the remainder of the experiment. Take all data necessary to calculate R. Is it necessary to measure θ itself? Do not forget the radius of the ball in measuring h_1 and h_2. Where should h_1 and h_2 be measured from? Use Eq. (7a) to predict R, together with uncertainty.

e.) Now check your prediction by seeing if the ball hits a quarter placed at the predicted landing point. Note the spot on the floor where the ball lands. Over this spot place an open page of your lab report form, fastened firmly to the floor with masking tape. On top of this, place a sheet of carbon paper (carbon side down) and on top of the carbon paper place a protective cover sheet (provided.)

Allow the ball to roll down about ten times. If the scatter in landing points is more than two or three centimeters, check for the presence of dirt in the tube. Measure the horizontal distance from the end of the tube to any convenient point on the taped down sheet and then mark on the taped down sheet a scale with 5 cm intervals that shows the range from the end of the tube. When you take up this sheet, you will be able to read the range for each shot directly from it.

f.) Tabulate the various values of R and calculate a mean range, with uncertainty.

g.) Compare the results of parts (d) and (f). What is the percentage discrepancy? Do the results overlap within the ranges of uncertainty? Does a systematic discrepancy appear or are all variations apparently random? To what physical process would you attribute any systematic discrepancy? (One possibility involves Δh_f and the possibility that frictional resistance is not the same at all speeds. There are others which you should be able to think of.) Briefly describe each such process and state what additional measurements would have to be made to decide whether the process really is a significant source of systematic error. Can the random variations in R be traced, in part at least, to any single physical process?

THE VELOCITY OF A BULLET

California Institute of Technology

Principles: Suppose a bullet is shot into an object and stops therein. If the object is initially at rest with respect to the laboratory, but free to move, the velocity of the bullet may be determined provided the mass of the bullet is known together with the mass of the object and its velocity immediately after impact. This velocity may be determined if the object is suspended as a pendulum and the height to which it swings is measured. Such an arrangement is called a ballistic pendulum.

The velocity of the bullet may also be determined by measuring the time required for it to go a known distance. This may be done with an electrical circuit by letting the bullet break two conductors. The time between the breaking of the conductors may be measured by determining the extent to which a capacitor is discharged during the time interval in question.

Let a battery, E, maintain a potential, V_0, across a resistor R (Fig. 1). The key, K_1, permits the capacitor, C, to be charged by the battery and discharged through the galvanometer.

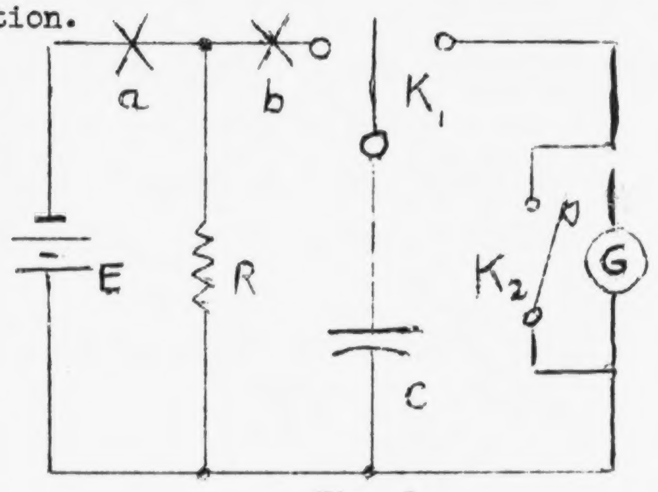


Fig. 1

The electrical charge from the capacitor flows very rapidly through the galvanometer and gives a sudden impulse to its coil. A galvanometer so used is called a ballistic galvanometer and is analogous to the ballistic pendulum, the chief difference being that the energy in the case of the pendulum is stored in the gravitational field when the displacement is a maximum, while in the case of the galvanometer the energy is stored in the torsion wire suspension.

The key, K_2, is used to damp the galvanometer motion.

If K_1 is first moved to the left and then to the right (Fig. 1), the galvanometer will give a maximum deflection proportional to the full charge on the capacitor. Let K_1 now be to the left. Let the bullet first break the circuit at "a" and a small time later, break the circuit at "b". In this time interval C will discharge through R. The remaining charge on C may then be compared with its initial charge by means of the galvanometer deflections.

If a capacitor with original charge Q_o discharges for a time t through a resistor R, the remaining charge Q, will be

$$Q = Q_o e^{-t/RC} \ .$$

R is in ohms and C in farads if t is in seconds.

Discussion: Compare your two determinations of velocity by discussing the sources of error and decide whether the disagreement is consistent with the expected errors. Do not just list the errors but give a reason for thinking each has a certain expected value.

Do you have a strong preference for one method over the other? Why?

A. CENTRIPETAL FORCE, B. CONSERVATION OF ANGULAR MOMENTUM

Cornell University

Your experiment and observations will involve conservation of energy, centripetal force, and conservation of angular momentum. You are already familiar with the first two concepts at this time of the term and the experiment will introduce you to the third concept listed above.

A. Consider the simple pendulum bob, mass m_1 on a light (negligible mass) wire. When the pendulum swings the tension in the wire is a maximum at the instant the bob is at the bottom of the swing. At this instant the tension is greater than the weight of the bob by an amount equal to the centripetal force needed to keep the bob moving in its circular path.

A force diagram of the bob at this instant is shown in Figure 1.

$$\sum_i F_i = m_1 a$$

$$T - F_w = \frac{m_1 v^2}{R}$$

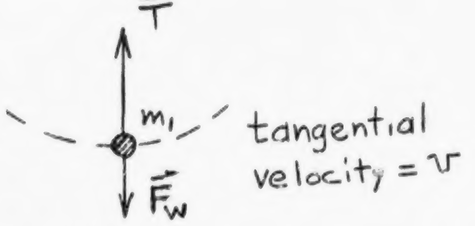


Fig. 1

$$T - m_1 g = \frac{m_1 v^2}{R}$$

$$T = m_1 g + \frac{m_1 v^2}{R} \quad (1)$$

Measurements of T at this instant for various values of v will determine the centripetal force acting on the bob.

To facilitate measurement of T you have an "equal" arm balance with the pendulum suspended at the end of one arm and a weight hanger is suspended at the other end. Two metal pins limit the amount of rotation of the beam to a few degrees. In operation, the bob is drawn back a given distance in the vertical plane of the balance arm and allowed to swing as a pendulum. Figure 2 shows the apparatus with the pendulum about to be released. Sufficient weights are placed on the weight hanger until the hanger end just fails to lift off of its stopping pin. When this condition obtains, and with equal arms for the balance, the total weight on the hanger side, $m_2 g$, equals the maximum tension, T, in the pendulum wire.

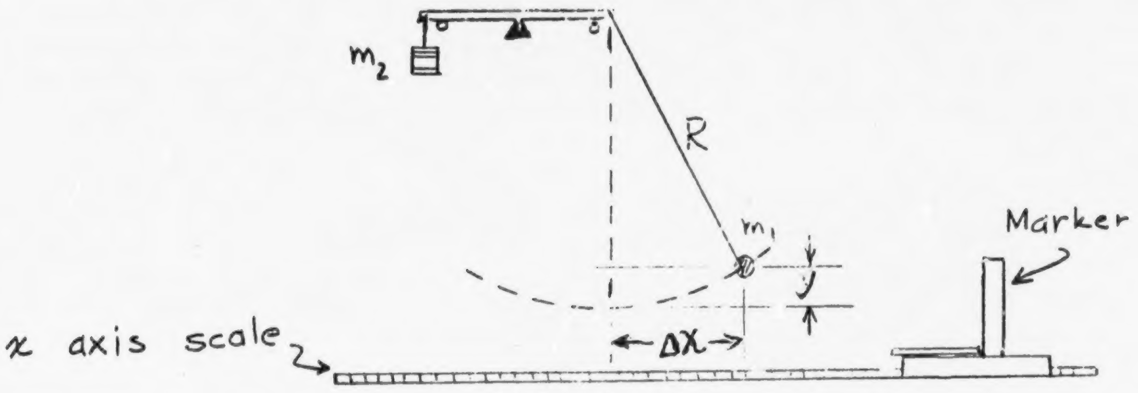


Fig. 2

If the mass m_1 starts from rest at height y above its equilibrium position the velocity of m_1 as it passes through its lowest position can be obtained from the Law of Conservation of Energy.

$$\frac{1}{2}m_1v^2 = m_1gy$$

$$v^2 = 2gy$$

Since, $T = m_2g = m_1g + m_1\frac{v^2}{R}$

then, $m_2g = m_1g + 2gym_1/R$

and, $m_2 = m_1 + \left(\frac{2m_1}{R}\right) y.$ (2)

A plot of measured values of m_2 vs. y is predicted, by Equation (2), to be a straight line with a slope of $2m_1/R$.

Procedure:

The first step is to determine the mass of the sphere used as the pendulum bob. Measure R, the length from the point of support to the center of the sphere. The bracket should be mounted so that the aluminum balance arm is horizontal when the arm rests on the stop pin closest to the weight hanger.

Next the marker (See Fig. 2) is brought up until the vertical side of the marker just touches the hanging sphere. This "zero" position of the marker is read on the meter stick which defines the x axis. The marker is then moved back to some new position on the x axis and clamped there during the ensuing trial measurements. Take data for 3 or 4 values of Δx without exceeding 50 cm. displacement of the marker.

Keeping the wire taut, the ball is drawn back until it touches the marker. It is released and allowed to swing. In repeated trials the load on the weight hanger is increased until the swinging pendulum just fails to raise the balance arm off of its pin. This total load, m_2, is recorded for each determination.

B. Elliptical Conical Pendulum.

You have seen the conical pendulum demonstrated in lecture, (See Fig. 4). If the initial velocity of the ball isn't just right, the ball will move in an approximately oval or elliptical path. Looking down on top of the pendulum, a plot of the position of the ball at equal time intervals, Δt, looks like this:

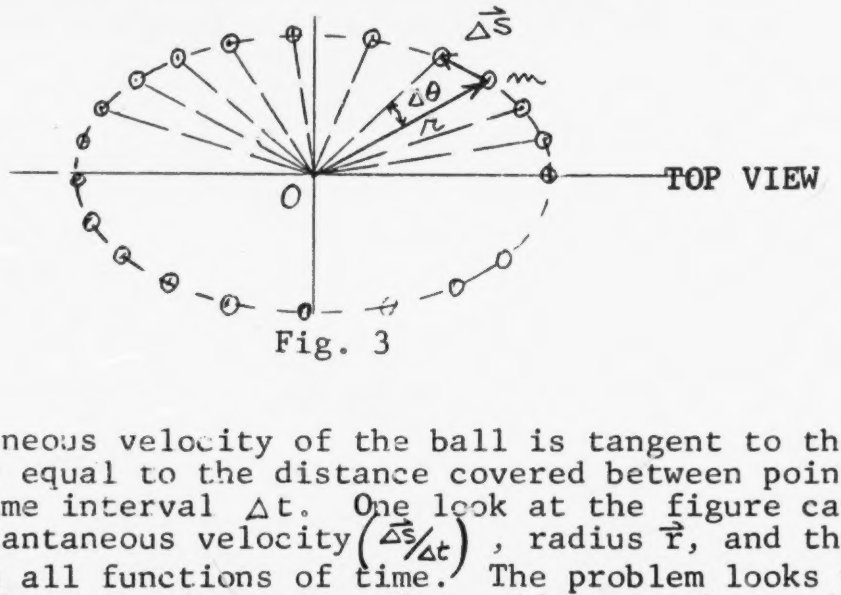
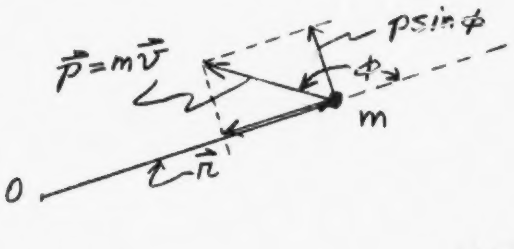


Fig. 3

The instantaneous velocity of the ball is tangent to the curve and approximately equal to the distance covered between points $\vec{\Delta s}$ divided by the time interval Δt. One look at the figure can tell you that the instantaneous velocity $\left(\vec{\Delta s}/\Delta t\right)$, radius $\vec{r}$, and the angular velocity $\left(\Delta\theta/\Delta t\right)$ are all functions of time. The problem looks very complicated but there is also some order evident in the motion. After you have had more experience in mechanics you will recognize that the forces acting on m exert zero torque about point 0 as seen in the Figure 3, and that, since the torque is zero, the angular momentum must be a constant. That is, the angular momentum of m about a vertical axis through 0 does not change, even though $\vec{v}$, $\vec{r}$, and $\left(\frac{d\theta}{dt}\right)$ do change as the point moves about.

What is meant by angular momentum? Consider the point mass m which has momentum $m\vec{v} = \vec{p}$ and is located $\vec{r}$ away from 0.



The angle between vector $\vec{p}$ and the vector $\vec{r}$ is ϕ. The component of $\vec{p}$ that is perpendicular to $\vec{r}$ equals $p \sin \phi$. The angular momentum of m about axis 0 is defined as the product of the perpendicular component of $\vec{p}$ and $\vec{r}$. That is,

$$|\text{Angular momentum}| = |(p \sin \phi)\, r|$$

$$\text{or,} \qquad L = r\, p \sin \phi \; .$$

We use the symbol L for angular momentum. It is this quantity, L, which is a constant of the motion in the elliptical conical pendulum.

The term torque, used above, is defined in the Appendix of this experiment along with the theory for the mechanics involved. The appendix is, at this part of the semester, optional reading. You are not expected to understand the theory for this experiment, and you will take an empirical point of view in studying angular momentum. Is the angular momentum a constant of this motion? Measure and see.

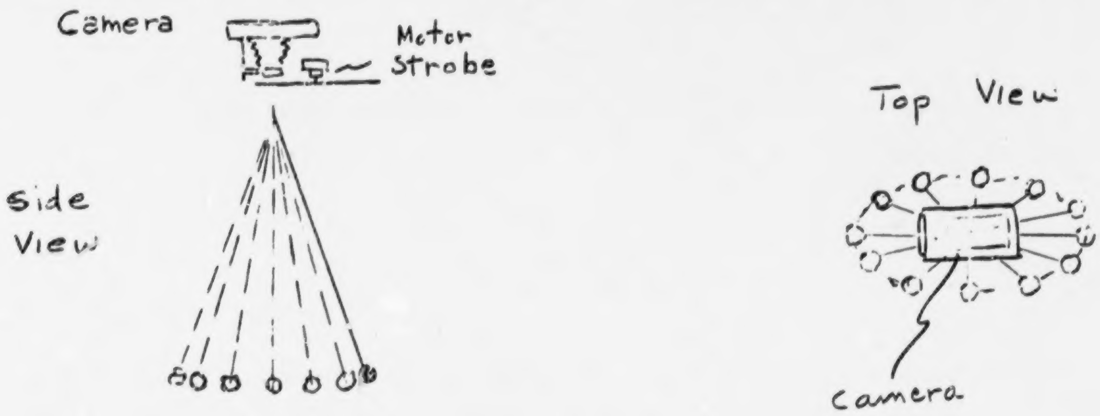


Fig. 4

Procedure (B).

You will photograph, with the strobe shutter, the elliptical conical pendulum from a point above the point of suspension. Try several "orbits" before exposing the film. You should keep the shutter open for just approximately one complete orbit. If the shutter is kept open for more than one orbit, the exposures will overlap and be difficult to distinguish. A small overlap or a gap is not serious. Take pictures of two orbits of different eccentricities. The negatives can also be used for data records. Record the length, ℓ, of the pendulum used.

Report:

Part A. Centripetal Force:

Plot a graph of m_2 vs. y. Compare the slope and the m_2 axis intercept at $y^2 = 0$ to the values predicted by equation (2). For one value of y calculate the speed of m_1 at the bottom of the swing, and them compare the excess weight on the hanger to the centripetal force, m_1v^2/R. No estimates of uncertainty need be made.

Part B. Conservation of Angular Momentum:

For at least six time intervals at various parts of the orbit of each photo, calculate the velocity during each interval from $\vec{v} = \vec{\Delta s}/\Delta t$. Using the definition of angular momentum given above, find the angular momentum during each time interval. What is the average angular momentum of the intervals you have examined for each orbit? What is the relative variation of each determination of angular momentum from the average value for each orbit?

Massachusetts Institute of Technology

Introduction

In this experiment we will investigate the relationship between work and energy in an isolated system consisting of a puck and two springs. (See Figure 1a.)

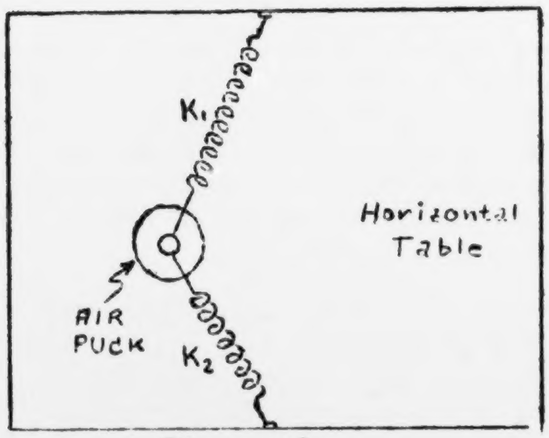


Figure 1a

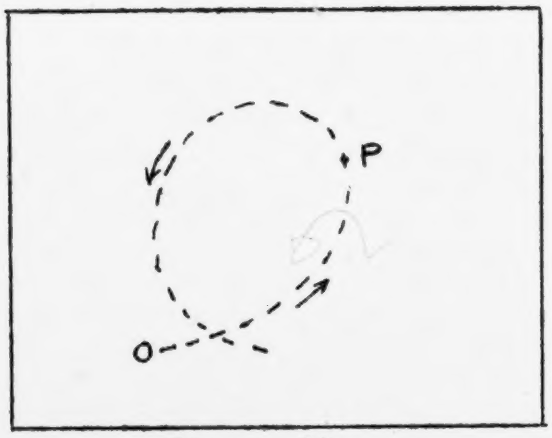


Figure 1b

If we displace the puck to a point O (Figure 1b) and release it with zero initial velocity it will follow a path such as the one shown. This trajectory is recorded on teledetos paper in the usual fashion by means of sparks occurring every 1/60th of a second. From the spark dots we can also find the velocity of the puck at any point along the trajectory.

With this data in hand we can determine whether the total energy of the system (puck plus springs K_1 and K_2) is conserved, i.e., whether it is constant in time. (An equivalent statement would be to determine whether the total energy is the same with the puck at O as it is at any other point, P, on the trajectory.) In order to do this it will of course first be necessary to carefully define what we mean by the total energy of the spring-mass system. This will involve both the kinetic energy of the puck at any point, $E = \frac{1}{2}mv^2$, and its potential energy V which it possesses at this point.* These concepts will be more fully discussed below. Our first experimental investigation will be to determine whether the total energy, $H = E + V$, is conserved.

---

* Actually, the kinetic energy of the springs, which are obviously in motion as the puck moves on its trajectory, should also be added into the total energy of this spring-mass system. We shall, however, neglect this rather small contribution in this experiment.

Our data will also allow us to look at the relationship between the work done on the puck by the springs and the corresponding increase in kinetic energy of the puck. (Note that in this part of the analysis of the data we consider the puck our system and the springs as external forces acting on this system.) Thus, by carefully defining (and then measuring from our data) the work done on the puck as it moves from O to point P under the influence of the two spring forces, we shall be able to ascertain whether the total work done on the puck equals the change in kinetic energy of the puck, i.e., whether $\Delta W = \Delta E$.

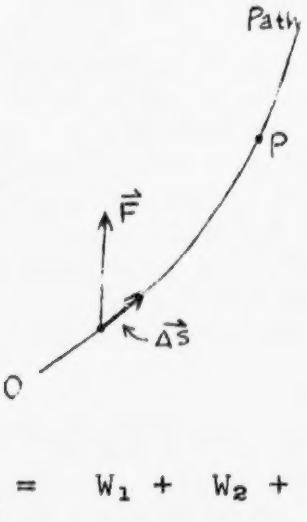
A. Kinetic Energy

A moving body, such as the puck in our experiment, has a kinetic energy $E = \frac{1}{2}mv^2$, where m is the mass and v the speed of the body. In our experiment we may measure this energy as a function of position of the puck along its trajectory by simply measuring the puck velocity at various points. We will see later that it is convenient to plot E versus S, the distance along the trajectory from the point θ, instead of versus the time, t.

It should be noted that kinetic energy, as well as all other types of energy, is a scalar, i.e., it has magnitude but no direction associated with it. Hence, if a system possesses various forms of energy a simple algebraic sum of these gives the total energy of the system. This is quite different, of course, from the vector addition which you needed in Experiment #2 to find the total momentum of the two-puck system.

B. Work and Kinetic Energy

Before considering potential energy it is useful to define the work done by a force $\overline{F}$ on an object such as our puck as it moves along its trajectory. (See Figure 2.) The work done by $\overline{F}$ in a small vector puck displacement of Δs is simply given by



$$W = \overline{F} \cdot \overrightarrow{\Delta s} = F_s \Delta s$$

where F_s is the average component of the force along the direction $\vec{\Delta S}$ and ΔS is a short, straight line segment, along the trajectory. Since the entire curved path can be approximated by such straight line segments, we can find the total work done on the puck as it moves from O to some point P, say, simply by adding the successive contributions:

$$W_{tot} = W_1 + W_2 + \dots = F_{s_1} \Delta s_1 + F_{s_2} \Delta s_2 + \dots = \sum F_{s_1} \Delta s_1$$

It should be noted here that, in general, the tangential force component F_{s_1} will be different for each segment Δs_1. Also, it may be noted that if more than one force is acting, as we have in our experiment since there are two spring forces being exerted, then it is merely necessary to find the tangential component of each force, add these algebraically (some may be negative - i.e., point in the $-\Delta s$ direction), and then multiply by Δs to obtain the work ΔW.

Now we wish to relate the work done $\vec{F}_s \cdot \Delta \vec{S}$ over the short segment to the change in kinetic energy of the puck. We can obtain this from $\vec{F} = \frac{d\vec{p}}{dt}$ as follows. If $\vec{v}$ is the average velocity of the puck over the short Δs segment, then $\Delta s = v \Delta t$. Thus

$$F_s = \frac{\Delta p}{\Delta t} = \frac{\Delta p}{\Delta s/v} = v \frac{\Delta p}{\Delta s}$$

and in the limit as $\Delta s \rightarrow 0$ we have $F_s = v \frac{dp}{ds}$, which can be seen to equal $\frac{d}{ds}(\frac{mv^2}{2}) = \frac{d}{ds}(E)$. Hence $F_s ds = dE$ or the increase in kinetic energy just equals the work done over the small displacement. In the particular case of our experiment this means that the kinetic energy of the puck at P could be found by connecting points O and P by a set of short, straight-line segments approximating the trajectory, finding the average tangential force for each segment and adding the corresponding work increments. This, of course, would be tedious and, instead, we make use of the mathematical fact that from a plot of these tangential forces versus displacement we know the area under the curve from $s = 0$ to $S = Sp$ equals the sum of the infinitesimal work increments.

C. Potential Energy

We can often "store work" in a system and we then say that it possesses potential energy. Thus, suppose we attach our puck to a single spring. Let us do work on this system by applying a force on the puck so as to (slowly) stretch the spring by an amount $r-r_o$ where r_o is the unstretched spring length. (Note that the puck is at rest after the stretching, so that we have given it no net kinetic energy.) The change in potential energy of this system is then defined as the negative of the work done in the stretching, i.e.,

$$V_2 - V_1 = -\Delta W.$$

V_2 is the potential energy of the system with the spring having length r and V_1 is that with a spring length r_o. If we now release the puck this stored work will go over into kinetic energy and, since the work equals the change in kinetic energy of the puck, we have

$$V_2 - V_1 = -(E_2 - E_1),$$

$$\text{or} \quad V_1 + E_1 = V_2 + E_2 = \text{const.} = H.$$

This quantity, kinetic energy plus potential energy, is called the total mechanical energy of the system. It is conserved (i.e., a constant) in all systems where no dissipative or non-conservative forces are present. (Frictional forces are non-conservative.)

Hence, if we measure the total energy of our system at point O, i.e., if we measure the kinetic energy by measuring the puck velocity and measure the potential energy by determining the elongation of each of the springs, then at any later point on the puck's path the total energy will be the same as it was at point O.

Note that the potential energy of a stretched spring is given by $V = \frac{K}{2}(r - r_o)^2$ while the force exerted by such a spring is $\bar{F} = -K(r - r_o)$. The total potential energy of our system for any given position of the puck is merely the scalar sum $V_{tot} = V_1 + V_2$ of the two springs. (See Sections 7-2 and 7-4 in your text.)

Answer the following questions before studying the experimental details which follow. If you are unable to answer these questions you should study the text, Ingard and Kraushaar, Chapters 6 and 7.

Consider Figure 1a. What can you say about the speed of the puck at the point at which the trajectory crosses itself? What can you say about the work the springs perform on the puck as it travels the entire closed trajectory?

II Experimental Procedure

To test the ideas presented in the previous sections you will analyze the motion of the puck-spring system. Because of the complicated nature of the experiment you should study the following instructions carefully.

A. It will be necessary to know the forces exerted by the two springs as well as the potential energies at several points along the puck trajectory. To minimize computation the procedure will be followed of measuring these quantities for several elongations of the two springs, then plotting four graphs from which F_1, F_2, V_1, and V_2 can be read directly for various total spring lengths.

To obtain the force curve for spring 1, for example, merely hang the spring from a support and record directly on a sheet of graph paper the total length r_1 for various weights. Draw a smooth curve through the points and properly label it. It will be necessary to do this also for spring 2 since the two will not be identical.

In order to obtain the energy of the spring versus its extension, we will "graphically integrate" the force curve. You will remember from the section on work, that the work, and hence the potential is the area under the force curve. This point is discussed more fully in Appendix II. Perform this integration and then plot potential energy versus spring length for each of the springs. Do this immediately, as you will be needing these graphs.

B. Release the puck so that it follows a path on the teledeltos paper which crosses itself. Pick ten to fifteen points along this trajectory. To determine the number and spacing of the points apply the criterion that a straight line between any two successive points approximates the puck trajectory between the points.

C. Measure the kinetic energy of the puck at each of your chosen points. Do this carefully since lines which will be drawn on the teledeltos paper in sections D and E will obliterate spark dots and you will have difficulty "re-checking" your kinetic energies later. Plot the kinetic energy as a function of position. Note that position values, S, can be measured with the flexible plastic scales since they can be bent to follow the contours of the trajectory.

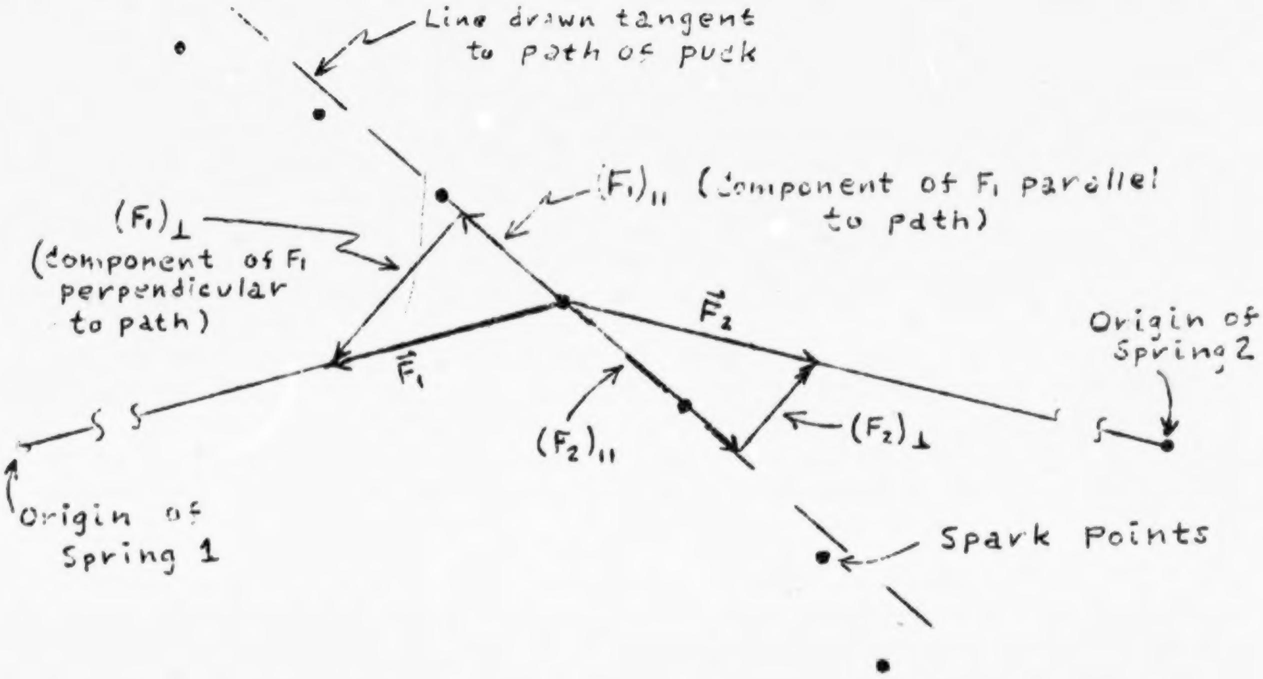
D. Find the net tangential force Fs at each of your points and plot it as a function of S. The procedure for doing this is outlined in Figure 3 in Appendix I. Then, using numerical integration as outlined in Appendix II, find the work performed on the puck by the springs. This should be plotted as a function of S.

E. Find and plot the potential energy of the puck at each point along its path versus S. Remember that the total potential energy is just the sum of that due to each of the springs.

F. You now have three important graphs: kinetic energy, work, and potential energy, each plotted as a function of the distance traveled by the puck along its path. How accurate are your points? Make a plot of the total energy as a function of distance. Is it a constant? Compare the three graphs, remembering the definitions of these quantities. What is the relationship between work and kinetic energy, for example? What is the relationship between work and potential energy? Compare your graphs, taking into account any experimental inaccuracies.

To reduce the amount of calculation involved in this experiment, we have suggested that you take much of your data directly on graph paper. You should thus bring a supply of rectilinear graph paper and a slide rule with you. In addition, you will be drawing many lines on the teledeltos paper, so bring a few colored pencils with you.

APPENDIX I



Line drawn tangent to path of puck
$(F_1)_{\parallel}$ (Component of F_1 parallel to path)
$(F_1)_{\perp}$
(component of F_1 perpendicular to path)
$\vec{F}_2$
$\vec{F}_1$
Origin of Spring 2
$(F_2)_{\parallel}$
$(F_2)_{\perp}$
Origin of Spring 1
Spark Points

Figure 3 We measure the force parallel to the path of the puck. Draw lines from the origins of the two springs to the point of interest. Draw arrows along each of these lines representing the spring forces. Use your triangle to draw the perpendiculars to the tangent line; this will yield the components of the forces perpendicular and parallel to the path of the puck, as indicated above. The total force parallel to the path may now be found by adding $(F_1)_{11}$ and $(F_2)_{11}$. The entire calculation should be done on the teledeltos paper, using colored pencils to prevent confusion. The magnitude of the spring forces may be read from your graphs of spring forces vs spring lengths.

APPENDIX II

Numerical Integration We are interested in calculating the work performed by a force acting on a body which moves some distance. If this distance is very small, we remember that

$$W=\vec{F}\cdot\overrightarrow{\Delta S} = F_{11}\Delta S$$

F_{11} is the force parallel to $\overrightarrow{\Delta S}$.

If the body makes many small displacements, ΔS_1, we see that the total work done in the movement is

$$W= F_{\parallel}(S_1)\Delta S_1$$

This is the area under the graph of F_{11} vs S (S is the total length along the path). See Figure 4

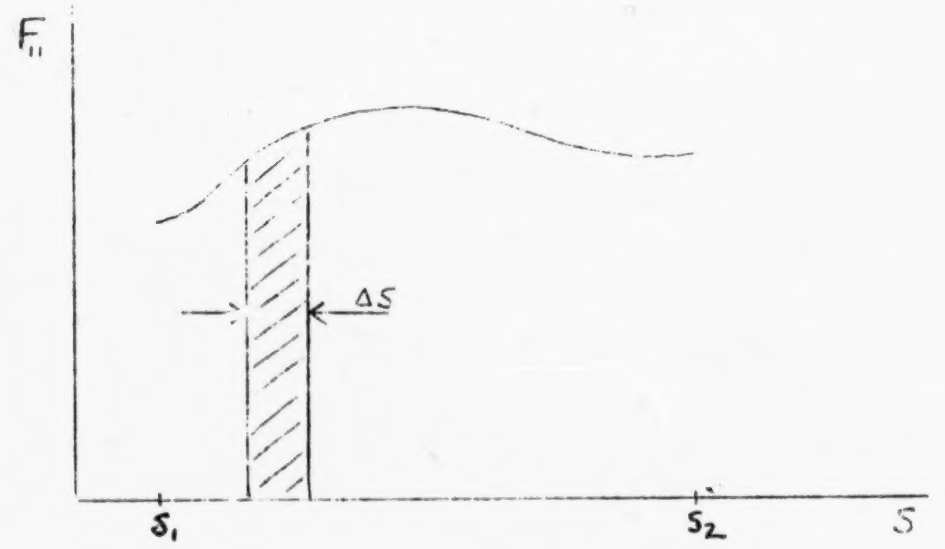


Figure 4

$F_{\parallel}\,\Delta S$ is the shaded area under the above graph. If the particle moves from S_1 to S_2, the integral of $\vec{F}.\vec{ds}$ is the total area under the curve between S_1 and S_2.

We may perform numerical integration by plotting $F_{\parallel}$ versus S on graph paper, and determining the area under the curve. If the figure is a simple geometric curve, eg. a straight line, calculate the area in as simple a way as possible.If the curve is complicated count the squares under the curve.

Be careful with units. What are the units of work and energy? How much work does each square on your graph represent?

VELOCITY WHEEL-DYNAMICAL ANALYSIS

University of Maryland

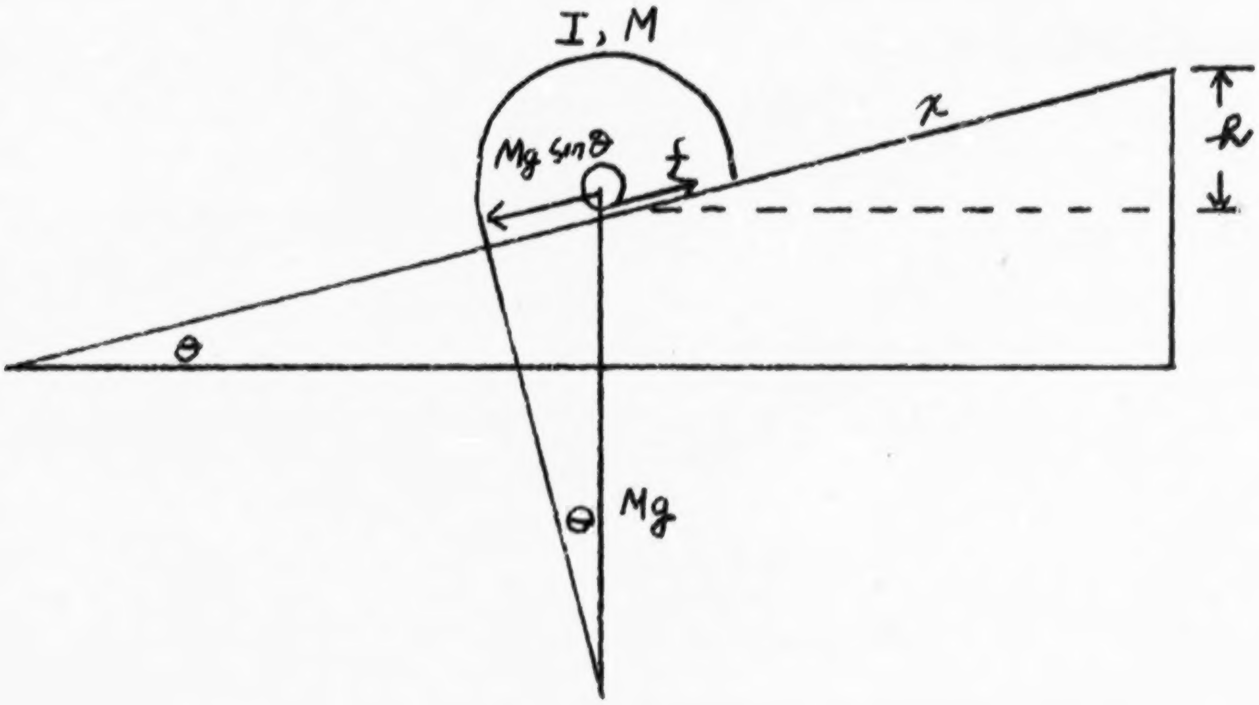
The preceding experiments with the Velocity Wheel apparatus dealt with the kinematics of the wheel's motion. In Experiment IV we found that the acceleration was constant. This acceleration was measured directly by measurements of length and time.

The purpose of this experiment is to obtain a value for the acceleration as predicted by Newton's laws of motion. We will use the second law and the known forces and torques acting on the wheel to predict its acceleration. A comparison of the directly measured acceleration and the acceleration predicted by Newton's laws would constitute a test of the validity of these laws.

I. Derivation of the Acceleration Formula

The acceleration formula can be derived directly from the equation of motion (Newton's second law), or by use of the principle of energy conservation. The quantities affecting the acceleration are:

1) The moment of inertia of the wheel about the turning axis, I
2) The mass of the wheel, M
3) The radius of the axle, r
4) The angle of the incline, θ



There are two forces acting upon the wheel. There is the force of gravity, Mg, which has a component, $Mg \sin\theta$ in the direction of motion. The direction of motion will be taken as positive. There is also a frictional force f, acting to impede the motion. The net force is the difference between these two. Then, according to Newton's 2nd law

$$1)\quad Mg\sin\theta - f = Ma$$

The frictional force must necessarily act at the point of contact between the rail and the axel. It therefore acts at a perpendicular distance r from the turning axis and produces a torque, r x f, on the wheel. In accordance with Newton's 2nd law for rotational motion

$$2)\quad rf = I\alpha = I\frac{a}{r}$$

Solving for f, in equation 2) and introducing the result into equation 1) we have

$$Mg\sin\theta - I\frac{a}{r^2} = Ma$$

Solving for a, we have for our final result,

$$3)\quad a = \frac{g\sin\theta}{1 + I/Mr^2}$$

The same result can be arrived at by making use of energy conservation. The wheel starts at rest. It drops through a distance h in moving a distance x along the incline and acquires a kinetic energy

$$\tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2 = Mgh$$

where

$$\omega = \frac{v}{r}, \quad h = x\sin\theta$$

Solving for v, we obtain

$$4)\quad v^2 = \frac{2gh}{1+I/Mr^2} = \frac{2gx\sin\theta}{1+I/Mr^2}$$

Now we make use of the fact the wheel is in constantly accelerated motion. Then,

$$v = at \quad ; \quad x = \tfrac{1}{2}at^2$$

Upon substitution into equation 4) we have

$$5)\quad a^2t^2 = \frac{2g(\tfrac{1}{2}at^2)\sin\theta}{1+I/Mr^2}$$

or

$$a = \frac{g\sin\theta}{1+I/Mr^2}$$

II. Determination of the Moment of Inertia

We will determine the moment of inertia of the wheel by measuring its frequency of oscillation in a torsion pendulum. In this arrangement, the axel of the wheel is rigidly coupled to a steel wire and is suspended by the wire so that it can rotate in the horizontal plane. In accordance with Hooke's law, the wire produces a torque on the wheel,

$$L = -C\theta$$

where C in the rorsional constant of the wire and θ is the angular displacement. Then, According to Newton's 2nd law

$$L = I\alpha = -C\theta$$

or

$$\frac{d^2\theta}{at^2} = -\frac{C}{I}\theta$$

The wheel will execute simple harmonic angular motion with a frequency

$$6)\quad f = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$$

Thus, if we knew C we could immediately calculate I. Rather than determining C directly we can calibrate with a disc of known moment of inertia. We will use a solid disc of uniform thickness. The moment of inertia of our standard is given by

$$7)\, I_o = \tfrac{1}{2} M_o R_o^2$$

Where M_o = Mass of disc
R_o = radius of disc

The frequency of the torsion pendulum with the standard is

$$8)\quad f_o = \frac{1}{2\pi}\sqrt{\frac{C}{I_o}}$$

Dividing equation 7) by equation 8) and squaring, we have

$$\frac{f}{f_o} = \frac{\frac{1}{2}\pi\sqrt{\frac{C}{I}}}{\frac{1}{2}\pi\sqrt{C/I_o}} = \sqrt{\frac{I_o}{I}}$$

$$9)\quad I = I_o\left(\frac{f_o}{f}\right)^2$$

Using equation 9) we can calculate the moment of inertia of our wheel from measured values of I_o, f and f_o.

III. Procedure

Obtain the same wheel and rail set used for the previous experiments

A. Measure the angle θ. Measure the distance between front leveling screws and rear contact point. The height of the step is exactly 2 centimeters. The appropriate ratio of these distances is the tangent of θ. Estimate the error in θ.

B. Measure the radius of the axel with a micrometer. The teaching assistant will instruct you in the use of the micrometer. Use the precision of the micrometer as an estimate of the error.

C. Weigh the wheel and the standard disc and determine the sensitivity of the balance in each weighing. Measure the radius of the standard disc.

D. Determine the frequencies f and f_0 of the torsion pendulum for the wheel and the standard disc respectively. Measure the time for 25 complete oscillations and from this, calculate the frequencies. Make at least two trials for each determination.

Calculate the moment of inertia of the wheel using equations 7) and 9). The error in I is given by

$$\Delta I = \Delta I_0 \left(\frac{f_0}{f}\right)^2 = \frac{1}{2}\left(R_0 \Delta M_0 + 2 M_0 \Delta R_0\right)\left(\frac{f_0}{f}\right)^2$$

The Errors in the frequencies should be negligibly small.

Substitute values of I, r, M, and θ into equation 3) and compute the acceleration in centimeters per second per second. Compute the error in a as follows:

$$\Delta a = a - \frac{g \sin(\theta - \Delta\theta)}{1 + \dfrac{I + \Delta I}{(M - \Delta M)(r - \Delta r)^2}}$$

where the errors $\Delta\theta$, ΔM, Δr, and ΔI are taken as the estimated error in each measured quantity. Compare this result

$$a \pm \Delta a$$

with the result obtained in Experiment IV. Comment.

ROTATIONAL DYNAMICS

The Florida State University

Purpose:

To study the rotational motion of two geometrically identical discs of different mass under the action of constant torque. The study will utilize both Newton's Second Law and the Principle of the Conservation of Energy.

Apparatus:

Rotational dynamics apparatus with steel and aluminum discs; 60 cycle alternating voltage supply for the impact timers; red-pressure-sensitive tape for recording time intervals; string, weight hanger and weights; six-inch ruler. A pan balance and weights should be available.

The basic apparatus consists of two discs mounted coaxially within a rugged frame; two impact timers mounted on the frame so the hammers strike the discs; and two pulleys, one mounted on the frame, and one on which the lower disc rides. The lower disc is easily changed: to remove it, lift the top disc and remove from beneath. The upper disc will not be used in the first part of the experiment.

Pressure sensitive tape can be mounted on either disc by pushing one end into the slot on the edge of the disc with the help of a card or piece of thin plastic. Wrap the tape around the disc, red side out, and force the other end into the slot.

One of the two buttons on the 60 cycle supply box will activate the lower impact timer, producing red marks on the "white" side of the tape at intervals of 1/60 sec. With practice you will be able to obtain records of rotational position versus time for almost exactly one revolution. Be sure not to take records extending over more than one revolution. (If the apparatus is adjusted so that the falling weight strikes the floor just before the disc completes one revolution, the noise may help you time the switch.)

A knot in the end of a string will catch in the slot of the pulley holding the lower disc, so that the string can be wound up on this pulley, extended out over the small nylon pulley on the frame and used to support a weight hanger and weights.

Part I:

For a range of different masses, m, on the weight hanger, take records of rotational position versus time for the steel disc. Plot the curves of the linear velocity and the position versus time; lable the curve carefully so it is obvious exactly what physical quantities are being plotted. Plot these points in such a way that the velocity and position points actually correspond in time.

Questions:

1) How can the plotting be done so that velocity and position points correspond?
2) What is the slope of the velocity curve, physically and numerically (with errors)?

From these results, plot a curve of 1/m versus 1/a, where a is the acceleration.

Questions:

3) What is the slope of this acceleration curve, physically and numerically?
4) What is the intercept on the a axis?
5) If this curve is not a straight line, does it curve in a direction consistent with the assumption that friction is present? If so, what part of the curve should be used to obtain an estimate of the moment of inertia of the disc?
6) Find the moment of inertia of the disc and estimate the error.

Part II:

From the data in Part I, plot distance versus the square of the velocity. Answer 3, 4, and 5 of Part I with regard to this curve. Be sure to state carefully the principles used in any calculations you use.

Part III:

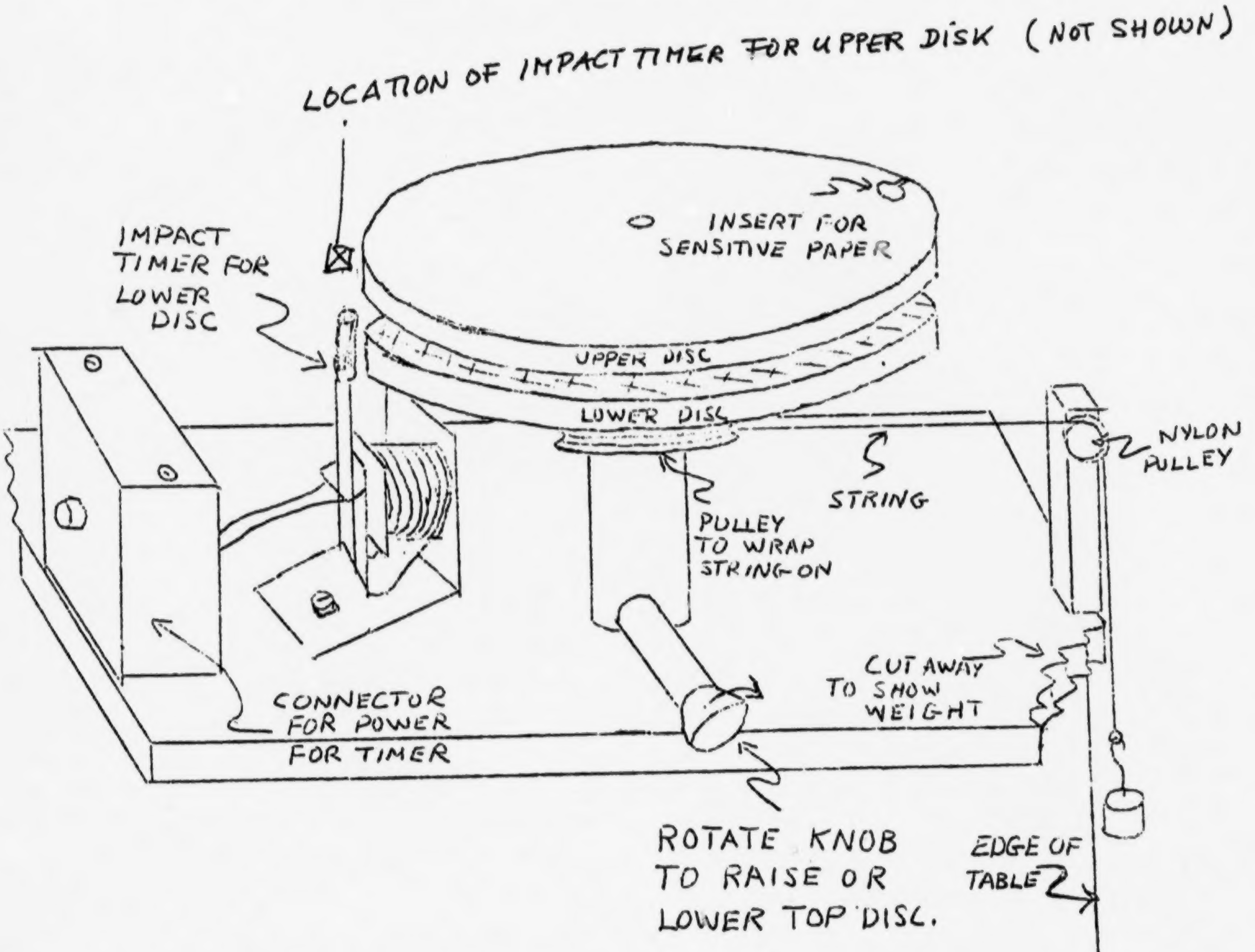
Repeat Part I with the upper disc resting on the lower and then with the aluminum disc alone.

Question:

Is moment of inertia an additive property? If the geometry is fixed, is it proportional to the mass?

WARNING:

Be sure you take all auxiliary measurements necessary for the interpretation of these experiments.


LOCATION OF IMPACT TIMER FOR UPPER DISK (NOT SHOWN)
IMPACT TIMER FOR LOWER DISC
INSERT FOR SENSITIVE PAPER
UPPER DISC
LOWER DISC
NYLON PULLEY
STRING
PULLEY TO WRAP STRING ON
CUT AWAY TO SHOW WEIGHT
CONNECTOR FOR POWER FOR TIMER
ROTATE KNOB TO RAISE OR LOWER TOP DISC.
EDGE OF TABLE

Object: To study the motion of the bob of a spherical pendulum by obtaining a space-time record of this motion. To analyze this space-time record and then show: (1) that angular momentum of the pendulum about its vertical axis is conserved and (2) that the orbit of the pendulum bob is an ellipse, provided that the maximum angular displacement of the pendulum is small ($<3°$).

Apparatus: A pendulum consisting of a long suspension wire (6-8 ft) attached to a spherical bob (1–2 lb), with a sparking electrode attached to the bottom of the bob; an adjustable plane table with leveling screws, recording paper (Teledeltos), spark timer, and connectors.

Theory: A simple pendulum is usually constrained to move in a fixed vertical plane. The path of the bob is, therefore, a circular arc whose radius is the length of the suspension. In this case the pendulum is said to have one degree of freedom, *i.e.*, to describe its position requires the use of only one variable. If we remove the constraint of motion in a fixed vertical plane for the simple pendulum, it then becomes a spherical pendulum. The path of the bob is now some sort of a curve on a spherical surface whose radius is the length of the suspension. The pendulum has two degrees of freedom, *i.e.*, to describe its position now requires the use of two variables. You should convince yourself of the validity of these statements by finding the position variables in each of the above cases. There are many ways of doing so, but no matter how it is done, one finds that it requires only one position variable for the simple pendulum and two position variables for the spherical pendulum.

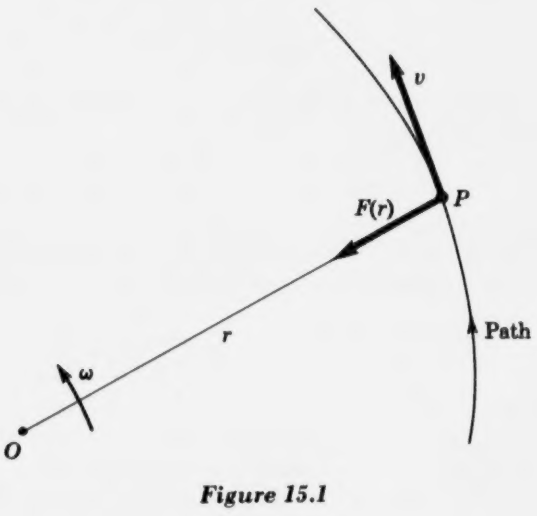


Figure 15.1

Motion about a center of force. The motion of the pendulum bob in this experiment is a particular example of a more general type of motion, one of great importance in many branches of physics and astronomy. This is the motion of a particle about a center of force.

Suppose a particle of mass m moves under the action of a single force F that is always directed toward or away from a fixed point O in space. The magnitude of the force may be any function of the distance r between the fixed point O and the instantaneous position P of the particle. Let the instantaneous velocity of the particle at point P be v. This velocity will be directed along the tangent to the path of the particle. See Fig. 15.1. What can we say about the motion of the particle moving under these prescribed conditions? Newton's laws of motion enable us to draw two conclusions about the motion. They are:

1. The path of the particle must lie in the plane defined by the radius vector r and the velocity vector v, *i.e.*, a plane through point O that includes both r and v. If the velocity vector v happens to lie along r, then, as a special case, the path of the particle must lie along the line OP. The validity of these statements is based on the observation that the acceleration of the particle must lie along the line OP. You should supply the additional details of the proof.

2. The angular momentum of the particle $mr^2\omega$ about an axis through point O and perpendicular to the plane of motion must remain constant. Here, ω is the angular velocity with which the radius vector r is turning about the axis through O. The validity of this statement is based upon the observation that the force F, acting upon the particle, never exerts any torque on the particle about the axis through O. Here, again, you should supply the additional details of the proof.

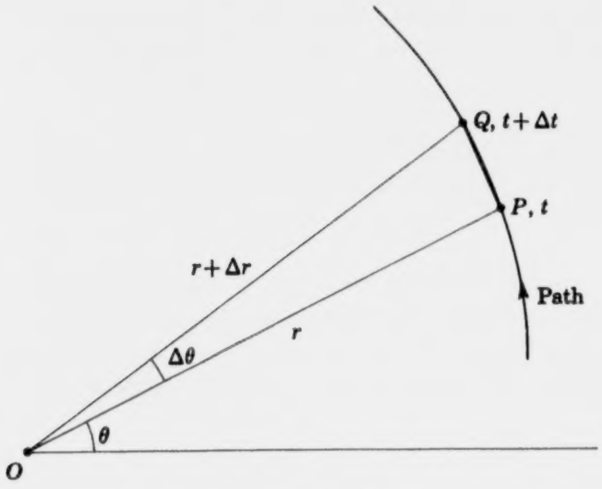


Figure 15.2

Since the mass of the particle is constant, we conclude that the particle must move in a plane about the point O in such a manner that $r^2\omega$ remains constant. This is equivalent to saying that the particle moves so that its radius vector r sweeps out equal areas in equal times. The proof of this is given in the following section.

In Fig. 15.2 let P be the position of the particle at time t. At this time, its radius vector is r, making an angle θ with some base line in the plane of motion. At time $t + \Delta t$, the position of the particle is Q. Its radius vector has increased in magnitude to the value $r + \Delta r$, and makes an angle $\theta + \Delta\theta$ with the base line. We assume that the time interval Δt is sufficiently small, thus the actual path of the particle in this small time interval very nearly coincides with its displacement PQ. The area ΔA swept out by the radius vector in time Δt is thus the area of the triangle OPQ. This area, by trigonometry, is equal to $\frac{1}{2}(r + \Delta r)(r) \sin \Delta\theta$. For small values of Δr and $\Delta\theta$, this expression reduces to $\frac{1}{2}r^2\Delta\theta$. Therefore

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \frac{\Delta\theta}{\Delta t} = \frac{1}{2} r^2\omega = \text{constant.} \tag{15.1}$$

This result will be recognized as Kepler's second law for planetary motion. Actually, it is a much more general law and holds for the motion of any particle moving about a center of force.

Orbit of the particle. It is impossible to determine theoretically the actual path of the particle (orbit) until we know how the force F on the particle depends on the distance r. In a great many cases (gravitational and electrical) the force is inversely proportional to the square of the distance r. In these cases it may be proved (the proof requires a knowledge of differential equations) that the orbit is a conic section (ellipse, parabola, or hyperbola) with the point O at one of the foci.

In another set of cases (elastic and mechanical) the force F is directly proportional to the distance r, and is a force of attraction. In these cases the orbit of the particle is always an ellipse (or one of its degenerate forms) with point O at the center. The motion is periodic; the period of the motion T is given by the well known relation

$$T = 2\pi \sqrt{\frac{m}{K}} \tag{15.2}$$

where K is the force constant, *i.e.*, F/r. Thus if the motion of the particle in its elliptical orbit is projected onto any straight line lying in the plane of the ellipse, the projected motion will always be simple harmonic motion. And finally, if rectangular coordinate axes are chosen so that they coincide with the major and minor axes of the ellipse, then the coordinates of the particle at any time t may be written

$$\left.\begin{aligned} x &= a \cos 2\pi \frac{t}{T} \\ y &= b \sin 2\pi \frac{t}{T} \end{aligned}\right\}. \tag{15.3}$$

Equations (15.3) are the parametric equations of an ellipse with semi-major axis a and semi-minor axis b. The period of the motion is T, *i.e.*, the time for the particle to make one complete circuit around the ellipse. The angle $2\pi \frac{t}{T}$ is the eccentric angle of the ellipse. By eliminating this angle in Eqs. (15.3), we obtain the standard equation for the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \tag{15.4}$$

The bob of a spherical pendulum has the motion just described, provided that its maximum angular displacement from the vertical is not too large. The argument is the same as that for a simple pendulum, *i.e.*, the displacement must be small enough to make the restoring force on the bob very nearly proportional to its displacement. It follows that the period of the spherical pendulum is equal to the period for the simple pendulum of the same length. Furthermore, its orbit is essentially a combination of two linear harmonic motions of the same period at right angles to one another. Equations (15.3) are merely mathematical statements of these facts.

Space-time record. In this experiment we verify the theoretical conclusions concerning the motion of a spherical pendulum. To do this, we obtain a space-time record of the motion of the bob (its center) on a sheet of recording paper placed just below it. The record produced by the spark timer consists of a set of dots, equally spaced in time, whose locus is the orbit of the bob.

An analysis of this record should show (1) that the motion of the bob is essentially motion about a center of force, and (2) that the orbit of the bob is an ellipse.

Method: Plaçe the tray (table) containing the conducting paper in its bottom immediately below the bob of the spherical pendulum. When the bob is at rest, the sparking electrode on the bottom of the bob should clear the paper by about 1 mm and should be near the center of the paper. Level the table. Make the necessary electrical connections and turn on the power for the spark timer so that it may warm up before being used.

Make a trial run without using the spark timer. Pull the bob of the pendulum back near the edge of the tray. In releasing the bob give it a moderate lateral push so that it moves over the paper in an elliptically shaped path, not a circle. The minor axis of the orbit should be about one-half to two-thirds that of the major axis. The bob should move smoothly around its orbit without any wobbling motion. At no point in the path should the sparking electrode actually touch the paper. Once you have succeeded in putting the bob into a satisfactory orbit, depress the sparking switch on the spark timer for one complete revolution of the bob. A small amount of overlap is not serious. Without moving tray or paper, bring the bob to rest, allowing it to hang in its normal equilibrium position. Again, use the spark timer to get the center of the orbit, a matter of considerable importance in the analysis. Repeat the above procedure with a new sheet of conducting paper. Each student will then have one space-time record to analyze.

There are several different ways of analyzing the data. We give one possible method. You may wish to use a different method, and may do so with the instructor's permission.

The record of motion of the bob in one complete period of motion is given on the paper as a set of dots lying along the orbit of the bob. The dot intervals correspond to equal time intervals (usually $\frac{1}{60}$ sec). In addition, the center of the orbit is given. The method of analysis that we describe requires the use of a particular set of rectangular axes; namely, one that has the x axis lying along the major (long) axis of symmetry of the orbit, and the y axis along the minor (short) axis of symmetry. These are not given directly on the record and therefore must be constructed. In order to construct the x axis draw a circle about the center of the orbit that has a diameter somewhat smaller than the approximate major axis of the orbit. This circle will intersect the orbit at four points, two of which lie near the right end of the major axis. Draw two lines from the center to these two points of intersection. Bisect the central angle formed by these lines. Use this bisector as the x axis. It should (very nearly) coincide with the major axis. Draw the y axis perpendicular to the x axis at the center of the orbit (the origin). This work must be done with great care since the verification of the elliptical nature of the orbit amounts to checking Eq. (15.4), which in turn assumes that the coordinate axes coincide with the major and minor axes of the ellipse. If they do not coincide, then Eq. (15.4) is transformed (by rotation of the axes) into a much more complex equation.

On the other hand, we need not worry about the orientation of the axes so far as areas are concerned, since these will be invariant under any rotation of the axes.

After having drawn in the axes, the next step is to choose convenient pairs of points on the orbit that represent equal time intervals, but that are still close enough together so that the orbit between any two points forming a pair is essentially a straight line. Of course, the original dots on the record satisfy this condition, but they are usually so close together and so numerous that an analysis using all of them would be exceedingly tedious. Further, it is only necessary to use pairs of points in the first quadrant since the orbit should be symmetrical with respect to the x and y axes. A convenient choice of pairs of points in this case may be 4-, 5-, or 6-dot intervals. Start near the end of the major axis (there may be no dot at the end). Choose a well defined dot on the orbit as a starting point. Label it 1. Make the chosen count (4, 5, or 6) along the orbit and label the last dot 2. Repeat the process, starting at dot 2 and labeling the last dot 3. Continue this process until you have virtually covered that part of the orbit lying in the first quadrant (see Fig. 15.3).

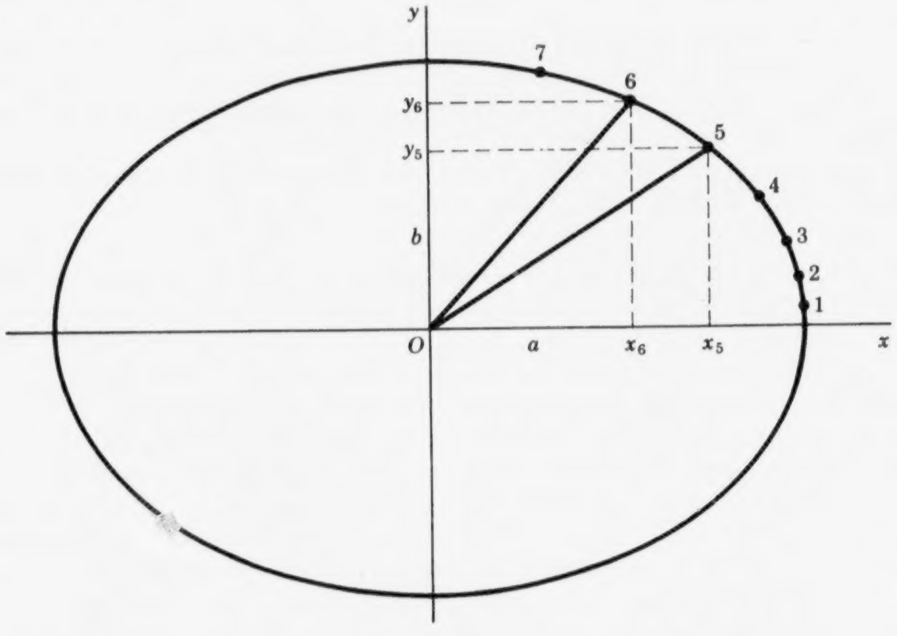


Figure 15.3

Determine the x and y coordinates of each of the labeled dots by direct measurement. For this procedure, it is most convenient to use a drawing board with a T-square and right triangle. Tabulate the results.

In order to verify the equal areas in equal times theorem, determine the area of each of the alternate triangles 012, 034, 056, and so on as shown in Fig. 15.3. Note that these triangles are completely independent, *i.e.*, they have no common side. The area of each triangle, *e.g.*, 056, may be computed most conveniently by use of the expression $\frac{1}{2}\,|\,x_5y_6 - x_6y_5\,|$ where x_5, y_5 are the coordinates of dot 5, and x_6, y_6 the coordinates of dot 6. Tabulate the areas of these triangles. They should be the same within the limits of experimental error. Estimate the approximate indeterminate error in at least one of the area determinations.

In order to verify the elliptical nature of the orbit, calculate the value of

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}$$

for each of the labeled dots. The value of this expression should be 1 for any point on the orbit, if the path is an ellipse and if the axes are properly chosen. Estimate the indeterminate error in the expression for one of the labeled dots.

Record: Tabulate your data and results in a systematic manner, using whatever method of analysis you wish. Do this directly on the recording paper if sufficient space is available.

QUESTIONS

1. Estimate the determinate error introduced into the value of the expression

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}$$

if the coordinate axes are out of alignment with the axes of the ellipse by 1°.

2. Show that the theoretical area of any one of the elliptical sectors that you have determined directly should be

$$\pi ab\,\frac{\Delta t}{T}$$

where Δt is the chosen time interval (calculus required). Compute this value and compare it with your measured values.

3. A more detailed analysis of the motion of a spherical pendulum shows that the orbit of the bob is not precisely a stationary ellipse, but rather one whose major and minor axes slowly rotate about the point O. How would you explain this in terms of the dependence of the period of a simple pendulum upon its amplitude of vibration?

OPTIONAL EXPERIMENTS

1. There are graphical methods of showing that the orbit in this experiment is an ellipse, methods that do not require construction of the major and minor axes. One of these methods is to draw six tangents to the ellipse so that a six sided polygon that circumscribes the ellipse is formed. The polygon need not be a regular one of equal sides. If the vertices of this polygon are labeled consecutively 1, 2, 3, 4, 5, 6, then lines drawn between points 1 and 4, 2 and 5, 3 and 6 will intersect at a common point. Try this construction.

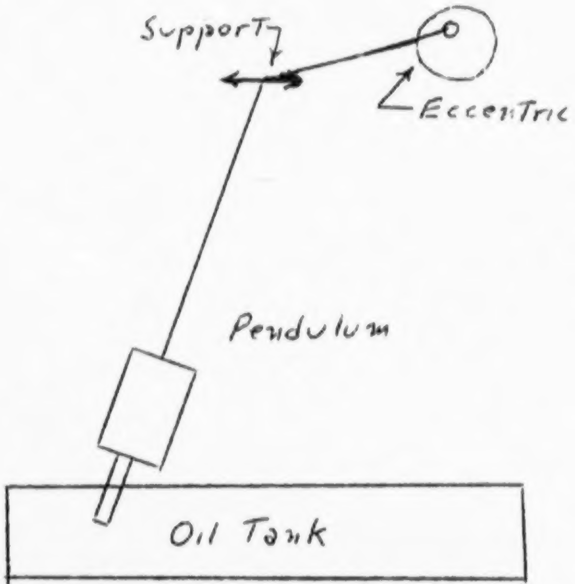
2. Let major and minor auxiliary circles be drawn about point O, the former having radius a and the latter radius b. Project the recorded dots on the orbit, vertically (parallel to the y axis) onto the major circle and horizontally (parallel to the x axis) onto the minor circle. These projected points should be distributed uniformly around each circle. Explain.

MECHANICAL RESONANCE

Stanford University

Purpose: To investigate the phase and amplitude of forced harmonic motion as a function of driving frequency and damping.

Procedure: The apparatus is illustrated schematically in figure 1. The support of a pendulum is moved horizontally by an eccentric connected to a variable speed motor. The damping may be varied by changing the depth to which the small rod (on the bottom of the pendulum bob) is immersed in the oil tank. The variable speed drive unit is composed of a constant speed motor and a variable speed motor coupled together by a differential unit. The output shaft speed is the algebraic sum of the speeds of the individual motor units. The maximum speed variation at the output is about 5%. Provision has been made for accurately measuring the rotation frequency of the variable speed unit, while that of the constant speed unit is given. The oscillation amplitude is measured by a mirror on the pendulum, a light beam and a scale. The relative phases of the pendulum and the excentric drive can also be compared.



Measure the amplitude and phase on a function of drive frequency for the cases of the small rod out of the oil, and immersed in the oil.

Derive the equation of motion of the pendulum (for small amplitude oscillations) and solve for the equilibrium amplitude and phase as a function of the driving frequency and amplitude, the frequency of free oscillations and the damping constant. Compare your results with your theoretical predictions.

With the small rod out of the oil, and for one drive frequency close to resonance, measure the oscillation amplitude as a function of time starting with the pendulum at rest. The oscillation amplitude will vary with time in a manner which depends on the driving frequency, the resonant frequency, and the damping constant. Derive the transient equation of motion for the amplitude and determine the resonant frequency and damping constant. Compare these results with those obtained by measuring the equilibrium amplitude versus frequency.

If you have time, measure the amplitude-time relation with the rod in the oil.

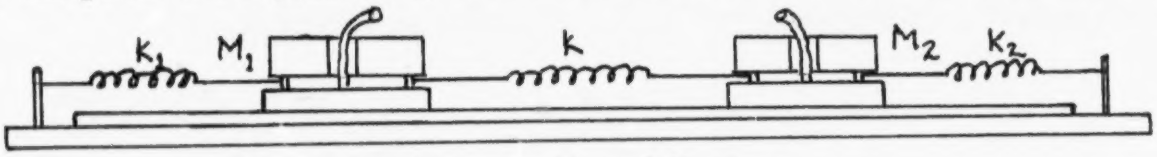
COUPLED LINEAR OSCILLATOR

University of Minnesota, Duluth Branch

Object: to test the solution for the principal modes of oscillation of coupled linear oscillators and to study the phenomenon of beats.

Apparatus. A small stand with a mirror scale and a weight pan is used for the determination of the force constant of each coil spring. The springs come in sets of 3 in each set. A set of springs is marked by a set of red dots on the loop at one end of the spring. The springs of a set each have the same number of red dots.

Two air pucks are set up with coil springs as shown below. When the air is turned on, the pucks are supported by a film of air and move in a friction-free manner.



Theory. The equations of motion for M_1 and M_2 are:

$$(1)\quad M_1\ddot{x}_1 + k_1x_1 + k(x_1 - x_2) = 0$$

$$(2)\quad M_2\ddot{x}_2 + k_2x_2 + k(x_2 - x_1) = 0$$

Where (x) represents the linear displacement of the masses and (k) the force constant of the respective springs.

The solution of these two linear homogeneous second order differential equations will be greatly simplified by taking the special case where $M_1 = M_2 = M$ and $k_1 = k_2 = k_o$.[1,2] This is a very symmetrical case for which the equations of motion are:

$$(3)\quad M\ddot{x}_1 + k_ox_1 + k(x_1 - x_2) = 0$$

$$(4)\quad M\ddot{x}_2 + k_ox_1 + k(x_2 - x_1) = 0$$

By addition and then subtraction of equations (3) and (4) the following equations are obtained:

$$(5)\quad M(\ddot{x}_1 + \ddot{x}_2) + k\,(x_1 + x_2) = 0$$

$$(6)\quad M(\ddot{x}_1 - \ddot{x}_2) + (k_o + 2k)(x_1 - x_2) = 0$$

These two equations may be simplified by introducing new coordinates:

$y_1 = (x_1 + x_2)$ $y_2 = (x_1 - x_2)$ and substituting these into equations (5) and (6).

$$(7)\quad M\ddot{y}_1 + k_oy_1 = 0$$

$$(8)\quad M\ddot{y}_2 + (k_o + 2k)y_2 = 0$$

Since the restoring force will be governed by Hook's law; i.e., $F = kx$, M_1 and M_2 may be expected to execute simple harmonic oscillations. Therefore let:

$y_1 = A_1\sin\omega_1t + B_1\cos\omega_1t$ and $y_2 = A_2\sin\omega_2t + B_2\cos\omega_2t$ for a solution of equations (7) and (8). The substitution of y_1 in equation (7) yields an explicit expression for ω_1:

$$-M\,\omega_1^2(A_1\sin\omega_1 t + B_1\cos\omega_1 t) + k_o(A_1\sin\omega_1 t + B_1\cos\omega_1 t) = 0$$

$$-M\,\omega_1^2 + k_o = 0; \text{ thus } \omega_1 = \sqrt{k_o/M}\,.$$

This is the case in which M_1 and M_2 each move in the same direction; i.e., in phase, with equal amplitudes and the coupling spring (k) is not elongated. The substitution of y_2 in equation (8) yields in a similar manner an explicit expression for ω_2: thus $\omega_2 = \sqrt{(k_o + 2k)/M}$.

This is the case in which M_1 and M_2 move in opposite directions; i.e., 180° out of phase, with equal amplitudes. The two angular frequencies, ω_1 and ω_2, are known as the principal modes of oscillation for coupled linear oscillators.

A brief intuitive consideration of the general solution for this special case, where $M_1 = M_2$ and $k_1 = k_2$, leads right into the phenomenon called "beats." Let the oscillations of the system be established by an initial displacement of M_1 from its equilibrium position a distance x_o. The general solution of the system will be:

$$x_1 = \tfrac{1}{2}(A_1\sin\omega_1 t + B_1\cos\omega_1 t + A_2\sin\omega_2 t + B_2\cos\omega_2 t)$$

$$x_2 = \tfrac{1}{2}(A_1\sin\omega_1 t + B_1\cos\omega_1 t - A_2\sin\omega_2 t - B_2\cos\omega_2 t)$$

At $t = 0$: $x_1 = x_o$, $x_2 = 0$, $\dot{x}_1 = 0$, $\dot{x}_2 = 0$ which reduces the general solution to:

$$x_1 = \tfrac{1}{2}x_o(\cos\omega_1 t + \cos\omega_2 t) = x_o\cos\tfrac{1}{2}(\omega_1 + \omega_2)t\,\cos\tfrac{1}{2}(\omega_1 - \omega_2)t$$

$$x_2 = \tfrac{1}{2}x_o(\cos\omega_1 t - \cos\omega_2 t) = x_o\sin\tfrac{1}{2}(\omega_1 + \omega_2)t\,\sin\tfrac{1}{2}(\omega_1 - \omega_2)t$$

These equations show that x_1 and x_2, under these particular initial conditions, will be a function of two angular frequencies, $(\omega_1 + \omega_2)$ and $(\omega_1 - \omega_2)$; and that there will be a 90° phase difference in the motion of M_1 and M_2. The amplitude of the higher angular frequency, $(\omega_1 + \omega_2)$, will vary at the rate $(\omega_1 - \omega_2)$. This particular type of motion of the masses is known as "beats."

Procedure.

Springs k_1 and k_2 should be the most identical of the three springs. Place a 2 kg mass on each puck and check to see that $M_1 = M_2$ to within .1% of M_1. The bottoms of the air pucks and the surface of the plate glass must be CLEAN.

1. Carefully measure the force constants of each of the springs: k_1, k_2, and k_3. Use the method of differences to find the average values of the spring constants.
2. Carefully measure the frequencies of oscillation of the principal modes of oscillation, f_1 and f_2, and if time permits the frequencies of the "beats" f_b.

Write-up.

Compare the values of ω_1 and ω_2 obtained from the measured values of f_1 and f_2 with the values of ω_1 and ω_2 computed as a function of (M) and (k). If you measured f_b, then compare ω_b with $(\omega_1 - \omega_2)$. A complete analysis of the probable errors in the results should be made and compared with the deviations between the directly measured and the computed values.

1. Pipes, L. A.; Applied Mathematics for Engineers and Physicists; pp. 172-177.
2. Broxon, J. W.; Mechanics; pp. 367-375.

MOTOR DRIVEN GYROSCOPE

Massachusetts Institute of Technology

1. What the Gyroscope is: The gyroscope is a rotating wheel which is mounted in a set of supports called gimbals so that its axis of rotation is free to point anywhere in space. It is pictured in Figure 1. The wheel, or rotor, is driven at a constant angular frequency of 150 revolutions per minute by an electric motor (what is ω in radians per second?). The gimbals are free to rotate about the center lines shown. The angular momentum of the rotor is in the direction defined by the metal "arrowhead" on the axle (direction of angular momentum is explained in Section 4). The gimbals are so constructed that all forces acting on the rotor due to gravity and the supports pass through center of mass of the rotor and do not cause any rotation. Any slight unbalance of the rotor can be detected by observing the rotor WITH THE MOTOR OFF to see if the axle remains horizontal. If it is not, ask your instructor for aid. Now, with this adjustment properly made, line up the two outer gimbal rings and tighten the setscrew to hold the inner ring while starting up the motor. You may release the ring by loosening the screw when the rotor is up to speed. The motor is now just compensating for the frictional effects on the rotor, and it will run at constant speed.

2. What the Gyroscope Does: The gyroscope, as an instrument, was first developed by Jean Foucalt around 1850. It was also incorporated into a geographical north seeking compass by Elmer Sperry around 1910, and has been recently used for fire control and inertial guidance by Charles S. Draper in the M.I.T. instrumentation Laboratory.

To discuss its operation, first let's degress for a moment to discuss three increasingly complicated examples of particle forces and motion.

I. When no forces act on a particle, it will move in a straight line (with respect to "inertial" space) with constant momentum (magnitude and direction).

II. When a particle is acted on by a force of constant magnitude and direction (such as gravitational force near the earth's surface) it moves in a parabolic path. Its momentum is not constant.

III. When the force is of constant magnitude, but continually changes its direction, when the body does, so as to remain always perpendicular to the momentum, we obtain circular motion. The momentum will be of constant magnitude but its direction will constantly change in space. An example of such a force is that provided by a string attached to the mass so as to move with it and this change the direction of the applied force appropriately.

When we observe the gyro, we note that when no external forces act in maintains its axle fixed in space. Foucault used this property (as well as his celebrated pendulum experiment) to demonstrate the earth's rotation.

Now, when we wish to discuss the effects of an external force on the rotor, we will simplify the discussion by considering the effect of an impulse of force for a short time. IN THE FOLLOWING DIAGRAMS THE GIMBALS AND SUPPORTS ARE NOT SHOWN FOR SIMPLICITY ONLY.

Figure 3 shows the rotor, and the momenta of 4 typical mass elements within the rotor are shown by the vectors $\overline{p}$. At a later moment these four mass elements will have moved to a new position in their circular path, impelled by the forces from the remaining elements of the rotor, since it is a rigid body. These internal forces act through the center of rotation of the wheel, and are not considered in what follows. The axis of the inner gymbal ring is shown by the center lines in these diagrams.

If we apply an impulse to the axle (by striking it a sharp blow, for example) an impulse will act on each element off the inner gimbal axis (why?) and in general these impulses are paralled to the axle. Two typical mass-element impulses are shown in Figure 4. The result of this impulsive force in not to lower the tip of the axle (in this simplified model) but instead, it rotates the direction of the plane of the rotor. If you actually go into the laboratory and hit the tip of the axle, the rotor will not only rotate its plane slightly as shown, but will also oscillate up and down. This oscillation is called "nutation", and we have not considered it in this simplified discussion.

Now, note Figure 5, which shows the rotor after the impulse. If we wish to continue an action similar to Case II of the particle motion examples, we must give the gyro another impulse AT THE SAME POINT IN SPACE (close to the point marked a in Figure 5) AND NOT AT THE TIP OF THE AXLE, WHICH HAS MOVED SINCE THE APPLICATION OF THE LAST IMPULSE. Note that, since the plane of the rotor has moved, we are now hitting nearer the gimbal axis and off the spin axle. The effect of our second impulse will be to rotate the plane of the rotor somewhat further (although not as far as the first impulse) and also to increase the spin of the rotor. As we continue this process the axle gets closer and closer to a position which is at right angles to its original position, and the spin gets faster and faster. Compare this to particle motion, Case II, where the particle velocity gets closed and closer to being at right angles to its original direction, and the velocity becomes greater and greater. This process, of applying a force at a point fixed in space even though the gyro axle tilts, is interesting to contemplate, BUT IS NOT WHAT IS DONE IN LABORATORY.

In the laboratory we will hang a weight on the end of the axle so that when the axle moves, the wieght moves also, and thus the point of application of the force moves with the gyroscope. This is analgous to Case III of particle motion, where the string moves when the particle moves, thus altering the direction of the applied force as the body moves.

Consider Figures 4 and 5 once more. If we move the point of application of the force so that it is always at the tip of the axle, the gyro axle tip will continue to move in a circle, at a constant rate. This motion is called "precession". Note that the spin of the rotor about its own axle is not altered in this case. This is analgous to Case III of particle motion, where, since the force is perpendicular to the velocity ("no-Work" force) it alters the direction (but not the magnitude) of the momentum.

3. Quantitative Considerations: Now, let's consider how fast this precessional motion will be. If the rotor is going at an angular velocity of ω radians/sec., a particle of mass m which is a distance r from the **axle** has a velocity ωr and a momentum $m\omega r$ (as shown in Figure 6). **An** impulse $F\Delta t$ is applied at a distance ℓ from the inner gimbal axis. Because the rotor is a rigid body, the axle does not bend when the impulse is applied. Instead, the force is transmitted to the particle, m, which we are considering. The particle feels an impulse, proportional to the original impulse $F\Delta t$, multiplied by $\frac{\ell}{r}$

Now, if we view the original momentum $\vec{p}$, the impulse, $\vec{J}$, and the new momentum, $\vec{p} + \vec{J}$, redrawn in Figure 7 in the plane of the paper, we see that the angle of $\Delta\phi$, (through which the plane of the rotor moves) is given by

$$\tan\Delta\phi = \frac{F\Delta t\ell/r}{m\omega r} \qquad (1)$$

Now, we can consider hitting the tip of the axle with a succession of small impulses, so that the angle ϕ is small at each motion. We can then make the very useful approximation that

$$\lim_{\Delta\phi \to 0} \frac{\tan\Delta\phi}{\Delta\phi} = 1 \qquad (2)$$

so we can than rewrite equation (1) for a succession of small changes in ϕ (approaching a smooth precession of the axle)

$$\Delta\phi \approx \tan\Delta\phi = \frac{F\Delta t\ell/r}{m\omega r} \qquad (a)$$

OR (3)

$$\underbrace{(m\omega r^2)}_{\text{angular momentum}} \underbrace{\left(\frac{\Delta\phi}{\Delta t}\right)}_{\text{precessional angular velocity}} = \underbrace{F\cdot\ell}_{\text{torque}} \qquad (b)$$

MOTOR DRIVEN GYRO

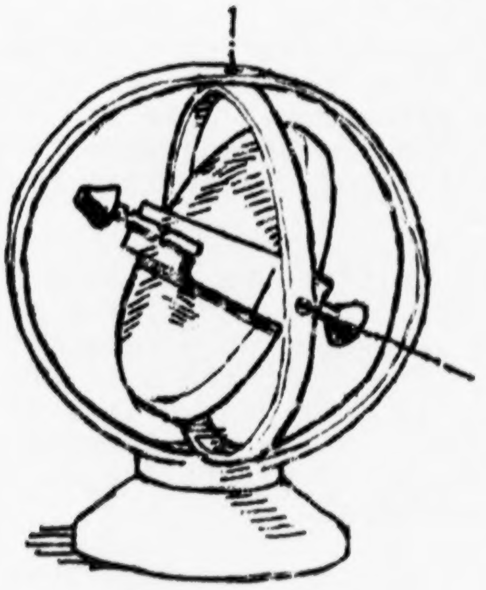


Fig. 1

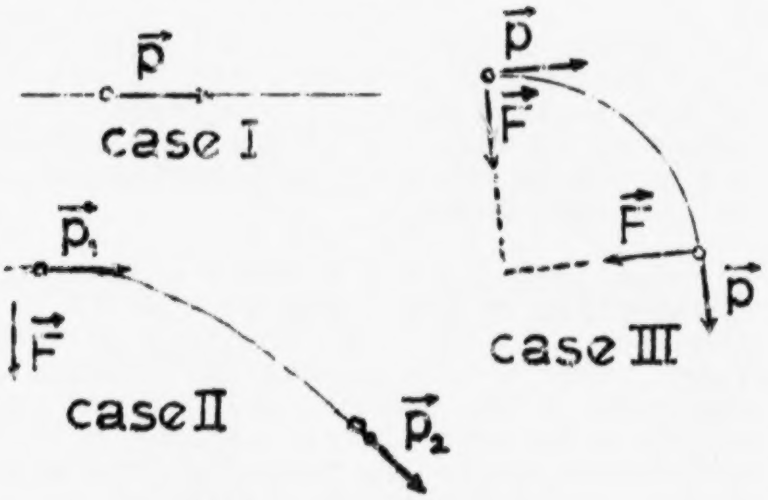

$\vec{p}$
case I
$\vec{p}_1$
$\vec{F}$
case II
$\vec{p}_2$
$\vec{p}$
$\vec{F}$
$\vec{F}$
$\vec{p}$
case III

Fig. 2

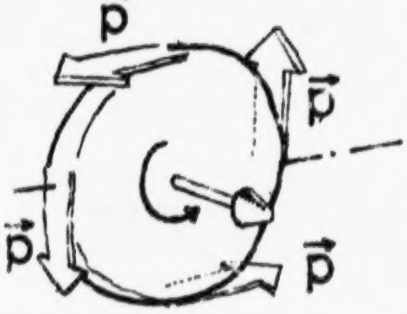

$\vec{p}$
$\vec{p}$
$\vec{p}$
$\vec{p}$

Fig. 3

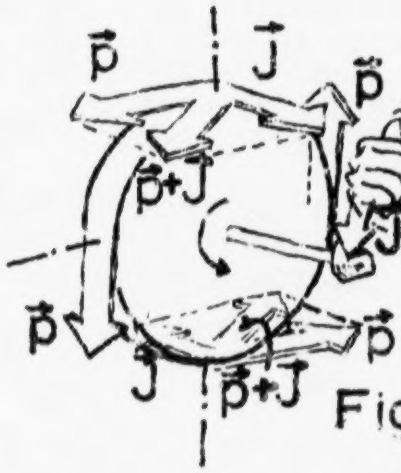

$\vec{p}$
$\vec{J}$
$\vec{p}$
$\vec{p}+\vec{J}$
$\vec{J}$
$\vec{p}$
$\vec{p}$
$\vec{J}$
$\vec{p}+\vec{J}$

Fig. 4

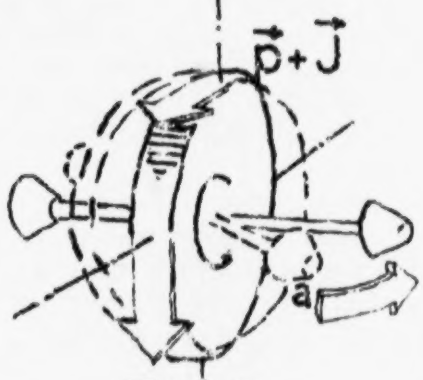

$\vec{p}+\vec{J}$
$\vec{a}$

Fig. 5

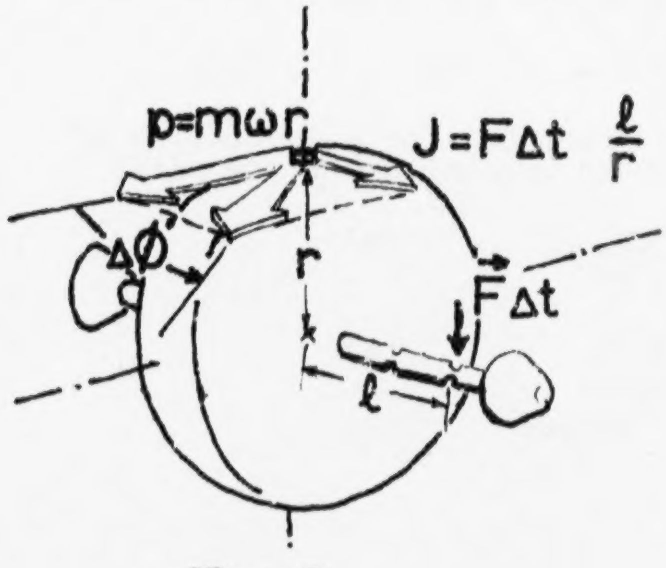

p=mωr
$J=F\Delta t\ \frac{\ell}{r}$
$\Delta\phi$
r
$\vec{F}\Delta t$
ℓ

Fig. 6

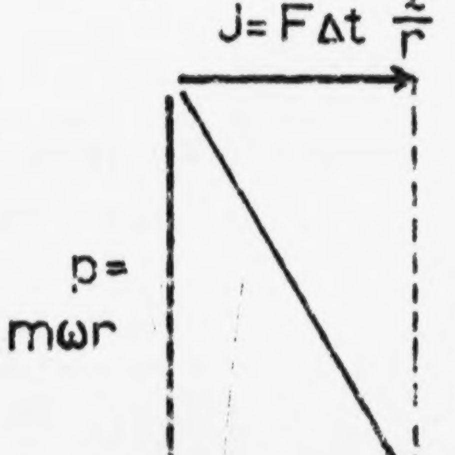

$J=F\Delta t\ \frac{\ell}{r}$
p=
mωr
$\Delta\phi$

Fig. 7

Even though $m\omega r^2$ is identified as angular momentum, it is only the angular momentum of one particle of the rotor. However, when we consider the effect of the impulse upon particles located at other points in the wheel, we would find that it is legitimate to write equation 3b with a sum of $m\omega r^2$ over all the particles in the rotor, so that we have the result (defining new symbols)

$$\overset{L}{\left[\Sigma\, m\omega r^2\right]} \qquad \overset{\frown}{\left(\frac{d\phi}{dt}\right)} \quad = \quad \overset{\tau}{F\ell}$$

Spin Angular Momentum   Precessional Angular Velocity = Torque (4)

4. Vector Formulation and Cross Products:

While the statement of equation 4 is correct, it leaves out the direction of the precessional angular velocity , and also becomes somewhat more elaborate if we attempt to consider a case where the gyro axle is not initially horizontal.

While it is not necessary to derive the equations of the gyroscope's motion using vector notation, it is more compact, since it includes information on the direction and sense of the various motions in a shorter form, once we know what the symbols mean.

The fundamental relation which we will use is written, in the vector form, as

$$\vec{T} = \frac{d\vec{L}}{dt} \qquad (5)$$

where $\vec{T}$ is the vector torque and $\vec{L}$ is the vector angular momentum. These quantities are defined in terms of linear motion vector quantities as

$$\vec{T} = \vec{r} \times \vec{F} \qquad (6)$$

and $$\vec{L} = I_o\, \vec{\omega}$$

where $$\vec{\omega} = \frac{\vec{u}_r \times \vec{v}}{|r|} \qquad (7)$$

(See Figure 8 and Figure 9)

In these equations, $\vec{r}$ is a vector from the center to the point of interest. $\vec{F}$ is the vector force applied at this point of interest, and I_o is the (Scalar, not vector) moment of inertia of the rotor about its axle. $\vec{v}$ is the velocity of a point of interest in the rotor, $\vec{u}_r$ is a unit (dimensionless, magnitude=1) vector parallel to the radius which points to this position of interest and $|\vec{r}|$ is the length of the radius to the position of interest.

Think of a case where we twist the axle of the rotor. Here the

$\bar{T}$ and $\bar{L}$ vectors are parallel. Thus, the torque will not change the direction of the angular momentum, but only its magnitude. This is not news, since we know that a torque applied in the same direction as the rotation of a rigid body will increase the angular velocity at a rate given by

$$T = I_o \frac{d\omega}{dt} \qquad (8)$$

Note that here we do not use vector signs because, since the $\bar{T}$ and $\bar{L} = I_o\bar{\omega}$ vectors are parallel, we are just considering their magnitudes (projected along the axis of spin).

When we attach a weight to the end of the gyro axle, however, we produce a torque vector which is perpendicular to the angular momentum vector. Furthermore, since this weight moves when the axle moves (thus continually altering the point of application of force and the plane of the $\vec{r}$ and $\vec{F}$ vectors) the torque vector remains perpendicular to the angular momentum vector. When the vector retains a constant magnitude, but merely has its tip rotate in a circle with an angular velocity of precession , the rate of change in L can be found from

$$\frac{d\vec{L}}{dt} = \vec{\Omega} \times \vec{L} \qquad (9)$$

and $d\vec{L}/dt$ is the torque vector $\vec{T}$. (See figure 10)

5. What to do in the Laboratory: Observe the motion of the gyroscope when given impulses (by hand) in various directions. Observe and measure the steady precessional angular velocity, Ω, when a known weight is hung on the axle. From this, find the moment of inertia of the rotor. Does the precessional angular velocity change if the axle is not rotating in a horizontal plane, but instead moves in a cone (as in Figure 10). According to the vector formulation, should Ω depend on the tilt of the axle (for a fixed weight and spin angular momentum)? Hold the axle steady in space (with a weight attached) and let go, observing the combination of precession and nutation oscillations of the axle. Do the nutation oscillations die out? Is their frequency dependent on their amplitude or independent of amplitude?

If you have time, attach a string to the axle arrowhead, stand a distance away and exert a steady tension on the string. Can you explain the motions you observe in terms of the vector theory?

Reference: Ingard and Kraushaar "Introduction to Mechanics, Matter, and Waves" Chapters 9 and 13.

97/98

MOTOR DRIVEN GYRO

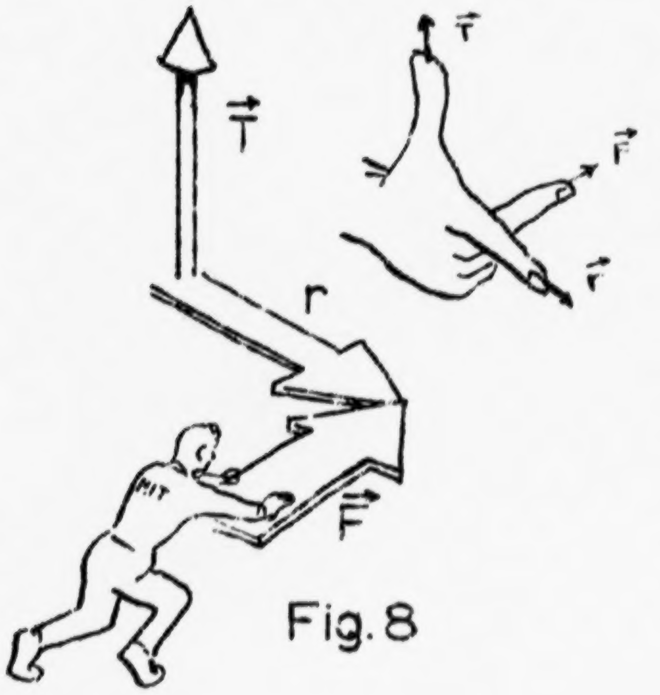

$\vec{T}$
$\vec{\tau}$
$\vec{F}$
$\vec{r}$
r
MIT
$\vec{F}$

Fig. 8

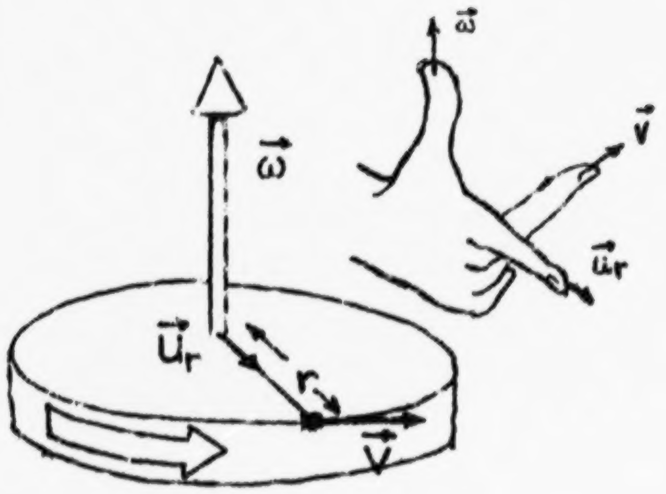

$\vec{\omega}$
$\vec{\omega}$
$\vec{v}$
$\vec{u}_r$
$\vec{u}_r$
r
$\vec{V}$

Fig. 9

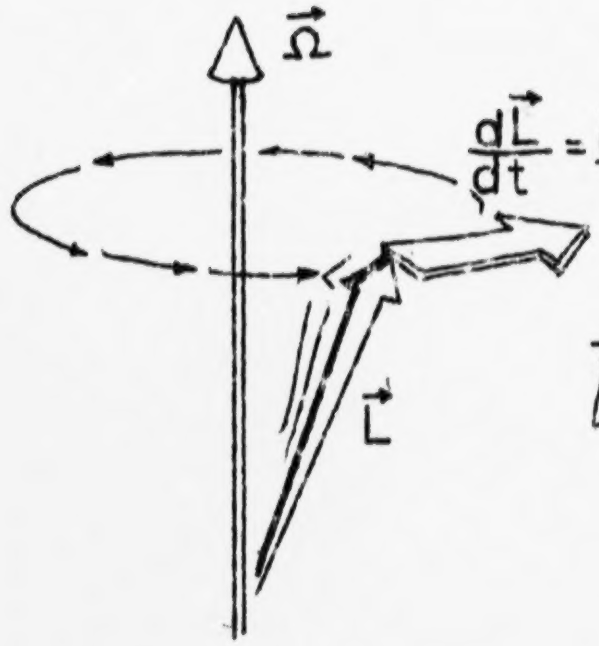

$\vec{\Omega}$
$\frac{d\vec{L}}{dt} = \vec{\Omega} \times \vec{L} = \vec{T}$
$\vec{L}$

Fig. 10

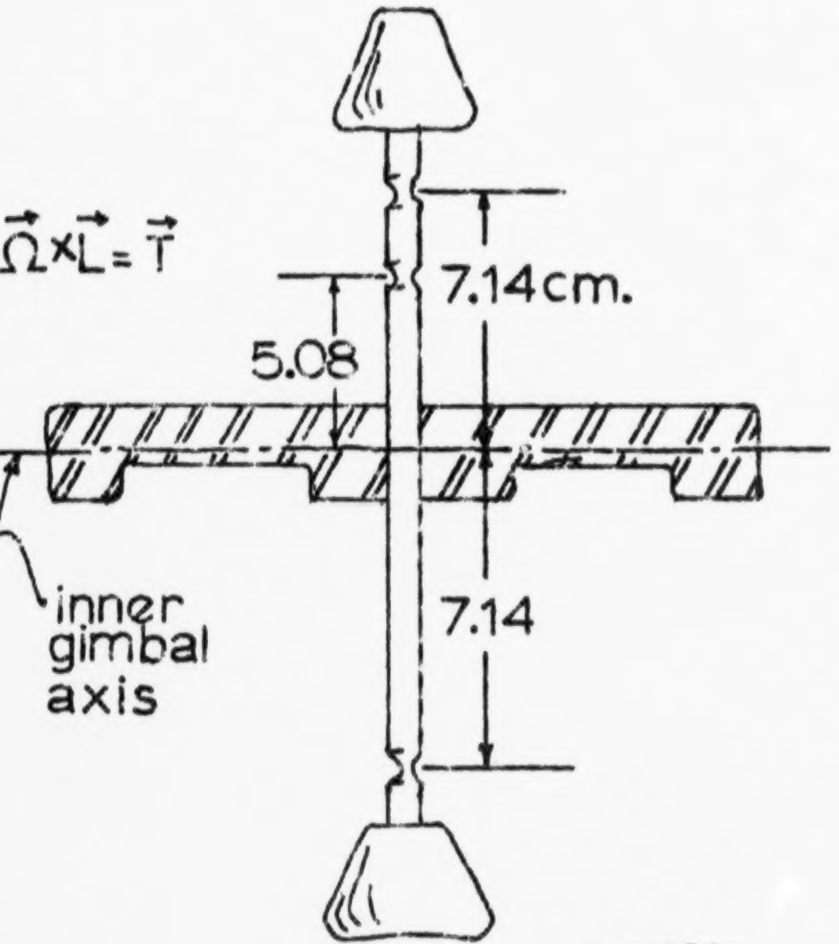

7.14 cm.
5.08
inner gimbal axis
7.14

Fig. 11

PRINCIPLES: A circularly symmetric body, free to rotate about its axis of symmetry, but having one point on its axis at a fixed location, is called a Maxwell top. It provides an excellent opportunity to study (1) accelerated angular motion and (2) the gyroscopic behavior of a spinning body. An air cushion provides a nearly frictionless bearing and airjets provide a constant torque at low angular velocities.

(a) There is a close analogy between linear and angular motion. One may list some corresponding relationships as follows:

Linear	Angular	Conditions
(1) $v = ds/dt$	$\omega = d\theta/dt$	General
(2) $a = dv/dt = d^2s/dt^2$	$\alpha = d\omega/dt = d^2\theta/dt^2$	General
(3) $S = S_o + v_o t + at^2/2$	$\theta = \theta_o + \omega_o t + \alpha t^2/2$	Constant acceleration
(4) $v^2 = v_o{}^2 + 2as$	$\omega^2 = \omega_o{}^2 + 2\alpha\theta$	Constant acceleration
(5) $v = v_o + at$	$\omega = \omega_o + \alpha t$	Constant acceleration
(6) $F = ma$	$\tau = I\alpha$	General
(7) $\Delta(mv) = \int F dt$	$\Delta(I\omega) = \int \tau dt$	General
(8) $T = mv^2/2$	$T = I\omega^2/2$	General
(9) $t_o = 2\pi\sqrt{m/k}$	$t_o = 2\pi\sqrt{I/K}$	Simple harmonic motion
(10) $F = ks$	$\tau = K\theta$	Hooke's law

The moment of inertia, I, may be found experimentally by using a body of known moment of inertia as follows: Let body 1, Fig. 1, be hung by a torsion rod and let it execute torsional oscillations. The period is given by (9) above, namely

$$t_o = 2\pi\sqrt{I/K} \qquad (11)$$

where K is defined by (10) above and is the torque required to give the rod a twist of one radian. Let body 2, having a known moment of inertia, I_2, be placed on 1. The new period will be

$$t_o' = 2\pi\sqrt{\frac{I_1 + I_2}{K}} \qquad (12)$$

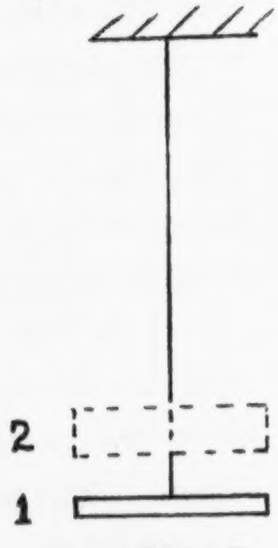


Fig. 1

From (11) and (12),

$$I_1 = \frac{I_2}{\left(t_o^{1}/\,t_o\right)^2 - 1} \qquad (13)$$

For a hollow cylinder of mass m, inner radius b and outer radius "a", the moment of inertia about its axis of symmetry is

$$I = m/2\ (a^2 + b^2) \qquad (14)$$

(b) Gyroscopic effects are quite complicated to analyze in general. This is especially true at low angular velocities. The two principal motions of the axis of rotation are called precession and nutation. Precession is the slow conical motion of the axis of rotation about a vertical line. Nutation is the rapid variation or "wobble" of the axis of rotation of the top about the average motion of precession.

For the case where no nutation is present, the rate of precession is easily calculated. If M is the mass of the top and h is the distance of the center of mass below the pivot point, then the rate of precession is given by,

$$\Omega = Mgh/I\omega \qquad (15)$$

where ω is the angular velocity of the top.

PROCEDURES: (a) In studying the relationship between torque and angular acceleration, one may compare the angular acceleration, α , calculated from (6) with that calculated from (3). Thus in (6) the torque, τ , due to the airjets may be found from the lever arm and the force. I may be found as explained above. To find α from (3) one may find the time required for 1, 4, 9, 16 -------revolutions, each time starting from rest. How closely do your two values of α agree? Do you find any systematic effects entering? Discuss.

(b) To study precession, it is necessary to find the distance of the center of mass of the top below the center of the ball, i.e. below the pivot point. This may be done most accurately by finding the position of the sliding mass, m, that gives no precession. Then from Fig. 2, md = Mh.

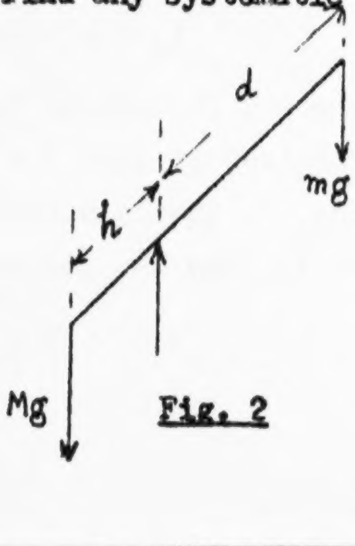


Fig. 2

The angular velocity, ω, may be held constant by adjusting the exhaust while monitoring the speed of rotation with a stroboscope. You should arrive at two values of the rate of precession, Ω ; one measured with a stopclock and the other computed from (15). You should also have a good estimate of the relative errors entering into each quantity. As explained in the introduction to these laboratory notes, the agreement of your two values should be consistent with the errors in each determination. Let the errors entering into the quantities on the right side of (15) all be independent of each other. Then, if the relative errors in the five quantities are E_1, E_2, ..., E_5, the relative error, E, in Ω is given by:

$$E^2 = E_1{}^2 + E_2{}^2 + - - - - + E_5{}^2.$$

Do you think any systematic errors need to be considered? If so point out where and how they might enter.

THE SYMMETRICAL TOP

Dartmouth College

The symmetrical top is one of the most interesting and difficult of the soluble problems of rigid body dynamics. Nutation is complicated and difficult to observe experimentally, because it is rapidly damped out by friction. Precession, however, can be explained by a simple approximate treatment, and is easily observed experimentally. First, the precessional motion of the heavy symmetrical top with apex fixed will be studied. Then the behavior of the top, whose axis is magnetized, will be investigated when an alternating magnetic field is applied perpendicular to the gravitational field. This behavior is interesting because it is exactly analogous to the problem of nuclear magnetic resonance absorption, which is currently yielding much information about nuclear magnetic moments and about electric and magnetic interactions in solids and liquids.

Apparatus
- Heavy Symmetrical Top, with magnetized axle
- Motor Driven Support (variable speed), which maintains the spin of the top but permits motion about the apex
- Helmholtz Coils
- Alternating Current Supply (variable strength and frequency)
- Tachometer (photo-diode, oscilloscope, audio oscillator)
- Stop watch, ruler, etc.

References
- R. J. Stephenson, Mechanics and Properties of Matter (Wiley, 1952), §6.21
- A. Sommerfeld, Theoretical Physics (Academic Press, 1952), Vol. I, Ch. IV.
- F. Bloch, Phys. Rev. 70, 460 (1946).
- G. E. Pake, Amer. J. Physics 18, 438 (1950).
- E. L. Hahn, Physics Today, Nov., 1953, p. 4.

The actual gravitational field in the z-direction is equivalent, in its effect on the precession of the top, to a constant magnetic field,

$$H_z = H_o = mgd/M,$$

where m is the mass of the top, M its magnetic moment, d the distance from the center of gravity to the apex, and g the acceleration of gravity. It will simplify the comparison with nuclear resonance if H is used in the analysis of the motion. Measure the precession frequency for a variety of values of the spin velocity. The value of H can also be changed (by changing d), but the top is unstable except for small d. Calculate the theoretical value, and compare with the observed one.

Caution: The Alnico axle of the top is very brittle, so be careful not to drop the top when putting it on the support or removing it.

Now suppose an alternating field of peak value $2H_1$ is applied in the x-direction. Write the equations of motion for the top. Two solutions are commonly used. Bloch's solution

$$M_x = M_o \cos \omega t/(1 + \delta^2)^{\frac{1}{2}}$$

$$M_y = - M_o \sin \omega t/(1 + \delta^2)^{\frac{1}{2}}$$

$$M_z = M_0 \delta /(1 + \delta^2)^{\frac{1}{2}}$$

where

$$\delta = (H_0 - \omega/\gamma)/H_1$$

is useful in describing the nuclear induction experiments. Another, which is only good at the frequency $\omega = \gamma H_0$ (the natural precession frequency), is useful in describing the "spin-echo" experiment:

$$M_x = M_0 \sin \omega_1 t \sin \omega t$$

$$M_y = M_0 \sin \omega_1 t \cos \omega t$$

$$Mz = M_0 \cos \omega_1 t$$

where

$$\omega_1 = \gamma H_1.$$

(Notice that Bloch's solution when $\delta = 0$ is different from this - why is that) Verify that these are solutions, and interpret them, assuming $H_1 << H_0$.

Now compare these solutions with the behavior of the top when the magnetic field is turned on.

Caution: Do not turn the SPIN VELOCITY or the FIELD FREQUENCY controls above "60."

Pay particular attention to the range near resonance. The product MH_1 can be found experimentally by measuring the period of oscillation of the magnetized axle when it is suspended horizontally by a thread in the magnetic field. The alternating magnetic field is actually a square wave, rather than a sine wave. To obtain quantitative agreement the first harmonic of the square wave can be calculated, ignoring higher harmonics (why?).

The tachometer for measuring the spin velocity is a photo-diode (RCA 7224), which gives a voltage proportional to the amount of light falling on it. The top is painted half black and half shiny, so that when the diode is near the top an alternating light signal is reflected into the diode. The resulting alternating voltage is compared on an oscilloscope with the variable frequency output of an audio oscillator. (It may be necessary to calibrate the oscillator frequency against line frequency.)

The field frequency is measured using an electrical counter which is actuated every time the field current reverses (by means of a mechanical commutator). The number of counts is therefore twice the number of periods.

Molecular Physics, Heat, Sound

SPECIFIC HEAT OF GRAPHITE

University of Colorado

The law of Dulong and Petit says that the atomic heats (the product of the specific heat and the atomic weight) of most substances are constant, being approximately 6 calories per gram atomic weight per degree centigrade. Carbon is an exception to this rule, having a specific heat that is neither constant with temperature, nor does it have the proper value, its atomic heat being much lower than that of other elements.

The purpose of this experiment is to measure the specific heat of carbon (graphite) as a function of temperature, and to learn to use thermocouples for the measurement of temperatures.

A large graphite block hangs in a copper can in a water bath, the can being a constant temperature environment for the graphite. When the graphite is at a higher temperature than its surroundings, it will lose heat by conduction, convection and radiation at a total rate that is a function of the temperature of the graphite and of its surroundings. If the surroundings are maintained at tap water temperature by a circulating bath, and if this is a constant temperature, then the power loss of the graphite should be a function only of the temperature of the graphite.

The first part of the measurements consists of putting electrical energy into the graphite by means of a resistance heater that is inserted in an axial hole in the graphite. The rate of energy input must be measured electrically, and for any given power input the graphite temperature will rise until an equilibrium is reached in which the power loss equals the power input. This temperature should be measured for each of several power inputs, covering equilibrium temperatures in the range from slightly above room temperature to about 400° C. (check this with the instructor) The second part of the work consists of raising the power input to the graphite slightly above that required for the highest temperature point, allowing the graphite temperature to rise a few degrees above the highest temperature previously recorded, and then one quickly,

a) Turns off the power input.
b) Carefully removes the electric heater from the graphite
c) Puts a cork in the hole in the shield through which the heater was withdrawn, and
d) Starts at once to take a cooling curve of temperature vs. time of the graphite, covering the full range of temperatures that were measured in the first part of the experiment.

Special Instructions.

Before coming to the laboratory you should work out the theory of the measurements so that you can explain to the instructor how the results of these measurements will yield values of the specific heat of graphite at different temperatures.

You will be using thermocouples to measure temperatures so you will need one of the Type K millivolt potentiometers, a standard cell, a good galvanometer, and an ice bath for the reference junction. The ice bath is prepared from crushed ice or snow mixed with a little water and kept in the pint thermos bottle. It is important that the bottle be kept full of snow or ice which is wet with water, rather than having water with ice floating in it, for the reference junction must be at 0° C. not at an unknown temperature between 0° C. and 4° C.

Data on voltage vs. temperature for the chromel-alumel thermocouple is available in the Handbook of Chemistry and Physics, and in other sources. If you still feel the need to calibrate the thermocouple, you can get one point easily by arranging a boiling water bath. Since the thermocouple output is nearly linear as a function of temperature, one calibration point will suffice.

In addition to the regular items in the report, you should compare your curve with the measurements of others. Check with measurements in one or more of the references given, or compare with the results in the International Critical Tables.

Optional. If one would like to extend the measurements to lower temperatures, one may use a liquid nitrogen bath, or a dry ice bath, with the experimental apparatus being suspended in the large Dewar flask rather than in the water bath can. Check your plans with the instructor before going ahead with this.

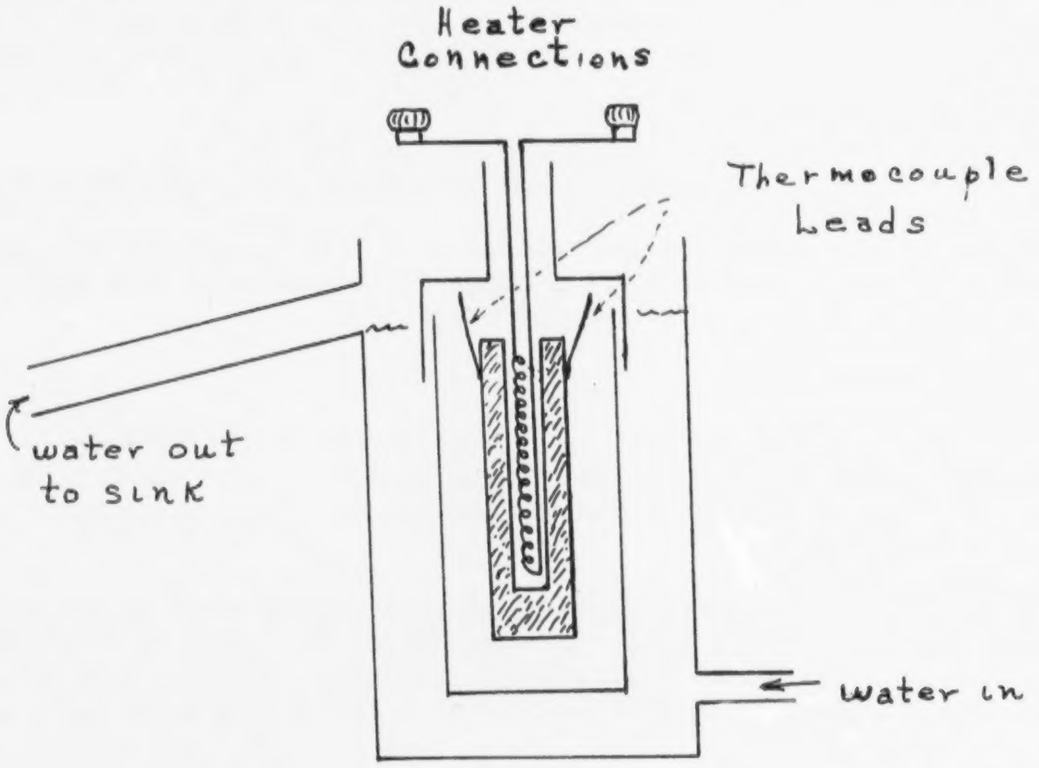
References.

This Method. M. Zemansky, "Heat and Thermodynamics"

This Experiment. E. Frank, Am. Jour. of Phys. 9, 227, 1941.

The Specific Heat of Carbon.

Richtmyer and Kennard, Chapter on Specific Heats.
W. Nernst, Ann. d. Physik, 36, 395, 1911.
V. V. Tarassov, Compt. Rend. (U.S.S.R.), 46, 110, 1945.
W. DeSorbo, W. W. Tyler, Phys. Rev. 83, 878, 1951.
R. W. Gurney, Phys. Rev. 88, 465, 1952.
and others.



California Institute of Technology

Principles: In the Millikan oil drop experiment, one of the important quantities you assumed as known was the numerical value of the viscosity of air. You will find tables giving the value of this quantity. The usual value quoted is $(0.1813 \pm 0.0001) \times 10^{-3}$ poises = $(0.1813 \pm 0.0001) \times 10^{-4}$ MKS units at 20°C.

One of the great triumphs of the kinetic theory of gases was the prediction that, as long as the mean-free-path of the molecules is small compared with the dimension of the apparatus, the viscosity is independent of pressure. This would certainly not be expected, a priori if one thinks of a gas as a perfectly continuous, homogeneous fluid. The apparatus used in this experiment consists of concentric cylinders whose radii differ by approximately 1 cm. Thus one would expect from kinetic theory that the viscosity of the enclosed gas would be constant down to, say, a pressure where the mean-free-path is something like 1 mm of mercury. Now, the mean-free-path of the molecules for air at room temperature is given approximately by

$$\lambda = 6 \times 10^{-6} \cdot \frac{p_o}{p} \text{ cm} \quad ,$$

when p_o is taken as 1 atm. Thus for $\lambda = 0.1$, we would expect the viscosity to remain substantially independent of pressure down to a pressure of 50 to 100 microns (1 micron = 10^{-3} mm of Hg. A new unit of pressure, called the Torr, is defined as 1 mm of Hg. Thus 1 micron = 10^{-3} Torr). Over a range of pressure of at least 10^4 in density, the viscosity is predicted by kinetic theory to remain essentially constant!

Viscosity, together with conduction of heat and diffusion, is one of the so-called transport phenomena of a gas. In the case of viscosity it is momentum that is transported.

Let $\bar{v}$ be the average velocity of the molecules of the gas whose density is ρ. Let λ be the mean-free-path of the molecules. Then, a consideration of the transport of momentum across a surface perpendicular to the velocity gives for the coefficient of viscosity,

$$\eta = \frac{1}{3}\rho\bar{\upsilon}\lambda \quad . \qquad (1)$$

Now $\bar{\upsilon}$ varies as the square root of the absolute temperature. The quantity $\rho\lambda$, for an ideal gas, is independent of pressure. This is the prediction we mentioned earlier.

When the mean-free-path becomes comparable with the dimension of the apparatus, the viscous drag of one cylinder on the other, in your apparatus, begins to decrease. Just how it decreases with pressure depends on the given apparatus.

Viscosity is measured by the drag of one surface on another. Thus in Fig. 1 let the top plate be moving with a constant velocity υ with respect to the lower plate. If the area of the plates is A and their separation d, then the force on the plates will be given by

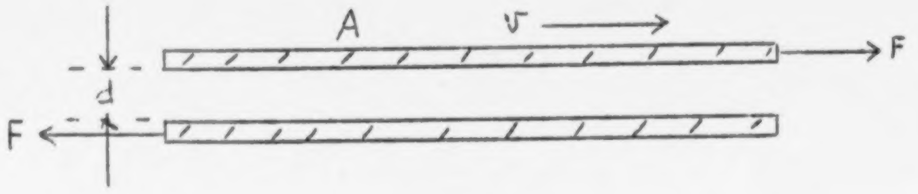


Fig. 1

$$F = \eta \frac{A\upsilon}{d} \qquad (2)$$

In the apparatus you will use, the gas is contained between concentric cylinders. The inner cylinder is suspended on a torsion wire and the outer cylinder may be caused to rotate at a controllable speed. To eliminate end effects, the inner cylinder has concentric guard cylinders of the same diameter as the moveable one, and separated from it by a small distance. The dimensions of the parts as well as the torsion constant may be accurately determined so that absolute measurements may be made.

For the cylindrical case, the coefficient of viscosity is given by,

$$\eta = \frac{R_2^2 - R_1^2}{R_1^2 R_2^2} \cdot \frac{ktd}{16\pi^2 \ell D}$$

where:

R_2 = radius of outer cylinder (mean value)

R_1 = radius of inner cylinder (mean value)

k = torsion constant of suspension

t = period of rotation of outer cylinder in r.p.s.

d = scale deflection

ℓ = effective length of inner cylinder (length measured to mid-points of gaps).

D = mirror-scale distance.

A schematic diagram of the apparatus is shown in Fig. 2.

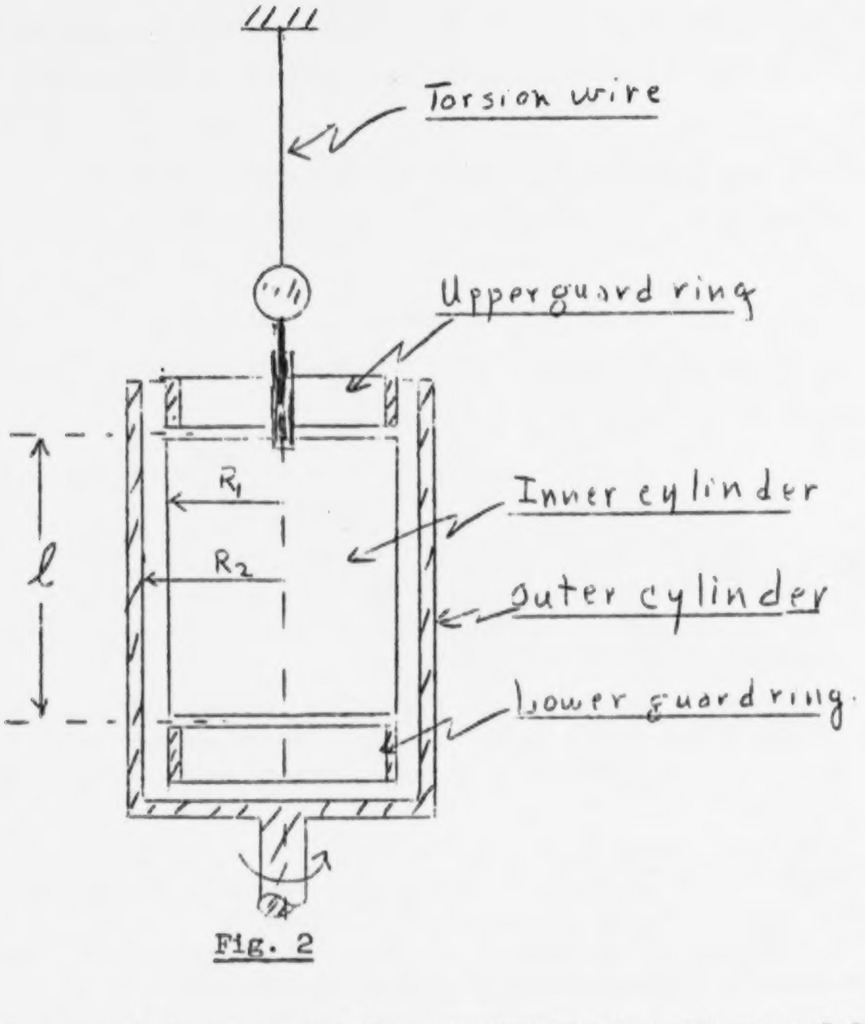

Torsion wire
Upper guard ring
Inner cylinder
Outer cylinder
Lower guard ring
R_1
R_2
ℓ

Fig. 2

If the outer cylinder rotates too rapidly, turbulence is set up and laminar flow no longer exists. The speeds necessary for this condition to occur are much higher than those you will use with this apparatus.

The torsion constant may be determined in the usual way by adding to the moveable system a mass of known moment of inertia. A measurement of the torsional period in the two cases, with and without this added mass, then permits the torsion constant to be calculated.

PROCEDURES: Note: Read carefully the directions supplied with this apparatus, noting the details of construction. The dash-pot of oil just below the inner cylinder gives the necessary damping. Note how you lift the inner cylinder, thus removing the damping vanes from the oil. A further lifting engages the mass of known moment of inertia.

Do not work with this apparatus unless you are acquainted with the internal construction. Ask your instructor.

In operating the apparatus, you will probably find it desirable for the person observing the deflection to also control the speed of the motor. Careful control, observation and measurement will pay off in accuracy attained.

There are a number of experimental quantities that you can vary such as speed of rotation, pressure of the gas and kind of gas.

In plotting viscosity vs pressure, you may find it convenient to use semi-log paper which allows data for several decades change in pressure to be plotted on the same piece of paper.

REFERENCES

Lachlan Gilchrist, Phys. Rev., 2nd Series, Vol. 1, No. 2, 124 (1913)
--- An Absolute Determination of the Viscosity of Air
(Original paper. Discusses theory of method, describes apparatus and measuring techniques employed.)

Ertle Leslie Harrington, Phys. Rev., 2nd Series, Vol. 8, No. 6, p. 738 (1916)
--- A Redetermination of the Absolute Value of the Coefficient of Viscosity of Air
(Extension of Gilchrist paper. Description of modified apparatus and techniques employed.)

R. A. Millikan, Phys. Rev., 2nd Series, Vol. 21, No. 3, p. 217 (1923)
--- Coefficients of Slip in Gases and the Laws of Reflection of Molecules from the Surfaces of Solids and Liquids.
(Discussion of phenomena, derivation of equations, comparison of various methods of measurement of viscosity and slip.)

Leland Johnson Stacey, Phys. Rev., 2nd Series, Vol. 21, No. 3, p. 239 (1923)
(Experimental results obtained with rotating concentric cylinder apparatus.)

Karl Stillman Van Dyke, Phys. Rev., 2nd Series, Vol. 21, No. 3, p. 250 (1923)
(Extension of Stacey paper.)

ULTRASONICS

Michigan State University

I. Introduction and Theory

In the field of acoustics one deals with mechanical waves at all frequencies in all substances. Architectural acoustics aim to improve buildings and rooms with respect to hearing conditions and adequate protection against noise. The subjects of communication acoustics and electroacoustic instrumentation have to do with speech transmission through telephone, radio and intercommunication systems and with the design of microphones, loudspeakers and apparatus for measuring sound. Still other phases of acoustics encompass the science of musical sounds and instruments, the behavior of the ear and the properties of speech, and the control of noise and vibration in machines.

In all of these branches of acoustics one is primarily concerned with audible sounds. However, it is apparent that acoustical vibrations can be utilized in many other ways. Sound waves can be used to sonograph the inner structures of matter which are opaque to light. Sound waves can penetrate many solids and liquids more readily than X-rays or other forms of electromagnetic energy. This sound can expose a tiny crack imbedded many feet in metal, where detection by any other means might be commercially impracticable if not impossible. By acoustic techniques we can measure elastic constants of solids; non-isotropic stresses and inhomogenieties can be analyzed. The molecular structure of many organic liquids can be inferred from sound measurements.

Rates of energy transfer among gas molecules and chemical affinity of gaseous mixtures can be determined by using sound waves.

From the point of view of sonics, the distinction between audible and inaudible frequencies is rather arbitrary. Some frequencies as low as a few hundred cycles per second (cps) have been used commercially for homogenization, and sonic techniques for measuring dynamic mechanical properties of viscoelastic materials have been extended considerably below 1 cps. The range from 1 to 10 megacycles per second (mcps) is used widely in flaw-detection equipment, and laboratory research in properties of metals and liquids has pushed the upper frequency limit to several hundred megacycles per second. To the range 10^4 - 10^9 cps, one sometimes assigns the name "ultrasonics."

The present experiment is concerned with ultrasonic diffraction effects in liquids, and the use of this diffraction for measuring velocities of sound in liquids.

Light passing through a transparent medium which carries standing waves of sound is diffracted in a manner similar to the way it is diffracted by an optical grating. This effect is caused by the periodic compressions and rarefactions in the sound field. The amount of interaction between light and matter is proportional to the number of atoms or molecules which are present in a given volume element. According to Fig. 1, a slit S is illuminated by a monochromatic light source LS.

By means of two converging lenses L_1 and L_2 an image of the slit is formed in a plane at A. The lenses are so adjusted that the light between them is parallel as it passes through the flat glass walls of a small test cell T. A standing sound wave represented by a thick interrupted line is set up in the cell between a quartz crystal transducer Q and a reflector, with parallelism between the optical axis of the system and the wave fronts of the sound. In order to produce the desired diffraction effects, the width of the light beam must be larger than the wavelength of the sound. This condition can be met easily at ultrasonic frequencies above 10^6 cps.

The angles of diffraction are given by

$$\sin \theta_n = n\lambda_o/\lambda = n\lambda_o f/c \qquad (1)$$

where θ_n is the angle between the direct slit image and the diffracted image of the nth order; λ_o is the wavelength of the light; λ the wavelength, c the velocity, and f the frequency of the sound wave. If F is the normal distance between lens L_2 and the plane A, and D is the distance between the zero order and the nth order slit images measured in the plane A, we may write, assuming $F \gg D$

$$c \approx n \lambda_o f \; F/D. \qquad (2)$$

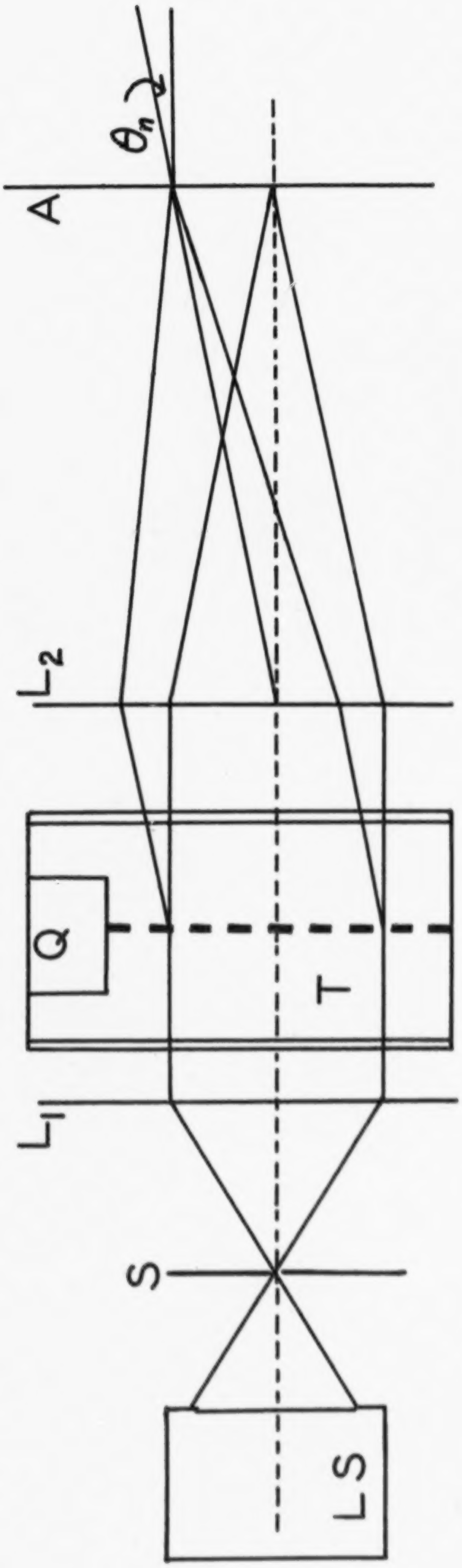
II. Procedure.

The experimental apparatus consists of several lenses, a test cell, several liquids, an ultrasonic generator (frequency range ~ 5-15 mcps), a quartz crystal and a light source (mercury arc lamp). In addition, a wave meter is needed for making measurements on the frequency of the output of the ultrasonic generator.

References:

1. Der Ultraschall	L. Bergmann, S. Hirzel Verlag, Zurich (1949)
2. Sonics	T. F. Heuter and R. H. Bolt, John Wiley & Sons, New York (1955)

Fig. 1. Optical Analysis of Standing Waves


LS
S
L1
T
Q
L2
A
θn

Electricity and Magnetism

ELECTROSTATIC FORCES AND THE ABSOLUTE MEASUREMENT OF VOLTAGE

Massachusetts Institute of Technology

A. Outline of the Experiment

In this experiment you will;

(1) use the electrostatic balance to study electrostatic forces,

(2) calibrate a voltmeter in absolute units without the use of any electrical standards, and

(3) use the calibrated voltmeter to measure the open-circuit potential of a dry cell.

B. Theory of the Parallel Plate Capacitor

The fundamental law of electrostatics is that two point charges, q_1 and q_2 separated by a distance r, exert on one another a force F given by the relation

$$F = k \frac{q_1 q_2}{r^2} . \tag{1}$$

k is a constant whose magnitude and physical dimensions depend upon the choice of units for the measurement of mass, length, time, and charge. In the MKS system used in this course, F is in newtons, q is in coulombs, r is in meters, and $k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{nts} - \text{m}^2}{\text{coul}^2}$. This law was stated by Charles Coulomb in about 1785 after a series of direct measurements on the forces between isolated static charges. It is difficult to make accurate measurements of this sort on the exposed charge of a torsion pendulum because of the influence of the charges which are induced around the room by the charges being studied. Therefore, the best way to test Coulomb's Law is to derive from it theoretical predictions which can be tested in accurate experiments.

The Kelvin balance, or absolute electrometer, consists of a parallel-plate capacitor and an equal-arm balance with which the force on the upper capacitor plate can be measured. The capacitor is specially constructed so that a simple and nearly exact theoretical analysis can be made of it on the basis of electrostatic theory. The Kelvin balance, therefore, can be used to test the predictions of electrostatic theory. If the predictions were not confirmed by measurements within the limits of experimental error, the theory of electrostatics would be disproved.

The applications of Coulomb's Law to the attraction between two charged discs runs into considerable mathematical complications. We shall first consider the forces which an infinite plane charge distribution of density σ coulombs/meter^2 exerts on an element of charge on the axis of a similarly charged plate a distance x away, as shown in Figure 1.

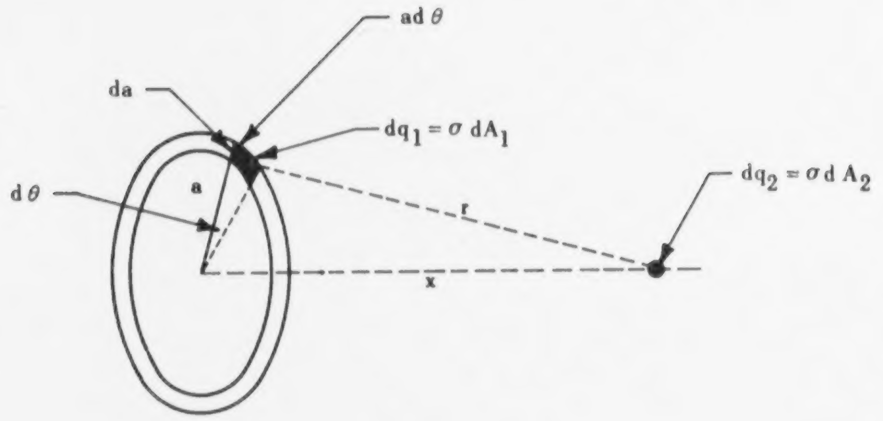


Figure 1.

If the charges on the plates are of opposite sign, the electrical force between them, and at the center, will be along the x-axis. We have for the x-component of the electrical force exerted by the infinite plate on $dq_2 = \sigma dA_2$

$$d\,F_{ex} = \int_{\text{over plate}} \frac{\sigma\, dA_1\ \sigma\, dA_2}{4\pi\epsilon_o\ r^2}\ \frac{x}{r} = \frac{\sigma^2 dA_2}{4\pi\epsilon_o}\ x \int_0^{2\pi} \int_0^{\infty} \frac{d\,a\ .\ a\,d\theta}{(x^2+a^2)^{3/2}} \tag{2}$$

$$= \frac{\sigma^2 dA_2}{2\,\epsilon_o}\ x \int_0^{\infty} \frac{a\,d\,a}{(x^2+a^2)^{3/2}}\ ,$$

which may be integrated to give

$$d\,F_{ex} = \frac{\sigma^2 d\,A_2}{2\,\epsilon_o}\ x \left[-0 + \frac{1}{x}\right] = \frac{\sigma^2 dA_2}{2\,\epsilon_o}\ .$$

If we call $F_e{}^1$ the force per unit area which is exerted on the right-hand plate of Figure 1, then

$$F_e{}^1 = \frac{d\,F_{ex}}{d\,A_2} = \frac{\sigma^2}{2\,\epsilon_o}\ . \tag{3}$$

The electric field between the plates is

$$E = \frac{\sigma}{\epsilon_o}\ .$$

Therefore, eq. (3) can be rewritten

$$F_e{}^1 = \frac{\epsilon_o E^2}{2}\ . \tag{4}$$

or in terms of the potential difference between the plates $V_{ab} = E x$,

$$F_e^{1} = \frac{\epsilon_o V_{ab}^{2}}{2\ x^2} \quad . \qquad (5)$$

Near the edges of a finite parallel-plate capacitor the charge distribution is not uniform and the electric field lines bulge out and are no longer straight. This makes the force per unit area near the edges very difficult to evaluate. In the Kelvin balance this difficulty is removed by dividing the top plate into two parts – an inner round plate and an outer annular "guard ring." Only the force on the inner plate is measured. Since, to a high degree of accuracy, the field lines over the inner plate are parallel, and uniform in density, the region over the inner plate is almost exactly equivalent to a finite region within an infinite plane condenser to which the above discussion pertains. We may therefore use the above expression for F_e^{1}, the force per unit area, to evaluate the force over the entire inner plate which is

$$F_e = F_e^{1}\ A. \qquad (6)$$

Actually, there is a slight crowding of charge near the edge of the movable plate due to the presence of the gap between the inner-plate and the guard ring. Eq. (6) for the total force may be partly corrected for this effect by including in A the area of the plate plus one half of the area of the gap between plate and guard ring.

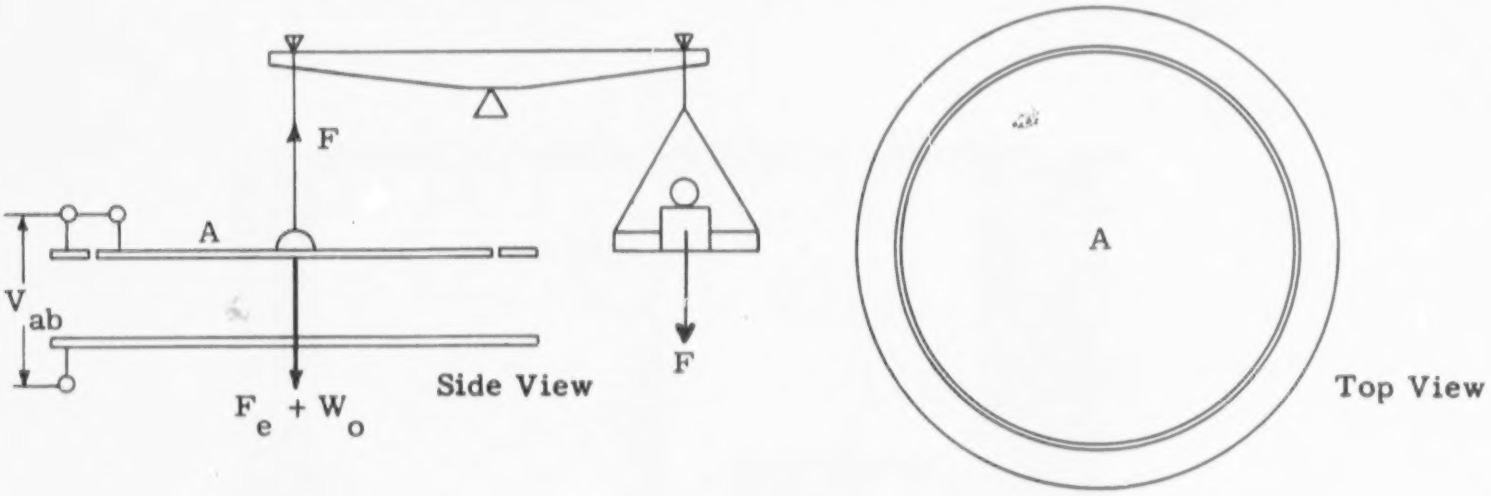


Figure 2

In order to measure the force F_e, the inner plate is suspended from an equal-arm balance. Let F be the force required to balance the charged capacitor, and let W_o be the unbalanced weight of the uncharged plate and associated equipment. Then the electric force F_e is

$$F_e = F - W_o = F_e^{1}\ A = \frac{\epsilon_o\ V_{ab}^2}{2\ x^2}\ A. \qquad (7)$$

If electrostatic theory based on Coulomb's Law is correct, then a plot of F versus $(1/x^2)$ for constant V_{ab} should be a straight line intersecting the F-axis at W_o.

Let us suppose that the inverse quadratic relation between F and x is confirmed. We can then establish an absolute electrostatic measure of voltage. By rearranging Eq. (7) we obtain

$$V_{ab} = \left(\frac{2}{\epsilon_o A}\right)^{1/2} (F - W_o)^{1/2}\, x, \tag{8}$$

which is an expression for the voltage in terms of measurable quantities and the known constant ϵ_o, whose choice determines the size and physical dimensions of the unit of charge. Once an absolute standard of voltage has been established, other electrical quantities, such as charge, using Eq. (3), or current, using the definition $i = dq/dt$, or resistance, using Ohm's law, can be determined in absolute units.

C. Apparatus

Kelvin balance (Figures 2 and 3)
Set of balance weights
Two precision resistors (20 megohms and 0.1 megohm)
Unknown voltage source
Low-voltage voltmeter to be used as a nullmeter, "G"
Tap switch
Red and black leads of various lengths
Power Supply

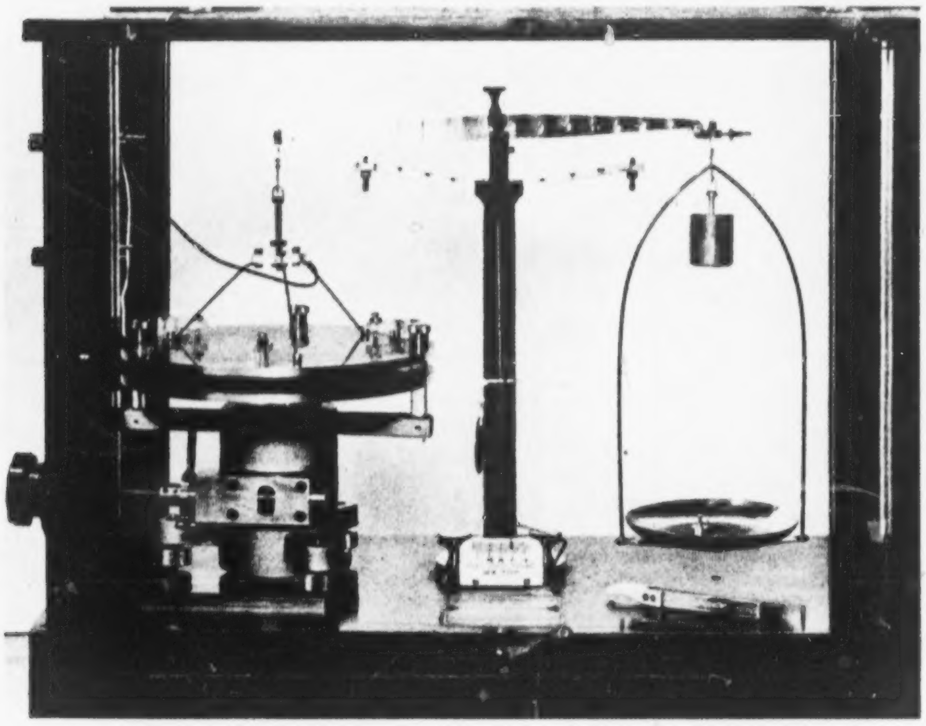
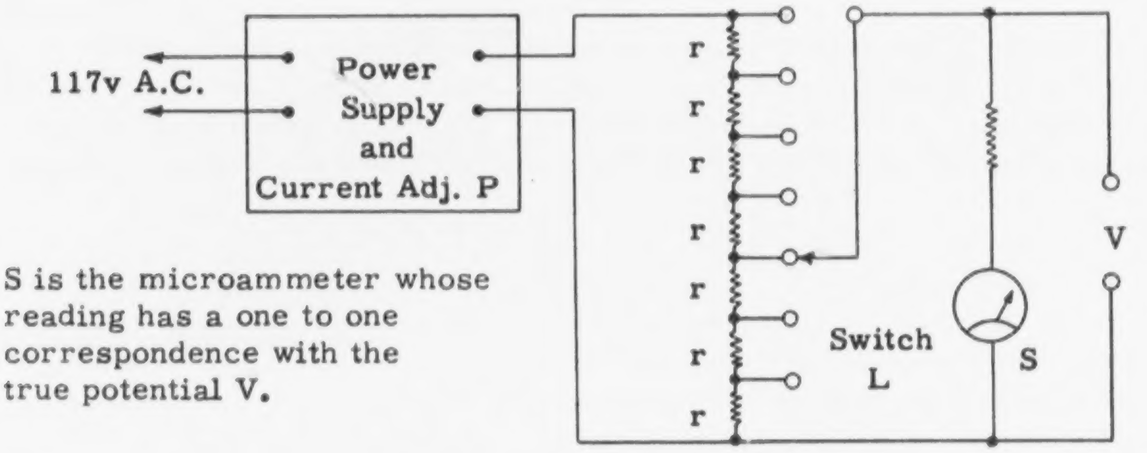


Figure 3

The air capacitor of the Kelvin balance is provided with a movable top plate which is centered in a guard ring as explained in Part B, and illustrated in Figures 2 and 3. Three brass screws supported from the guard ring prevent the top plate from rising. If the capacitor is adjusted correctly, the movable plate will touch all three retaining screws at the same time, and it will then be in the plane of the guard ring. The capacitor is provided with a device for setting the separation of the guard ring and the lower capacitor plate at 0.3 cm, 0.6 cm, or 0.9 cm. Each Kelvin balance is provided with a large brass weight, which almost balances the weight of the movable capacitor plate. The exact balancing weight at zero voltage can be computed from a graphical analysis of the data.

A high voltage power supply converts A.C. into D.C. at voltages up to 3500 volts. This supply provides the voltage which is to be measured by the Kelvin balance. The high voltage output is controlled by a coarse adjustment (L) with seven numbered "click" positions, and a fine adjustment (P) which is continuously variable. There is a one-to-one correspondence between the reading S on scale S and the high voltage output potential; one of the purposes of the experiment is to graph this correspondence. The click position of L determines the maximum S which can be obtained by varying P. The power supply circuit will explain this operation.



S is the microammeter whose reading has a one to one correspondence with the true potential V.

Figure 4
(Power supply schematic circuit)

The fine adjustment P varies the current continuously; the coarse adjustment L varies the resistance stepwise. Since resistance is defined as the quotient of potential and current, the coarse adjustment varies the maximum S stepwise.

WARNING: TO AVOID DAMAGE TO THE INSTRUMENT, THE METAL PARTS OF THE ELECTROSTATIC BALANCE SHOULD NEVER BE TOUCHED WHILE THE HIGH VOLTAGE POWER SUPPLY IS ON.

The electrostatic balance should be touched only after the high voltage is turned off and the capacitor properly discharged in order to prevent shock from residual charge. Of course, no bare electrical connections should be touched after the circuit is set up.

PROCEDURE

A. Discussion of the Experiment

We have described quantitatively the operation of the Kelvin balance by Eq (7), involving F, x, and V_{ab}. The most direct method of testing Eq (7) would be to hold V_{ab} constant (i.e., hold S constant) and measure F_e for various values of x. However, only S can be continuously varied with the apparatus available in the laboratory; therefore, we use an indirect method.

Holding x fixed at one of three possible settings, we measure S for several values of F. The operation is repeated two more times using the other two fixed values of x. The intercept of each of these three graphs on the F axis should be the same and equal to the quantity W_o. We can then study the dependence of F_e on x by drawing a line of constant S (i.e., constant V_{ab}) across the three graphs and noting the relation between the values $F_e = F - W_o$ and x at the points of intersection.

B. Balancing with an Unstable Equilibrium

The balance condition $F = F_e + W_o$ of the Kelvin electrostatic balance corresponds to an unstable equilibrium, like that attained in the measurement of the breaking strength of a rope on a testing machine. Before the rope breaks the dial of the machine will indicate how much force is being applied, but after the rope breaks the force can no longer be adjusted. The breaking strength is indicated by the maximum dial reading before the rope breaks. For precision the critical force must be approached slowly.

The force applied to the ropes by the testing machine and read on its dial corresponds to the continuously variable F_e. Whenever the critical F_e is exceeded it can no longer be adjusted, for the plates are already moving closer together, decreasing x and increasing F_e. (See Eq (7).) Therefore, the reading of S which corresponds to the balance of the electrical and mechanical forces is its maximum reading before the capacitor plate starts to move.

Since the balance condition is an unstable equilibrium, special precautions must be taken to permit accurate measurement of S and reproducible balance readings. A recommended balance procedure follows:

(1) Be sure that there is no friction between the movable capacitor plate and its guard ring. If there is friction, have your instructor adjust the balance.

(2) Turn on the high voltage and wait for S to reach a steady state, turn P to its minimum position, and set L to what you think is its critical range. Adjust P to just below your guess for critical S, and increase S slowly until the capacitor plate starts to fall. Then, remembering the maximum S, quickly turn P to its minimum and switch off the high voltage. (It is better to avoid changing L near the critical scale reading because the shock may vibrate the capacitor plate and cause the plate to fall prematurely.)

(3) Make sure that the high voltage is off, and discharge the capacitor by shorting it out temporarily. Specifically, remove the lead from the positive terminal of the high voltage, touch it to the post of the negative terminal, and replace it. This will prevent shock to the experimenter from residual charge on the capacitor.

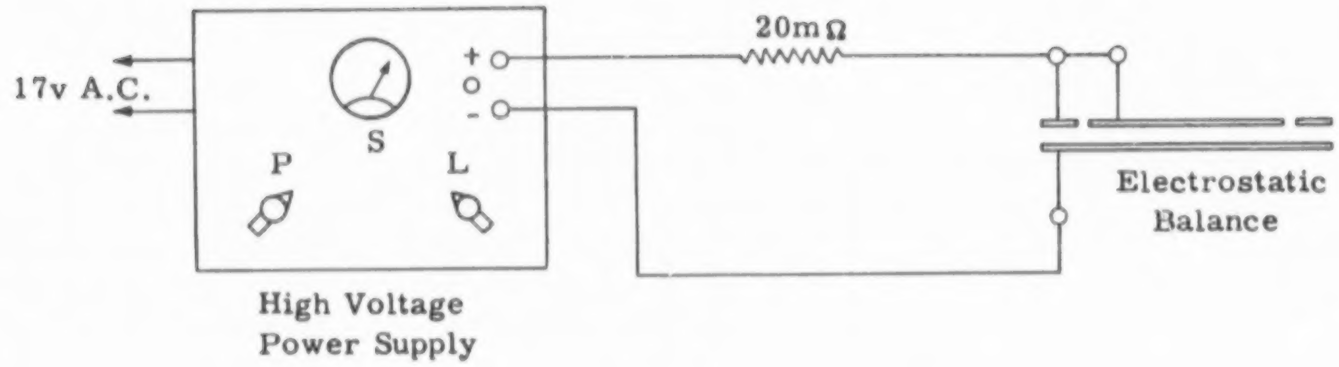


Figure 5

THE EXPERIMENT

A. Investigation of Electric Force

(1) Set up the circuit in Figure 5 and have it checked by your instructor. Use the 20-megohm resistor as a protective resistor.

(2) Measure S as a function of F for the three possible values of x. Record your date directly on a graph of S^2 vs F. Follow the procedure given above in "Balancing with an Unstable Equilibrium." Draw the best curves through the data points and determine W_o. What does the shape of the curves indicate regarding the linearity of scale S?

(3) Plot $F_e = F - W_o$ vs $(1/x^2)$. Is the theory of electrostatic forces verified?

(4) Draw a calibration curve of S vs V_{ab}.

B. Measurement of an Unknown Voltage in Absolute Units

Allow at least twenty minutes for this part.

Using the absolute calibration of the high voltage source, measure the "open-circuit" voltage of an unknown battery and express the results in statvolts and in MKS-volts.

Since the unknown voltage is much smaller than the minimum voltage which can be measured accurately with the Kelvin balance, provision must be made for comparing the unknown voltage with a precise fraction of the calibrated voltage. Two Resistors, R_1 and R_2, are connected in series across the terminals of the high voltage source, so that, according to the principle of the voltage divider,

$$V_1 = \frac{R_1}{R_1 + R_2} \cdot V_{ab} = \frac{1}{1 + \left(\frac{R_2}{R_1}\right)} \cdot V \; . \qquad (9)$$

V_1 is the open circuit potential across R_1. V_{ab} is the calibrated potential across the terminals of the high voltage supply. The only quantity which must be known in order to determine $\left(\frac{V_1}{V}\right)$ is the ratio of the resistances.

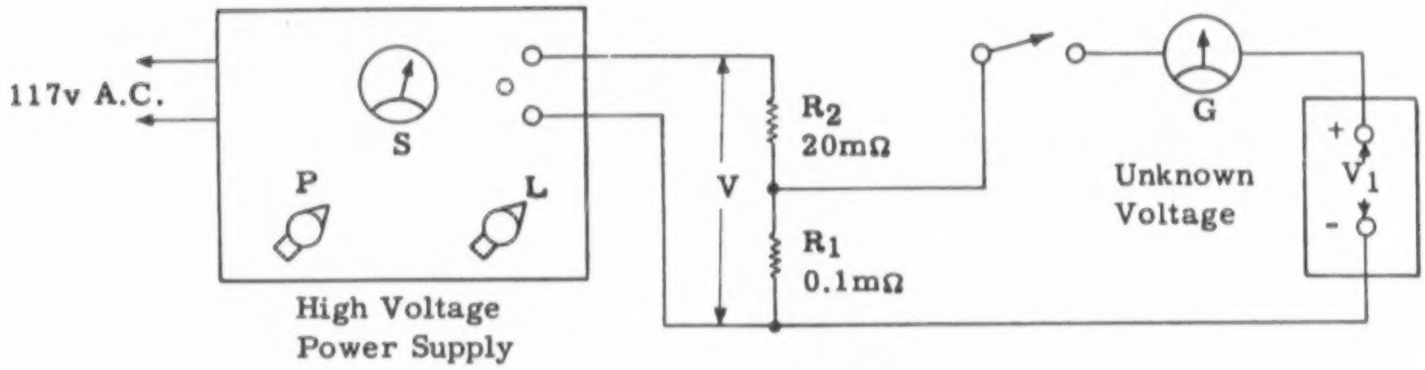


Figure 6

In order to compare the open-circuit potential of both the known and the unknown voltage, a null-type comparison is made. In the circuit shown in Figure 6, the known and unknown potentials oppose each other. When they are equal no current flows between them, as indicated by a zero deflection on the voltmeter.

References

1. F. Bitter, Currents, Fields, and Particles, pp 65.

2. J. C. Slater and N. H. Frank, Electromagnetism, Appendix II, pp 205-211.

3. N. H. Frank, Introduction to Electricity and Optics, 2nd Ed., pp 37-46, 63-72, especially Appendix I, pp 423-425.

4. Harnwell, Principles of Electricity and Magnetism, 2nd Ed., p 575; also the Appendix on "Units and Standards."

Supplementary Problems.

1. From Ohm's law, derive Eq (9) in Part B of the experimental section.
2. What quantities must be measured in this experiment?
3. Why is it better to plot V_{ab}^2 versus F rather than V_{ab} versus $F^{1/2}$?

MAGNETIC FIELDS

Columbia University

Reference: Halliday and Resnick, Chapters 33, 34, and 35.

Object: To determine the strength of the field in the gap of a magnet by measuring, (I) The force on a current-carrying rod, (II) the change of magnetic flux through a coil when the coil is inserted into the field, and (III) the curvature of a current-carrying wire kept under fixed tension in the field.

Caution: (1) Reduce the current to a minimum -- i.e. increase the series resistance to a maximum - before opening the circuit of the magnet coils.

(2) Remove wrist watches before placing hands near magnet gaps.

PART I - FORCE ON A CURRENT-CARRYING WIRE

In a field of magnetic induction $\vec{B}$, the force on a straight wire carrying i amperes of current is

$$\vec{F} = i\vec{\ell} \times \vec{B} \tag{1}$$

where the vector $\vec{\ell}$ has magnitude equal to the length of the wire and direction along that of the current. In this part of the experiment, an electromagnet will be used, oriented so that the uniform magnetic field in the air gap is horizontal. If current is passed through a straight horizontal conductor which is hung in the gap, perpendicular to the field direction, the resultant force will be vertical and can be measured by means of a balance. The balance to be used is illustrated in Fig. 1. (It is a simplified version of the precision apparatus used in the Bureau of Standards experiment described on pages 729 and 730 of Halliday and Resnick.)

The current in the magnet coils is supplied by the circuit shown in Fig. 2. (Note Caution (1).)

The current in the conductor is supplied by the circuit shown in Fig. 3.

Measurements: For the maximum obtainable magnet current, determine the weights needed to bring the balance to equilibrium for several different values of current through the conductor. (Note that it is easier to make the final adjustment on the current, once weights have been selected for the approximate current value.) Repeat for two different (i.e. lower) values of the magnet coil current.

Computations: Present the measurements of force vs. current graphically, (on a single sheet of graph paper, if you wish), and from these graphs find the value of B at each of the three values of the magnet coil current.

PART II - INDUCED EMF ACROSS A COIL

A more common method of measuring magnetic induction is to use a ballistic galvanometer to measure the charge passed through a small search coil when the coil is inserted into or removed from the field. The instantaneous emf induced across each turn of the coil equals $d\phi/dt$, i.e. it is proportional to the speed at which the coil is moved. The net effect, however, integrated over the time during which the coil is moved, produces a deflection of the damped galvanometer, which depends only on $\Delta\phi$ the net change in flux through the coil, N, the number of turns in the coil, and R, the total resistance of the coil and galvanometer circuit.

The "fluxmeter" which is used in this experiment uses a finely suspended galvanometer with no mechanical restoring force. The motion is damped only by the induced back emf in the galvanometer coil. The indicating needle remains stationary after each deflection and can be returned to zero only by passing charge through the coil. For most of the time during which the galvanometer moves, no current is flowing in the circuit, since the galvanometer coil moves at such a speed that the instantaneous back emf just equals the emf induced in the search coil. Since the net charge passed through the circuit is negligible, the sensitivity is not strongly dependent on the circuit resistance. The deflection of the fluxmeter, however, is proportional to the net change in flux through the coil - i.e. to the value of emf integrated over the time of coil motion.

A convenient and accurate method for calibrating the "fluxmeter" is to measure the needle deflection for a known change in "flux linkages" i.e. a known $N'\Delta\phi$. At the middle of a long solenoid there is a uniform field inside the windings and no field outside. If a search coil with N' turns is wound around the outside of such a solenoid, and the current in the solenoid is changed by Δi amps, the flux linkages are changed by

$$N'\Delta\phi = N'A\Delta B = N'A\mu_0 n\Delta i \qquad (2)$$

where A is the area inside the solenoid and n is the number of turns per unit length.

Since the field in the gap of the iron electromagnet is much greater than that in an air core solenoid carrying reasonable current, the same search coil can not be used for the measurement of the field in the electromagnet gap and for the solenoid calibration of the same fluxmeter scale. Instead, two coils can be connected in series with the fluxmeter, one of few turns and small area to measure the large field and another of many turns and larger area to surround the solenoid. If both coils are kept in the circuit for both measurements, the resistance of the circuit will remain constant, and the known flux linkages through the larger coil can be used to calibrate for the measurement by the small coil (of known area and number of turns) of the field of the electromagnet.

Measurements: Use the circuit shown in Fig. 4.

(1) Measure the fluxmeter deflection on inserting and withdrawing the small search coil from the gap of the electromagnet for the same three values of magnet coil current used in Part I.

(2) Calibrate the same fluxmeter scale for several values of current change through the solenoid.

Computation: Check to see whether the fluxmeter deflection is proportional to Δi through the solenoid, i.e. to $\Delta \phi$. Compute the value of B in the electromagnet gap for each of the magnet coil currents. Compare the values of B with those measured in Part I, and indicate whether agreement is within the experimental errors.

PART III - FLOATING WIRE

Consider a light flexible wire held horizontally under a tension T. One end of the wire is fixed while the other runs over a pulley to a weight which supplies the tension as shown in Fig. 5. Assume that there is a uniform vertical magnetic field, B, over the region of the wire. If a current, i, is passed through the wire, then there will be a horizontal force normal to the wire at every point. As a result, the wire will form the arc of a circle, as indicated in Fig. 6. Prove (before coming to laboratory) that the radius of curvature of the circular arc is

$$\rho = \frac{T}{Bi} \qquad (3)$$

In this experiment, B will be determined by measuring ρ, T, and i. Actually, this "floating wire" technique has a more common application in determining the trajectory of charged particles in magnetic fields. A particle of charge e, with horizontal linear momentum equal to p, will move in a circle of radius ρ in a vertical magnetic field B, where

$$\frac{p}{e} = B\rho \qquad (4)$$

Prove this result.

From equations (3) and (4), we see that in purely magnetic fields, the floating wire will follow a trajectory identical to that of a beam of charged particles if $p/e = T/i$. It is not necessary to perform a floating wire experiment to predict the simple circular orbits of charged particles in uniform magnetic fields: but the wire measurement is most useful when the beam of particles passes through non-uniform field regions, such as those in magnetic lenses.

For the maximum magnet current used in part I and for several combinations of T and i in the wire, measure the wire's radii of curvature. Compute a value of B from each of these measurements. Compare this value of B with those for the same magnet current made in parts I and II.

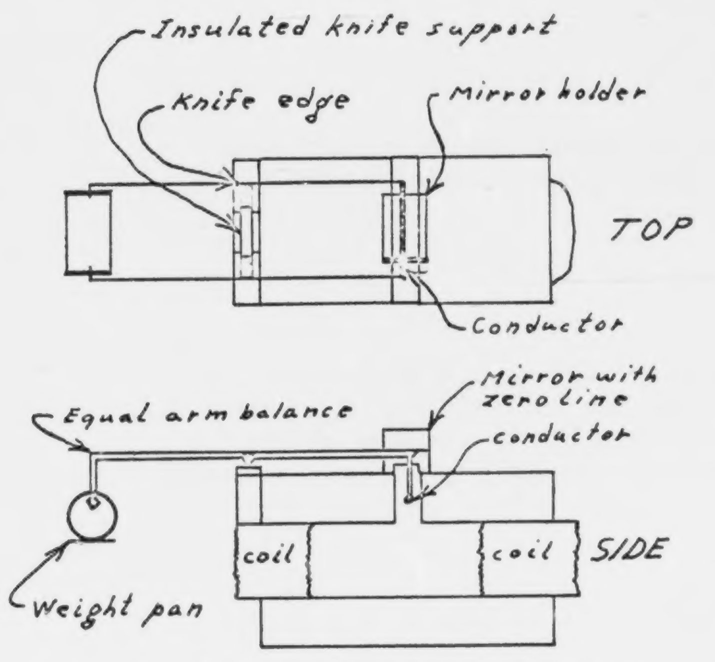

Insulated knife support
Knife edge
Mirror holder
TOP
Conductor
Mirror with zero line
Equal arm balance
conductor
coil
coil
SIDE
Weight pan

Fig. 1. Important Magnet Features.

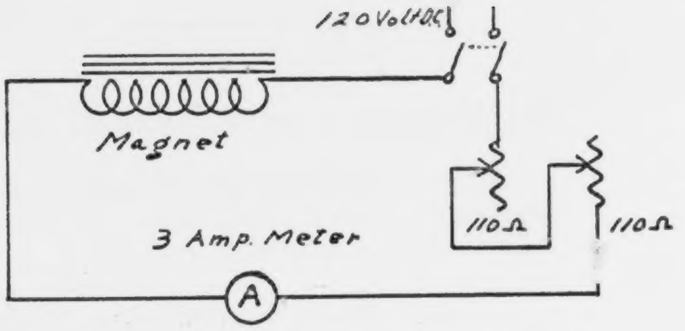

120 Volt DC
Magnet
110Ω
110Ω
3 Amp. Meter
A

Fig. 2. Magnet coil circuit. Check polarity of meter, switch, and coils. Note that rheostats are in series.

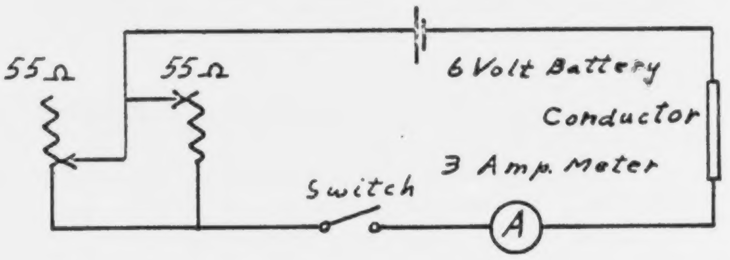

55Ω
55Ω
6 Volt Battery
Conductor
3 Amp. Meter
Switch
A

Fig. 3. Conductor circuit.

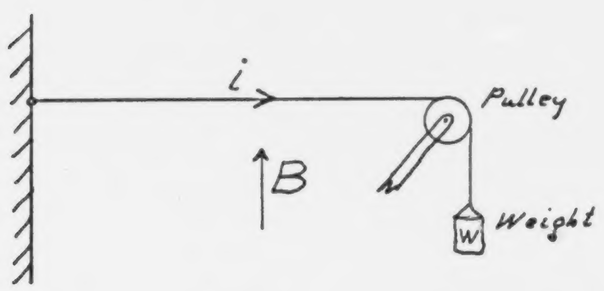

i
Pulley
B
W
Weight

Fig. 5. Wire in tension in field B.

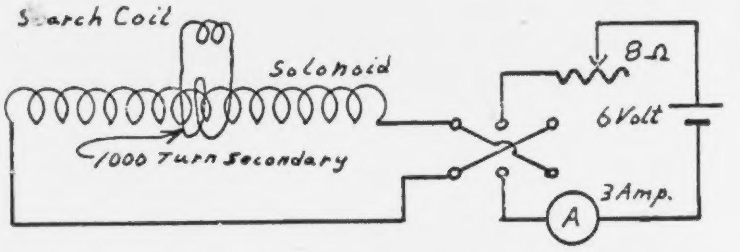

Search Coil
Solonoid
8Ω
6 Volt
1000 Turn Secondary
3 Amp.
A

Fig. 4. Search coil calibration circuit.

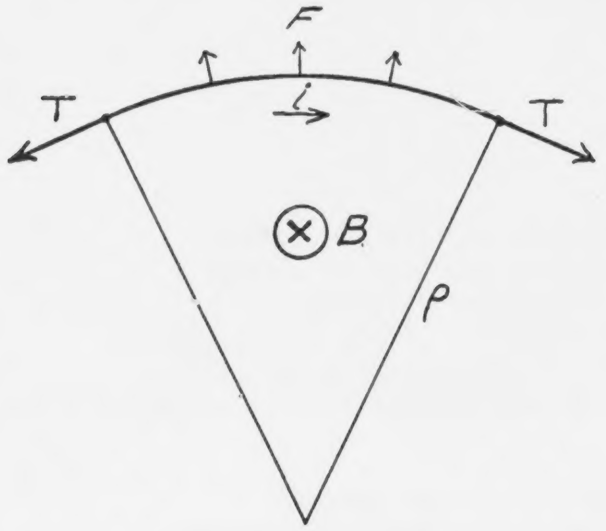


Fig. 6. Force on a conductor in a uniform magnetic field B (into the paper) causes it to form an arc.

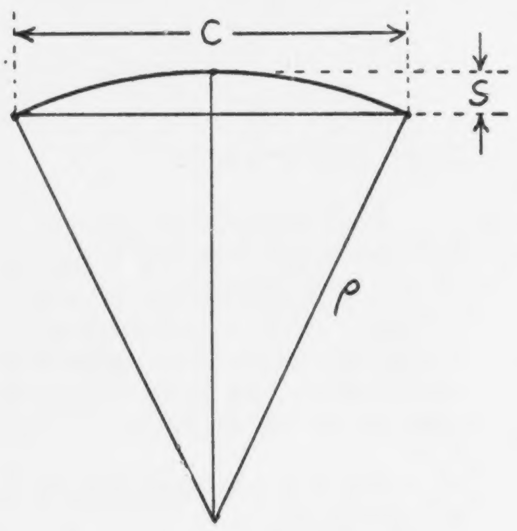


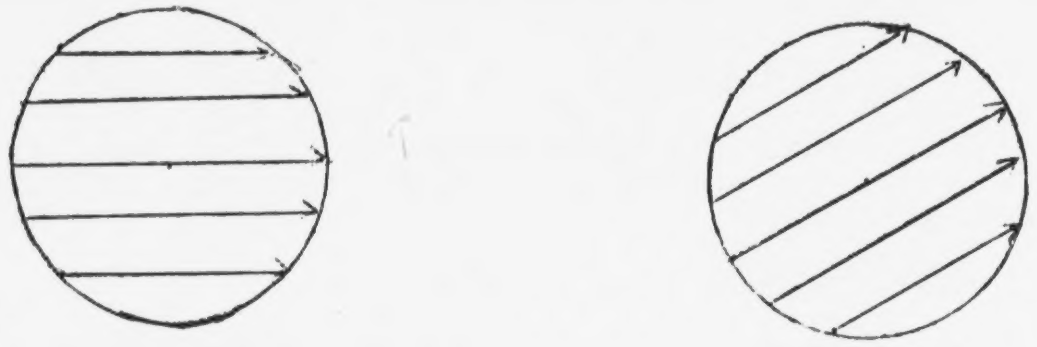
Fig. 7. The apparatus will let you measure the chord C and the sagitta S. From these you may compute ρ.

EDDY CURRENTS

Harvard University

Faraday's Induction Law applies to extended conducting media as well as to linear circuits. If the magnetic flux linking a closed path in a conductor is changing, an e.m.f. will be induced and a current will flow in the closed path. In the core of a transformer, for example, such eddy currents can cause undesirable losses.

In this experiment, a conducting cylindrical shell is placed in a magnetic field perpendicular to its axis. The field inside the cylinder is unchanged by the presence of the conductor. If the cylinder is now rotated about its axis, eddy currents flow in the shell, parallel to the axis of rotation. The magnetic field produced by these currents causes a rotation of the field direction inside the cylinder. (Fig. 1)



cylinder at rest

cylinder rotating about axis of symmetry

Fig. 1

THEORY

Let the cylinder have radius a and wall thickness da, and assume that it rotates with angular velocity $\underline{\omega}$ about the z-axis. The cylinder is of infinite length and is immersed in a field B_o as shown in Fig. 2. Any charges in the conductor will experience a force

$$F = q\,(\vec{v} \times \vec{B}),$$

where $\vec{v}$ is the velocity of an element of the conductor. This is equivalent to an electric field

$$\vec{E} = \vec{v} \times \vec{B}. \qquad (1)$$

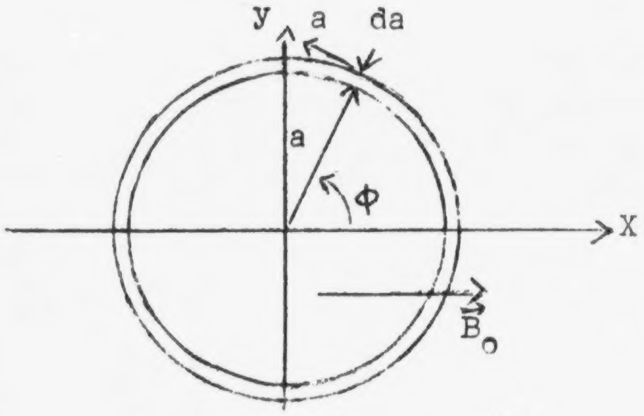


Fig. 2

For the geometry of Fig. 2, $\vec{E}$ is directed along the z-axis:

$$E_z = -\omega a\ B_o \cos\phi$$

The medium which comprises the cylinder is assumed to obey Ohm's law:

$$\vec{J} = \sigma\vec{E}, \qquad (2)$$

so that the current density at any point on the shell is

$$\vec{J} = J_z = -\sigma\omega a B_o \cos\phi\ . \qquad (3)$$

The next step in the calculation is to find the magnetic field resulting from a current distribution of the form (3). This result is then superposed upon the original field to the problem. It is most convenient to proceed by using the magnetic vector potential, defined by

$$\vec{B} = \text{curl}\ \vec{A} \qquad (4)$$

To find the vector potential at any point inside the cylinder, it is necessary to first find the vector potential dA for an infinitely long wire carrying a current.

$$di = J_z ds = -\sigma\omega a B_o \cos\phi\ \ (a d\phi\ da) \qquad (5)$$

The magnetic field of an infinitely long straight wire carrying a current i is, in cylindrical coordinates (R, θ, z),

$$\vec{B} = \hat{u}_\theta\ \frac{i}{2\pi R}$$

In cylindrical coordinates the curl of a vector is

$$\text{curl } A = \frac{1}{R}\begin{vmatrix} \hat{u}_R & R\hat{u}_\theta & \hat{u}_z \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_R & RA_\theta & A_z \end{vmatrix}$$

For the infinite wire this gives three equations:

$$B_R = \frac{1}{R}\left(\frac{\partial A_z}{\partial\theta} - R\frac{\partial A_\theta}{\partial z}\right) = 0$$

$$B_\theta = \left(\frac{\partial A_R}{\partial z} - \frac{\partial A_z}{\partial R}\right) = \frac{\mu_o i}{2\pi R}$$

$$B_z = \frac{1}{R}\left(\frac{\partial}{\partial}\frac{(RA_\theta}{R} - \frac{\partial A_R}{\partial\theta}\right) = 0.$$

These three equations can be satisfied by picking

$$A_R = 0$$

$$A_\theta = 0 \qquad (6)$$

$$A_Z = -\frac{\mu_0 i}{2\pi} \ln R$$

Returning to the main problem, and using the coordinate system indicated in Fig. 3, the vector potential at the point (r, θ)

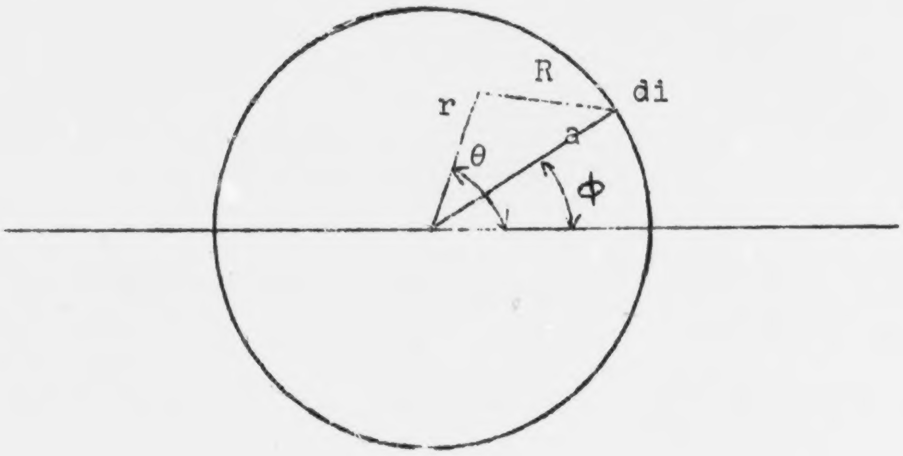


Fig. 3

is given by

$$\vec{A} = A_Z = dA_Z = \frac{\mu_0 \sigma \omega B_0 a^2 da}{2\pi} \int_0^{2\pi} d\phi \, \ln R \cos\phi \qquad (7)$$

It is now necessary to make use of an expansion, valid for $r < a$.

$$\ln R = \ln a - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{r}{a}\right)^m (\cos m\theta \cos m\phi + \sin m\theta \sin m\phi). \qquad (8)$$

(You may derive eq. (8). Hint: $R^2 = a^2 + r^2 - 2\,ar \cos(\theta - \phi)$. Note that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots\ldots$

and use the necessary trigonometric identities.) This expansion may be applied to eq. (7).

$$A_z = \frac{\mu_0 \omega B_0 a^2 da}{2\pi} \left[\ln a \int_0^{2\pi} \cos\phi \, d\phi - \sum \frac{1}{m} \left(\frac{r}{a}\right)^m \left\{ \cos m\theta \int_0^{2\pi} \cos\phi \cos m\phi \, d\phi + \sin m\theta \int_0^{2\pi} \cos\phi \sin m\phi \, d\phi \right\} \right] .$$

All the integrals vanish except $\int_0^{2\pi} \cos^2\phi \, d\phi = \pi$. Hence

$$A_z = \frac{-\mu_0 \sigma \omega B_0 a \; da}{2} \; r \cos\theta = \frac{-\mu_0 \sigma \omega B_0 a \; da}{2} x \; . \quad (9)$$

The field inside the cylinder, due to the sheet of current, can now be calculated.

$$B_x = \frac{\partial A_z}{\partial y} = 0$$

$$B_y = - \frac{\partial A_z}{\partial x} = + \frac{\mu_0 \sigma \omega B_0 \; a \; da}{2}$$

Superposing the original B_c, and denoting the angle of rotation of the field by α,

$$\tan \alpha = \frac{B_y}{B_c} = \frac{\mu_0 \sigma \omega a \; da}{2} \; . \quad (10)$$

EXPERIMENTAL TECHNIQUES

The The apparatus for verifying eq. (10) is fairly simple. A small magnetic dipole is fastened to the end of a soda straw, the other end of which is suspended by a fine silk fiber. A galvanometer mirror is attached to the soda straw at the upper end, and the entire assembly is placed in a glass tube to eliminate air currents. Various size piece of copper or brass tubing are slipped around this assembly and rotated on a phonograph turntable. A pair of Helmholtz coils provide the original field. A galvanometer lamp and scale are used to measure the angle of deflection, which is twice the angle of rotation of the mirror.

The turntable has three speeds. If one makes measurements on any given cylinder(s) at more than one speed an estimate of some of the systematic errors may be obtained.

The angular velocity is obtained with a stopwatch; since the cylinders vary in weight and are all somewhat heavier than phonograph records, this velocity should be measured for each cylinder.

If undue difficulty in measuring deflection angle is experienced as a result of the oscillations of the suspended magnet and mirror, it might be desirable to partly fill the shielding glass tube with water to damp these oscillations. Note also that these oscillations mean that the suspension exerts a restoring torque on the suspended dipole; you may find it interesting to consider what one does about this.

Conductivity is determined by measuring dimensions and resistance of the cylinders used. A vernier caliper is available for dimensional measurements. Resistance is measured with a Kelvin Bridge; for a discussion of its principle of operation, see Page and Adams, p. 187, or any book on electrical measurements. The Kelvin Bridge available for this experiment gives best results when used with an external power supply rather than the built-in battery; this permits the use of higher current and results in greater sensitivity. Current through the bridge should under no circumstances exceed 10 amperes, and good results can be obtained with somewhat less current.

In order to minimize drift arising from heating of bridge components, current should be allowed to flow only while a measurement is being made. Measurements should be made as rapidly as possible without sacrifice of accuracy.

The cylinders are held in a specially made bracket for the resistance measurement. One should be sure that all the necessary electrical connections are well made; it may be necessary to clean the cylinders where they contact the holder.

References: Halliday and Resnick, sections 38-1 through 38-4.

DISCUSSION

The Physical Process--In Exp. 16 we saw that an RCL circuit can have oscillatory transients, provided R is small enough. In the limiting case of $R \rightarrow 0$ we would have a continuous migration of energy between the capacitor when it is completely charged, (no current), and the field in the coil, when current flows through the coil (capacitor discharges). We have a sinusoidally oscillating current through the circuit and oscillatory changing voltage across the capacitor, which oscillates with a frequency:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

If R is bigger than zero,but small, we have the case of damped oscillation.

We can now imagine two cases:

(a) The circuit is cut as in Fig. 1 and attached to a generator.

If the A.C. generator is tuned to exactly the resonant frequency of the circuit, the A.C. voltage of the generator can always be in phase with the current in the circuit. The generator is pumping energy into the circuit, either to compensate for the dissipative losses in the resistance or even to increase the current and voltage amplitude. The circuit behaves, for this particular frequency of the generator, as a very low impedance circuit, and we say that the generator and the circuit are in resonance.

For any other frequency of the generator, its voltage is not in phase with the current; they are often in opposition (depending on the difference between the resonant and applied frequencies), so that the circuit is preventing the current from flowing from the generator. The circuit shows an increased impedance compared to that at resonance.

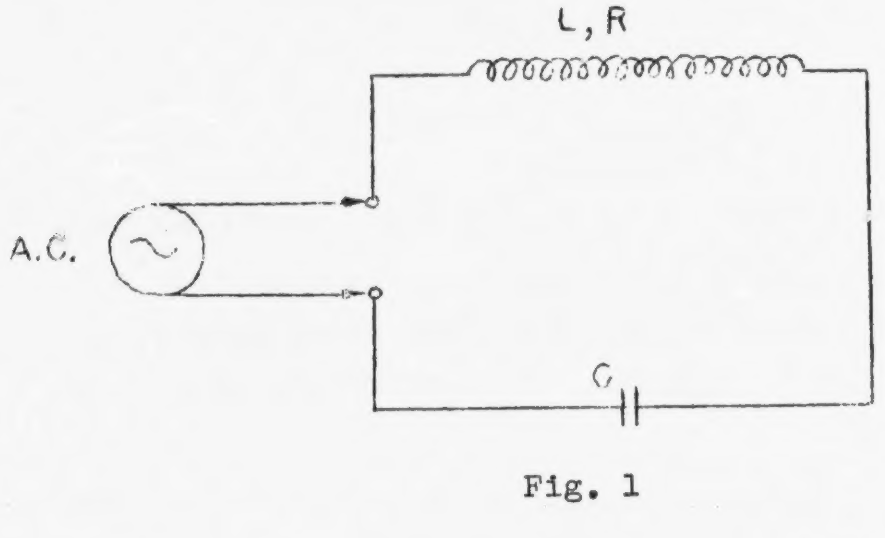


Fig. 1

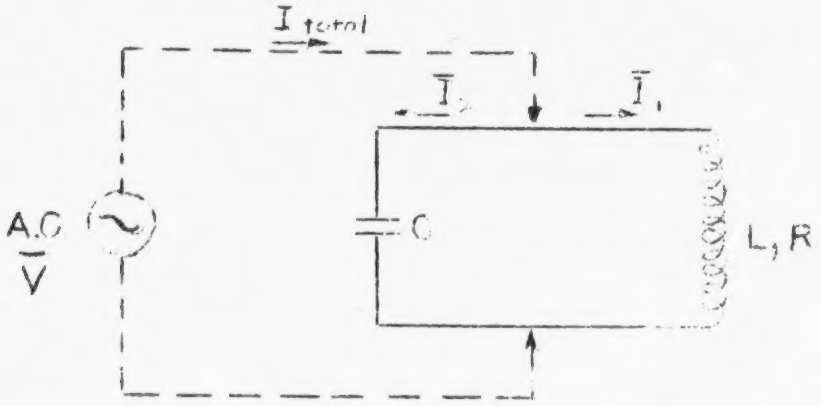


Fig. 2

(b) Now the generator is attached as in Fig. 2.

If the generator produces exactly the frequency of the transient oscillations of the current, the generator voltage will be in opposition to that across the capacitor, and almost no current will flow from the generator into the circuit. The circuit behaves like a high impedance and we have the case of 'anti-resonance'.

If the generator changes frequency, its voltage is no longer always in phase with the capacitor voltage and there is more chance for the current to flow from the generator through the circuit. Its impedance decreases.

The Mathematical Description

(a) The series resonant circuit--The vector equation relating the current I, the voltage V, and vector impedance Z, in the circuit shown in Fig. 1 is:

$$\overline{V} = \overline{I}\,\overline{Z} \qquad (1)$$

The scalar magnitudes are related by the formula:

$$I_m = V_m / Z = V_m / \sqrt{R^2 + X^2} \qquad (2)$$

where

$$X = \omega L - 1/\omega C \qquad (3)$$

and the vector $\overline{V}$ leads the vector $\overline{I}$ by an angle ϕ, where $\tan\phi = X / R$.

Consider the way in which I varies as ω is increased and V is held constant. At small values of ω, X is large and negative, hence Z is large, I_m is small and $\overline{I}$ leads $\overline{V}$. At large values of ω, X is large and positive, hence Z is large, I_m is small and $\overline{I}$ lags behind $\overline{V}$. In the neighborhood of the 'resonant' value

$$\omega = \omega_o = 1 / \sqrt{LC} \qquad (4)$$

I_m changes very rapidly with ω. At the critical frequency, $\omega = \omega_o$, the following relations are true:

1. The vector impedance is a pure resistance: $Z = R$.
2. The value of the scalar impedance is a minimum,

$$Z \text{ min} = R$$

3. The scalar current is a maximum

$$I_m = V_m / R$$

4. The current is in phase with the voltage, $\Phi = 0$.

The frequency $f_o = \omega_o / 2\pi$, is called the 'resonant' frequency of the circuit.

It follows immediately from equations (2), (3) and (4), by elimination of C, that:

$$Z = \sqrt{R^2 + (L^2 / \omega^2)\ (\omega^2 - \omega_o{}^2)^2} \qquad (5)$$

At frequencies in the neighborhood of f_o, denote $\omega - \omega_o = \Delta\omega$. Furthermore $\omega + \omega_o \approx 2\omega$ (approximate equality). Therefore

$$Z \approx \sqrt{R^2 + (2L\Delta\omega)^2} \qquad (6)$$

A measure of the behavior of the circuit, as far as the resonance is concerned, is the ratio of the width of the current curve at height $I = I_m / \sqrt{2}$, to the frequency at the peak of the curve, and is called the 'sharpness' of the resonance.

This width is obtained as follows: when $R = 2L\Delta\omega$, $I = 0.707\ I_m$ Therefore: $\Delta\omega = R / 2L$, and the width is:

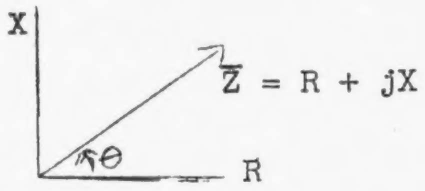
$$\text{width} = 2\Delta f \approx \frac{R}{2\pi L} \qquad (7)$$

The 'sharpness':

$$\frac{2\Delta f}{f} = \frac{R}{2\pi f L} = \frac{R}{\omega L} = \frac{1}{Q} \qquad (8)$$

is the reciprocal of the Q factor of the circuit, introduced in Exp. 16. A high Q circuit, therefore, has a very narrow resonance curve.

(b) The parallel resonant circuit--In order to analyze the parallel LCR circuit shown in Fig. 2, it would be possible, but overwhelmingly complicated, to use direct trigonometric methods analagous to those used to analyze the series circuit in Exp. 17. Instead, we introduce complex numbers as a method of representing vectors in a plane. In reference to the vector diagram, where resistance is plotted on the horizontal axis and reactance vertically, we define vector impedance as a complex number, $\overline{Z} = R + jX$, where $j = \sqrt{-1}$ and $X = \omega L - 1/\omega C$. That is, $|\overline{Z}| = (R^2 + X^2)^{1/2}$ and $\tan\theta = X/R$. Voltage and current which are also vector quantities with amplitude and phase angle, can similarly be considered as complex numbers, with a real value corresponding to phase angle of zero and a purely imaginary value corresponding to a phase angle of ± 90 degrees. For an A.C. circuit, we then have,



$$\overline{V} = \overline{I}\,\overline{Z} \qquad (9)$$

In the parallel circuit of Fig. 3,

$$\frac{\overline{I}_{total}}{\overline{V}} = \frac{\overline{I}_1}{\overline{V}} + \frac{\overline{I}_2}{\overline{V}} = \frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2} \qquad (10)$$

where $\overline{Z}_1 = R + j\omega L$ and $\overline{Z}_2 = -j(1 / \omega C)$. The reciprocal of a complex number can be found by multiplying through by the complex conjugate:

$$\frac{1}{\overline{Z}_1} = \frac{(R - j\omega L)}{(R - j\omega L)\,(R + j\omega L)} = \left(\frac{1}{R^2 + \omega^2 L^2}\right)(R - j\omega L)$$

Therefore

$$\frac{\overline{I}_{total}}{\overline{V}} = \frac{R}{R^2 + \omega^2 L^2} - j\left(\frac{\omega L}{R^2 + \omega^2 L^2} - \omega C\right) \qquad (11)$$

'Resonance' for the parallel circuit can be defined in two ways:

(1) Phase Resonance, where the total current is in phase with the applied voltage, i.e. the imaginary part of equation (11), vanishes:

$$\frac{\omega L}{R^2 + \omega^2 L^2} - \omega C = 0$$

or

$$\omega_o^{\text{phase}} = \omega = \left(\frac{1}{LC} - \frac{R^2}{L^2} \right)^{1/2} \qquad (12)$$

(2) Amplitude Resonance, where the amplitude of the total current is a minimum for a given amplitude of applied voltage. In order to find the relationship among ω, L, C, and R for amplitude resonance, the derivative of the amplitude of equation (11) (with respect to ω) can be set equal to zero. Solving for ω, (after considerable computation), we have

$$\omega_o^{\text{ampl.}} = \omega = \left(\frac{\sqrt{1 + 2R^2(C/L)}}{LC} - \frac{R^2}{L^2} \right)^{1/2} \qquad (13)$$

For a well made inductance, $R \ll \omega L$ over the frequency range for which it is designed. Under this approximation, equations (12) and (13) both reduce to equation (4); that is, the frequencies for both types of parallel resonance approach the frequency for series resonance, $f_o = \frac{1}{2\pi} \sqrt{1/LC}$.

PROCEDURE

Determination of phase relations:

In this experiment, the voltage across one portion of a circuit will be applied to the vertical deflecting plates and the voltage across another portion of the circuit to the horizontal deflecting plates of an oscilloscope. The resultant pattern is a Lissajous figure and may be analyzed to give information concerning the relative phase of the two voltages. Thus, if $V_1 = V_{o_1} \cos\omega t$ and $V_2 = V_{o_2} \cos(\omega t \pm \theta)$ represent the two voltages, an elliptical figure will be obtained as is indicated in Fig.3.

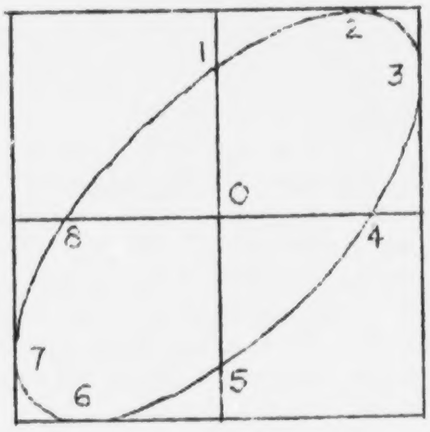


Fig. 3

In Fig. 3 the point 0, the geometric center of the ellipse, is taken as the origin of a coordinate system. At the points 1, 4, 5, and 8 the ellipse crosses the coordinate axes, and at the points 2, 3, 6, and 7 the tangents are vertical or horizontal. If (x_1, y_1) represent the position of point 1, etc., then the student may show, as an exercise, that:

$$\sin\theta = \frac{y_1 - y_5}{y_2 - y_6} = \frac{x_4 - x_8}{x_3 - x_7} \qquad (14)$$

and

$$\cos\theta = \frac{y_3 - y_7}{y_2 - y_6} = \frac{x_2 - x_6}{x_3 - x_7}$$

If θ is nearly 90° the major axis of the ellipse is almost vertical and the expression for cos θ is the most useful. If θ is nearly 0° or 180° the ellipse is very narrow and the expression for sin θ is the most useful. Note that when the two voltages are in phase, the Lissajous figure becomes a straight line.

PART I - Series Resonant Circuit

Connect the series CLR circuit shown in Fig. 4, where C = .01 microfarad, L is a small R.F. choke, R is a 47-ohm resistor, and the A.C. source is a variable frequency signal generator.

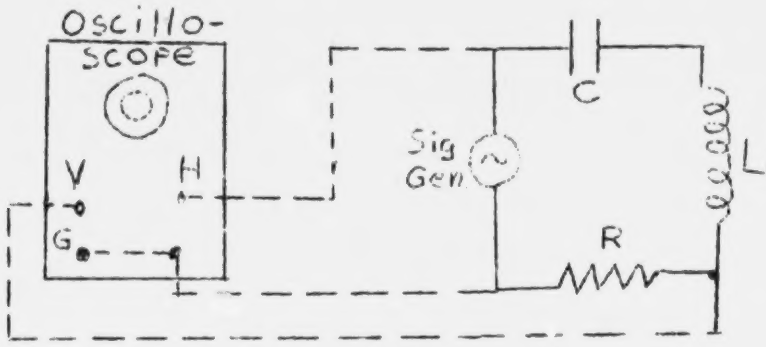


Fig. 4

Measurements

Vary the frequency of the signal generator and take data for resonance curves, by measuring both the amplitude and phase of the voltage across R, (i.e. the current), compared to the applied voltage, as a function of frequency. Note that the voltage output of the signal generator can be kept constant by adjusting the generator amplitude for constant horizontal deflection of the oscilloscope at each frequency. Take closely spaced points in the most interesting region of the curves.

For a larger value of R, take data for a resonance curve of amplitude vs. frequency. (Do not bother with phase measurements.)

Reconnect the oscilloscope to measure the phase and amplitude difference between V_L and V_C at the resonant frequency using the smaller value of R.

Measure the D.C. resistance of the coil with a Simpson Ohm-meter.

Computations

Plot the resonance curves on a single sheet of graph paper. Explain briefly.

Find the value of L from the value of the resonant frequency.

Determine the Q of each of the circuits from the widths of the amplitude resonance curves.

Calculate the resistance of the coil from the Q-values.

Calculate the resistance of the coil independently, from the measurement of phase difference between V_L and V_C, using the previously determined values of L and ω_o.

Calculate the resistance of the coil from the voltage across R (known resistance) at resonance.

Compare the calculated values of the coil resistance.

Part II - Parallel Resonant Circuit

Connect the parallel resonant circuit shown in Fig. 5 with the same L and C as in Part I, but with R = 1000 ohms.

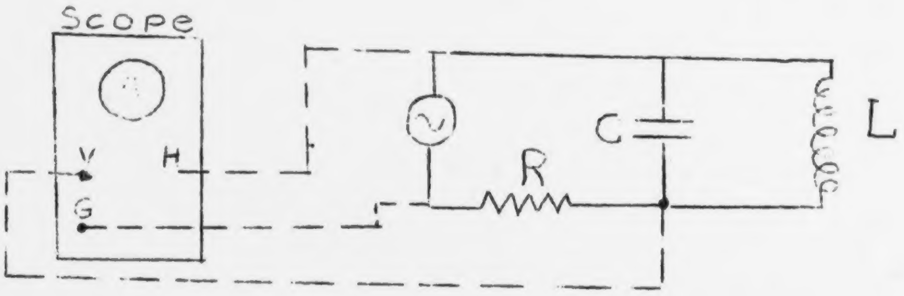


Fig. 5

Take data for resonance curves, measuring both amplitude and phase of V_R compared to the applied voltage.

Add a variable resistor in series with L. Measure the frequency (not the whole curve) for amplitude resonance and phase resonance at several interesting values of the resistance.

Plot the resonance curves and explain your results.

Electronics

THERMIONIC EMISSION

Massachusetts Institute of Technology

Part I: Construction of vacuum tube.

A. The basic design for the tube is as shown by the tube on display.

The main features are:

Header "A"	Phillips Gauge and getter
Header "B"	Diode - note that more than one filament is used, in case of burnout.

Composition:

Header "A" - Phillips Gauge - plates - cut from sheet nickel - wire loop - nichrome wire

Getter - Barium

Header "B" - Diode - plates - cut from sheet nickel filaments - thoriated tungsten

Remarks

1. One should become familiar with the use of the spot-welder before working with the headers, finding the appropriate Variac setting for the thickness of wire to be welded.

2. Care must be taken in working with the headers that the glass seals do not crack as this may introduce leaks.

3. The plates of the Phillips Gauge should not be too near the cathode ring, as shorting will occur.

4. For the Thermionic Emission experiment it is important that the entire filament(s) be easily visible.

5. Do not kink the tungsten wire, as this will cause rapid burnout of the filament.

B. After the tube is built, assemble with solder glass and leave it to be baked in the oven.

C. Once the tube is baked, seal-off procedure is as follows:

1. Close off one end.

2. Evacuate as low as the vacuum system will go.

3. Outgas the filaments and the getter. (Heat till a dull red glow is visible for about two minutes. Do not fire the getter! If metallic deposit forms on the side of the tube, the getter is too hot).

4. Seal off the other end carefully.

5. Fire the getter. (Heat till it glows bright red and leave it till it burns out, leaving deposit on the side of the tube).

Final pressure should be (as read by Phillips Gauge) less than 10^{-5}mm Hg.
Note in connecting Phillips Gauge that the black lead goes to the plates.

Thermionic Emission Part II

References:
1. Born, M. - Atomic Physics, pp 267-270, 273-276, 417-419
2. Harnwell and Livingood, Experimental Atomic Physics
3. Sears, F. W. - An Introduction to Thermodynamics, The Kinetic Theory of Gases, and Statistical Mechanics, Chap. 16, pp 325 - 335, 337-338
4. Handbook of Chemistry and Physics. Data on brightness of Tungsten. pp. 2744, 40th Edition

- -

The purpose of this experiment is to investigate the temperature dependence of the thermionic emission from thoriated tungsten. The thermionic current of a vacuum diode will be measured as a function of the filament temperature. The temperature of the filament is measured with an optical pyrometer.

Quantum statistics yields the following equation (Richardson's Equation) for the thermionic current, I, in the absence of space charge.

$$I = AT^2 e^{\frac{-q\phi_R}{kT}}$$

In this equation A is a dimensional constant, T is the absolute temperature, q is the magnitude of the charge of the electron, ϕ_R is the Richardson work function, and k is Boltzmann's constant.

In cases where the exponential term outweights the T^2 dependence, Nottingham has proposed the following revision of Equation (1):

$$I = a\ e^{-q\phi / kT}$$

In Equation (2) a is an empirical constant (a function of temperature which varies much more slowly than does the exponential term) and ϕ is the work factor.

For further details see the references and most any good book on physical electronics or atomic physics.

APPARATUS AND PROCEDURE

A circuit diagram is shown in Figure 1. The diode constructed should have a thoriated tungsten filament which is easily viewed for temperature measurement purposes. Alternating current is used to heat the filament since no precise measurements of the potential between filament and plate have to be made.

The tube filament should not be taken over 3300°K (a brightness temperature - as read by the pyrometer - of 2640° Centigrade) since the filament melts at 3643° Abs.

The filament temperature is read with the Leeds and Northrup optical pyrometer provided. In using the pyrometer to measure temperatures we must be aware of the following impediments:

(a) The filament temperature is not constant along its length (It loses heat by conduction through its ends). (How would you propose to investigate this?)

(b) The pyrometer used in this experiment is designed to measure the temperature of a "black body" - a "black body" being any system at equilibrium at a definite temperature, T, with all surrounding radiation. The filament is not a "black body" since it radiates much more energy than it absorbs. In addition, it loses energy in the form of kinetic energy of emitted electrons. For further details see Reference 2 above. There are tables available with which one can translate from the apparent "black body" temperature the pyrometer sees to the actual temperature of the filament. See Reference 5.

It is left to the student to decide on the particular procedure to be followed.

REPORT

Your report should include the following:

(a) Data, including both brightness and actual temperatures.
(b) Semi-log plots of both I/T^2 and I versus $1/T$.
(c) Discussion of results and errors.

OPTIONAL:

Plot the filament current (RMS), voltage (RMS), and resistance versus temperature for comparison with published data.

QUESTION:

What is the difference between a "space-charge Limited" and a "temperature-limited" plate current? Illustrate graphically.

Special Note

The reader is referred to the booklet Homemade High Vacuum Techniques, prepared by the Science Teaching Center, Massachusetts Institute of Technology, Cambridge 39, Massachusetts, for detailed information about the techniques of using solder glass and achieving a high vacuum by comparatively simple means.

Thermionic Emission

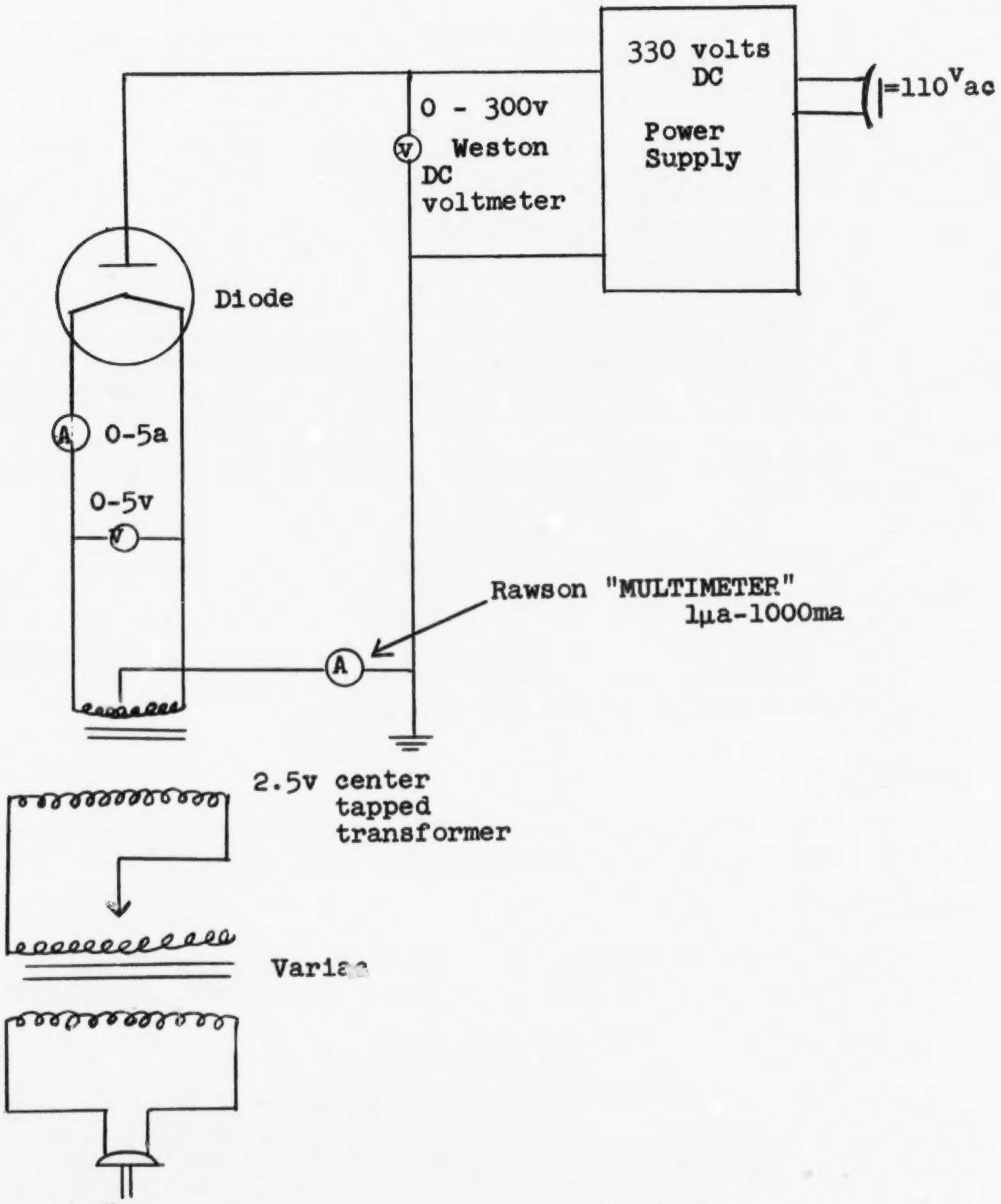

330 volts
DC
Power
Supply
=110^{v}ac
0 - 300v
Weston
DC
voltmeter
Diode
0-5a
0-5v
Rawson "MULTIMETER"
1μa-1000ma
2.5v center
tapped
transformer
Variac
110^v a.c.

Figure 1

TRANSISTOR CHARACTERISTICS

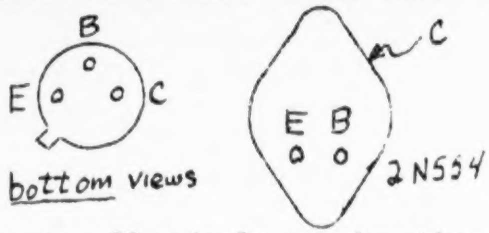
University of Delaware

REFERENCES: Jerrard and McNeill, Theor. & Exp. Phys., pp. 576-581 (experimental)
Shea, Princ. of Trans. Circ., Sec. 2.2.4, 3.1, 3.2, 3.8 (theoretical)
Chirlian & Zemanian, Electronics, pp. 185-189, 193-198 (introductory)

OBJECT: An introduction to transistors; to attain a feeling for the orders of magnitude of the currents and voltages involved in typical transistors; to measure the static characteristics of several transistors and to determine from them the current-amplification factor, input impedance, and output impedance of the transistor in very simple circuits.

APPARATUS: Power transistor and audio transistor, sockets, two ma-meters, μa-meter, VTVM, dry cells, Eico power supply, decade resistor box 1 ohm to 1 Meg.

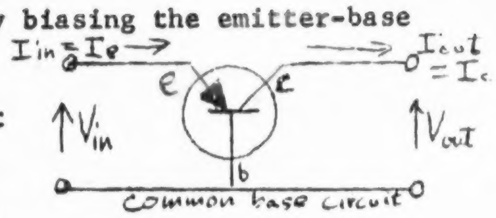
THEORY: See references for details. The transistors used in this lab are PNP germanium junction transistors. They consist of a wafer of germanium doped with traces of arsenic and indium to form three regions called the emitter, base, and collector regions, with the base sandwiched between the others. Three wires are attached to these three regions as shown (in the 2N554 the collector terminal is the metal case of the transistor.) The base region is doped negatively, the others positively, hence "PNP"; the surface between the emitter and base is thus a PN junction and the surface between base and collector is also a PN junction. Thus there are effectively two junction diodes which share a common N-region.

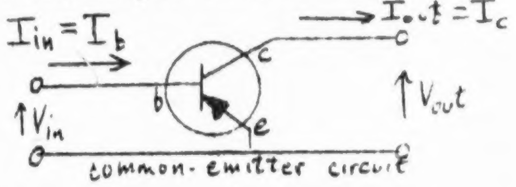


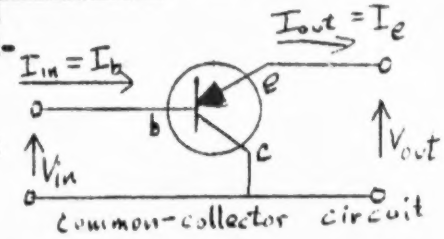
In a semiconductor both electrons and holes serve as charge carriers—these carriers are produced in pairs either by thermal excitation or by electrical injection (see next paragraph) of an electron into the conduction band from the valence band thus producing both an electron free to move in the conduction band and a hole free to move in the valence band. These holes and electrons will recombine when they diffuse close enough together and consequently an equilibrium condition is set up which obeys the law of mass action: $pn = \text{constant}$, where p and n are the concentrations of positive carriers (holes) and negative carriers (electrons), respectively. In a PNP transistor the base region is doped with arsenic and so this region has more negative carriers than positive carriers, that is the majority carriers are electrons while the minority carriers are holes ($n > p$) in the base-region; current in this region consists then predominantly of electron flow but since $pn = \text{constant}$ a small change in p produces a large change in n which means that if one injects a small current of holes into the base-region, this produces a large change in (electron) current: this is current amplification.

In order to produce this injection of holes into the base, the transistor is manufactured with one P-region of lower resistivity than the base-region (by doping the P-region more heavily) which results in efficient injection or "emission" of holes into the base by this region; this P-region is therefore called the emitter. Also, relatively light doping of the other P-region produces efficient collecting of the carriers that diffuse toward it from the base; this P-region is called the collector.

Transistor action is further facilitated by biasing the emitter-base junction forward and the base-collector junction backward. To see the advantage of this consider the common-base circuit in which the input circuit is connected between emitter and base while the output circuit is connected between base and collector (the base is "common" to both input and output circuits--hence the name "common-base circuit"). Since $I_b \ll I_e$ and $I_b \ll I_c$ in general, $I_e \approx I_c$ and the circuit has about unity current gain but the output circuit has much higher impedance than the input circuit--hence the whole circuit has considerable voltage and power gain. In the common-emitter circuit, an increase in impedance levels also exists, together with the current amplification mentioned before, so this circuit has great power gain as well as considerable current and voltage gain. Finally, in the common collector circuit, the impedance increase mentioned above is now turned around and just offsets the considerable current gain, yielding approximately unity voltage gain and only moderate power gain; however, the impedance decrease with no loss in voltage has some important applications such as matching impedances of transmission lines.







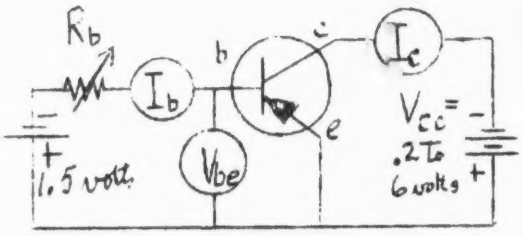
In this experiment the static characteristics of a transistor will be measured in the common-emitter circuit with V_{ce} and I_b treated as independent variables and with I_c and V_{be} as dependent variables; this choice is very practical and is well adapted to an intuitive understanding of the circuit. The curves yield directly the hybrid parameters defined by the matrix equation

$$\begin{bmatrix} v_{be} \\ i_c \end{bmatrix} = \begin{bmatrix} h_{bb} & h_{bc} \\ h_{cb} & h_{cc} \end{bmatrix} \begin{bmatrix} i_b \\ v_{ce} \end{bmatrix} .$$

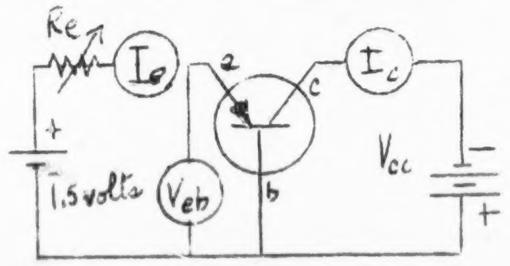
$$\therefore \quad \beta = h_{cb} \equiv \left.\frac{\partial I_c}{\partial I_b}\right|_{V_{ce}}, \quad h_{cc} \equiv \left.\frac{\partial I_c}{\partial V_{ce}}\right|_{I_b}, \quad h_{bb} \equiv \left.\frac{\partial V_{be}}{\partial I_b}\right|_{V_{ce}}, \quad h_{bc} \equiv \left.\frac{\partial V_{be}}{\partial V_{ce}}\right|_{I_b}.$$

The most important of the hybrid parameters, β, is called the "common-emitter current amplification" and can be in the range from ten to several hundred; $1/h_{cc}$ is called the "common-emitter output impedance with open input" ("open input" means I_{in} = constant) and is typically within a decade of 3K; h_{bb} is called the "common-emitter input impedance with shorted output" (shorted for AC, not DC) and is an order of magnitude smaller than the output impedance; h_{bc} is usually of little interest and not accurately measurable—it is called the common-emitter feedback factor and may typically be 10^{-3}—it is occasionally needed to anticipate the stability of a circuit.

The range of currents and voltages to be measured is limited on the high side primarily by the danger of overheating the transistor: the 2N554 can dissipate about 1 watt safely without a heat sink while the audio transistor can handle only 50 milliwatts; also the junctions will break down at 40 volts and 10 volts respectively. On the low current end the usefulness of the transistor is essentially limited when the portion of the output current due to thermal injection in the base is comparable to that due to injection of signal from the emitter; consequently we will measure I_{eo} , the collector current at zero base current, to determine the order of magnitude of this lower useful limit, and will investigate its dependence on temperature qualitatively.



PROCEDURE: Wire the circuit shown, being especially careful about polarities. Using the small audio transistor, set $R_b \geq 100$ K and $V_{cc} = 6$ volts, then reduce R_b until $I_c \approx 5$ ma; set I_b to a convenient round number via R_b, then observe $I_c(V_{ce})$ at constant I_b down to $V_{ce} = .2$ volts (note that $V_{ce} = V_{cc}$ except for the voltage drop across the milliammeter of about 1/10 volt full scale). Then observe V_{be} at two V_{ce} values, say 6 v and 3 v--the small change in V_{be} is important and (if it can be seem at all) should be recorded explicitly because it is more accurate than either V_{be} value separately: V_{be} is very small and requires careful zeroing of the VTVM. Repeat for smaller I_b values in convenient uniform ΔI_b steps. Then take two more sets of $I_c(V_{ce})$ with very low I_b--say 1/100 and 1/200 times the first I_b value. To determine how small an input current can compete with thermal injection, make $R_b = \infty$ and observe the collector current I_{co} with a microammeter. Hold a finger on the transistor case and observe the effect on I_{co}.



Study the common-base circuit in a manner similar to the above procedure: two milliammeters will now be needed: first compare them by placing them in series in the collector circuit and record any difference in their calibration, carefully noting which meter is to be used in the emitter circuit. Again, start with $R_e \geq 1$ K and $V_{cc} = 6$ volts and use R_e to reach $I_e \approx I_c \approx 5$ ma and observe $I_c(V_{cb}, I_e)$ for a half-dozen V_{cc} values and several I_e values (below 5 ma); then observe $V_{eb}(V_{cb}, I_e)$ for two V_{ce} values and a half-dozen I_e values; in the common base circuit the change in V_{eb} with V_{cb} should be measurable. Finally make R_e infinite and observe I_{eo} (with microammeter), again checking the effect of hand temperature.

To see the different order of magnitudes possible with large power transistors, replace the small transistor with the large one and replace the power supply with several dry cells. With a modest heat sink the 2N554 can dissipate several watts, so the collector current may be made as large as 1/2 ampere at $V_{ce} = 3$ volts. Make reasonably complete $I_c(V_{ce}, I_b)$ observations (half-a-dozen V_{ce} values and I_b values) for the common-emitter circuit, but the common-base measurements may be omitted if time is pressing.

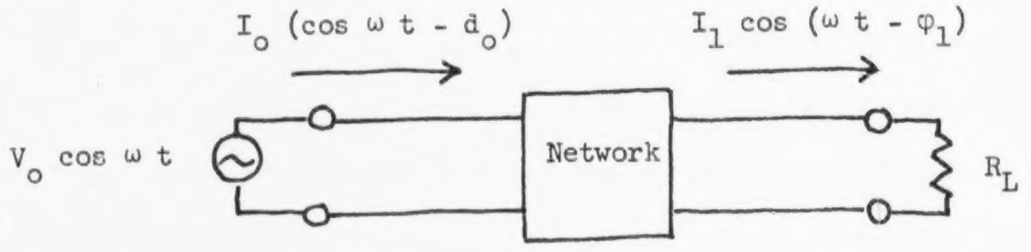
RESULTS: Make graphs of (1) I_c vs V_{ce} with I_b as parameter and (2) I_b vs V_{be} with V_{ce} as parameter for the common-emitter circuit; (3) I_c vs V_{cb} with I_e as parameter and (4) I_e vs V_{eb} with V_{cb} as parameter for the common-base circuit. From the common-emitter graphs determine β, h_{bb}, $1/h_{cc}$, (and if possible h_{bc}) at a point near the center of the curves. Do these quantities obviously vary significantly at other points on the graph? From the common-base graphs determine the analogous "input" and "output impedances" for the common-base circuit; compare these impedances with the common-emitter values. Determine β and $1/h_{cc}$ for the rather low I_b values suggested and compare with the values of these parameters at the more typical currents first measured.

University of California, Berkeley

In Part A of this laboratory we limited ourselves to what are called passive circuits. We studied the transient decay of a charged capacitor and found that the electrostatic energy of the capacitor was ultimately dissipated as heat in the circuit resistance. Similarly in the decay of an inductive current the energy associated with current flow through the inductor was ultimately dissipated in resistance. In the case of damped oscillations, although the energy was periodically exchanged between capacitor and inductor, it finally was degraded into heat. Put another way, whether we look at electrostatic energy or magnetic energy, or the sum of the two, there is an exponential decay of the form:

$$e^{-t/\tau}$$

where the characteristic time τ depends on the process involved. We can look at the same problem in terms of the response to a periodic signal. Let us imagine that we introduce a signal into some kind of network as shown below. We have a voltage $V = V_o \cos \omega t$ and a current $I_o \cos(\omega t - \varphi_o)$ at the input terminals.



If the network is terminated in a load R_L then the current flowing into the load may be given by $I_1 \cos(\omega t - \varphi_1)$. As long as the network is composed of

---

Copyright © 1963 Educational Services Incorporated. No part of this manuscript may be reproduced without written permission of Educational Services Incorporated.

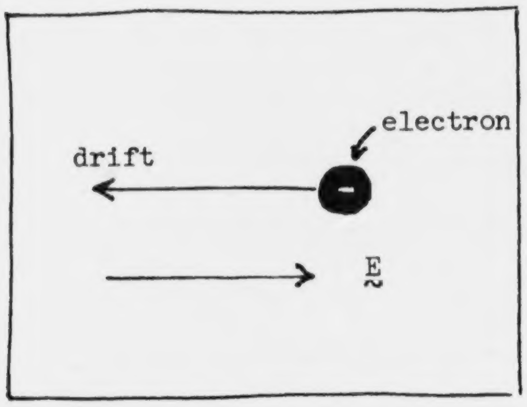
resistors and capacitors and inductors we can be sure that the power delivered to the load is smaller than or equal to the input power:

$$\frac{1}{2} I_o V_o \cos \varphi_o \leq \frac{1}{2} I_1^2 R_L$$

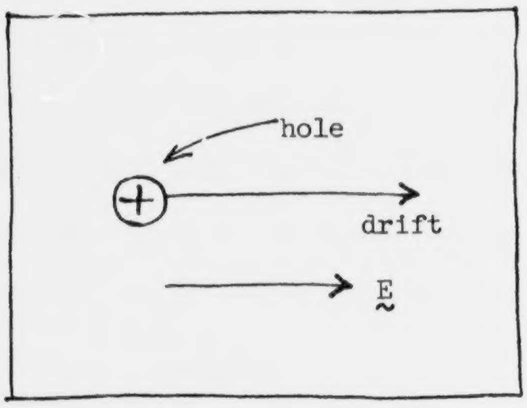
That is, a passive circuit contains no energy sources which in the transient case could lead to an exponential increase in stored energy or in steady state could lead to an increase in power transfer. If the circuit contains non-linear elements the output will appear not only at the fundamental but also at harmonics of the input signal. But the criterion of power transfer remains the same.

A circuit for which the transfered power exceeds the input power (or which under certain circumstances can lead to an exponential increase in stored energy) is called an active circuit. In its simplest form such a circuit contains an amplifier. Of course we believe in conservation of the total energy in the system. But this only means that the energy which is added to the signal at frequency ω must come from some other source, normally a battery or some other energy source. Until very recently the most common form of electrical amplifier was the vacuum tube. In such a device the current flowing from a heated cathode to an anode is controlled by the potential on a wire grid through which the electrons must flow. As long as the grid is at a potential which is negative with respect to the cathode, there will be no electron current flowing onto the grid. This means that with a negligible amount of power one can control a sizeable electron current. One may vary the grid voltage sinusoidally, developing a large signal power in the output of the vacuum tube. Such devices are largely being replaced by transistors, where the function of the grid is taken by an emitter junction and the function of the anode by a collector junction. The advantages of transistors over vacuum tubes are considerable. They are extremely small and reliable and do not require thermionic emission for the generation of charge carriers. For the purposes of this laboratory vacuum tubes have the advantage that a knowledge of the motion of charges in electrostatic fields is nearly all that is required in order to understand the behavior of a vacuum tube. We were able in Experiment A-1 to understand precisely the acceleration and deflection of an electron beam. Although the electrode configurations of a vacuum tube are more complex, the problems (at least in principle) are no different. If one is to really understand the behavior of a transistor one must know a good deal of quantum mechanics. Nevertheless we choose the transistor as the realization of an amplifier principally because it is replacing the vacuum tube and we should begin early to get an easy familiarity with its operation. Further since our emphasis here is on function rather than the mechanics of operation you should not be too distressed that much of transistor behavior you will have to take on faith. With the study of quantum mechanics, and particularly the behavior of electrons in periodic lattices, you may learn the principles of semiconductor behavior.

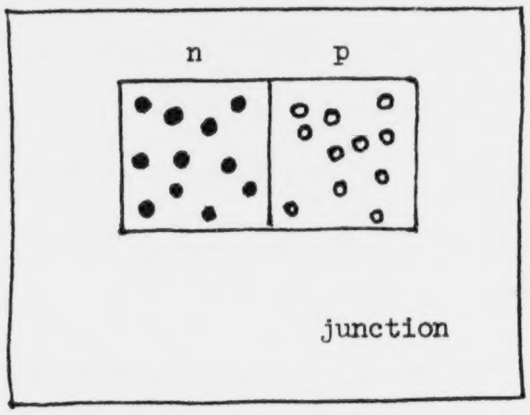
In particular you will have to take on faith that within a semiconductor, current may be carried by either electrons or holes. By chemical treatment of the semiconductor one may insure an excess of electrons or holes. The p-n junction diode, [whose properties we studied in A-10 is formed from a single crystal of semiconductor,] one half of which has an excess of electrons (n-type) and the other half an excess of holes (p-type). The "forward" direction is the direction in which electrons drift across the junction into the p-region and the holes drift in the opposite direction, their currents adding. The reverse direction is the direction for which the majority carriers drift away from the junction. The behavior of an electron in a solid is not very different from its behavior in vacuum and we need not be too surprised that it will drift opposite to the field. But what really is a hole? If it is a missing electron shouldn't it drift along with the other



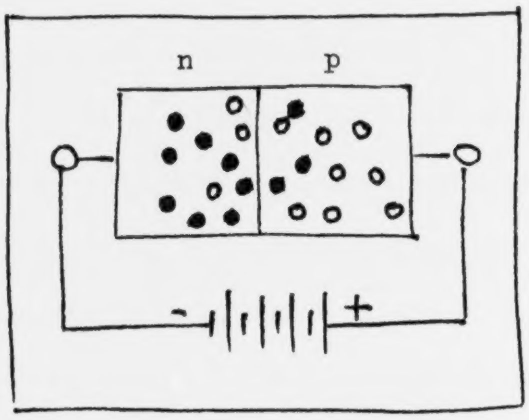
Semiconductors have two kinds of carriers: electrons, which have negative charge,



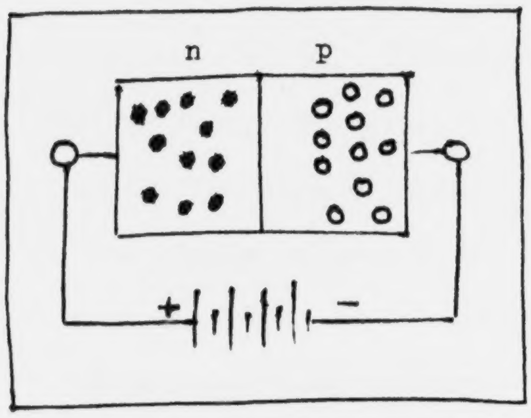
and holes, which are fiction for a deficit of electrons. Holes act as if they have positive charge.



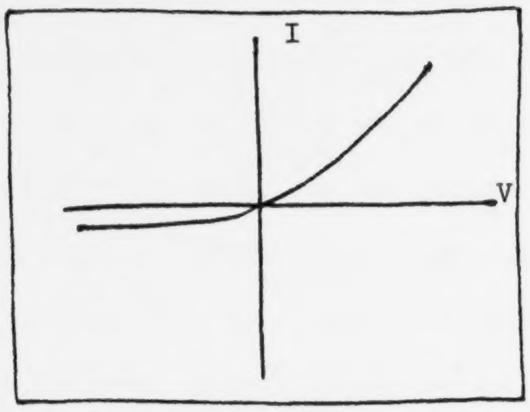
A semiconductor junction separates an n-type region, in which the majority carriers are electrons, and a p-type region, in which the majority carriers are holes.



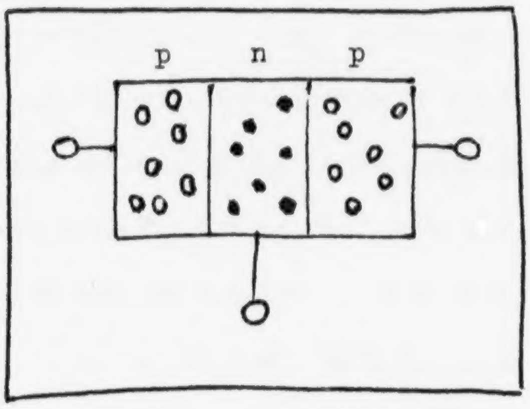
If we apply a positive potential to the p-type region we draw a heavy "forward" current across the junction. Electrons flow to the right and holes to the left.



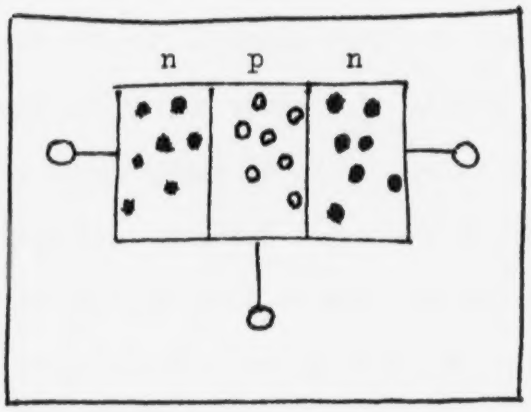
If we apply a negative potential to the p-type region we draw only a very light "reverse" current.



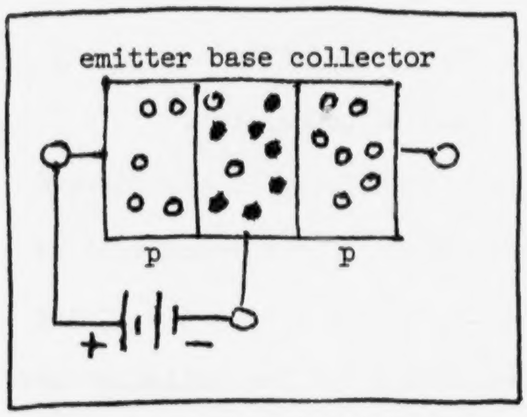
Thus a p-n junction is a current rectifier.



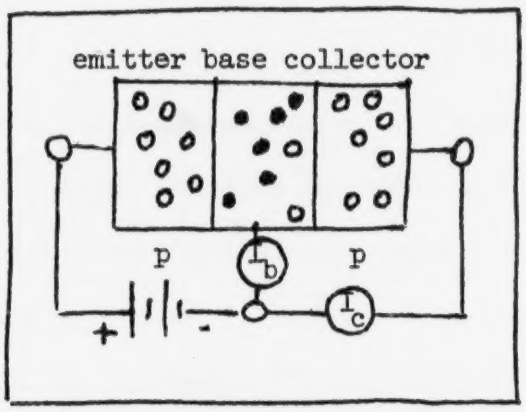
A transistor is formed from a pair of p-n junctions with either the n-type region in the center....



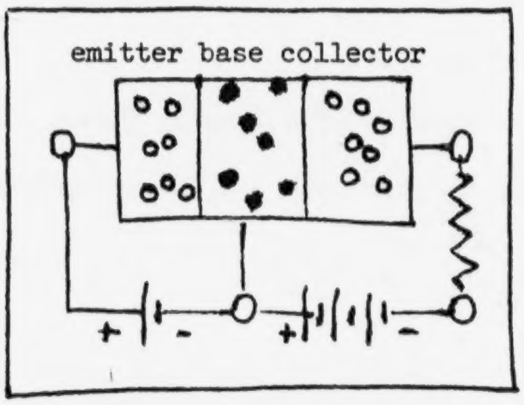
or with the p-type region in the center.


emitter base collector
p
p
+
-

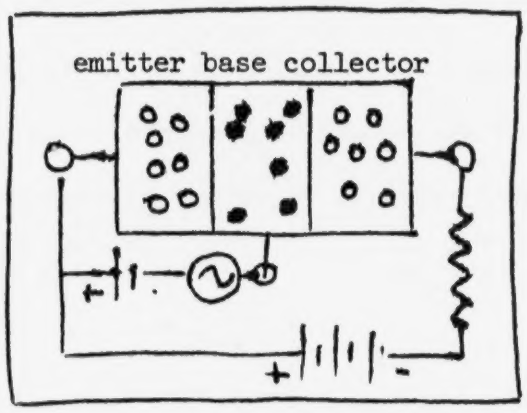
If we bias the one junction in the forward direction, we inject minority carriers into the base region.


emitter base collector
p
p
I_b
I_c
+
-

These carriers diffuse to the second junction, where they are collected. Ideally all the injected carriers are collected and there is no base current.


emitter base collector
+
-
+
-

If we place a resistor in the collector circuit we obtain a voltage proportional to the emitter current. The collector is biased in the reverse direction to prevent injection across the collector junction.


emitter base collector
+
+
-

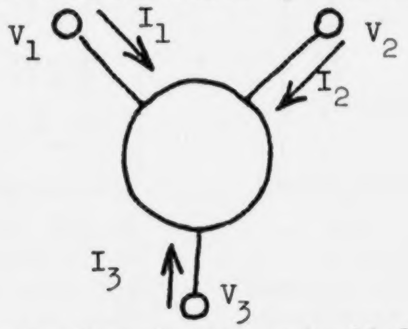
In the usual transistor amplifier we introduce a signal at the base and take the amplified signal from the collector.

electrons and thus opposite to the field also. One often finds a hole compared with a bubble in a glass of water, which moves in a direction opposite to the flow of water; or to an empty space in a line of cars, which moves back as each successive car moves forward. We must caution you that both of these analogies are incorrect; that a hole in a semiconductor is a child of quantum mechanics, and that you will have a satisfactory understanding of holes only after your study of quantum mechanics.

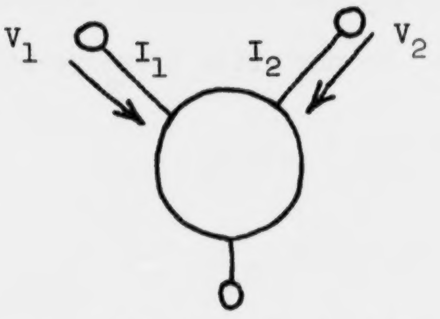
A transistor contains a pair of p-n junctions either in the sequence n-p-n or the sequence p-n-p. We will discuss primarily the n-p-n transistor, simply because we can talk mostly about electrons. But if one were to change the word electron where it appears to hole (and change the signs of all the potentials) one would have a description of the p-n-p transistor.

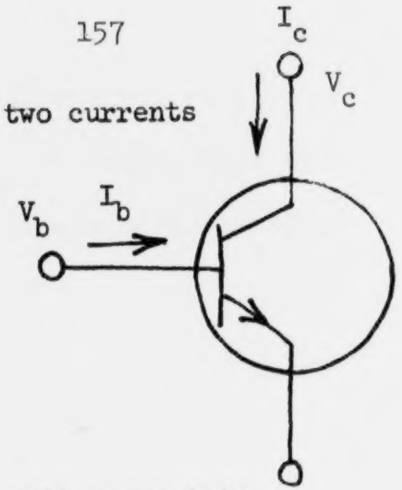
If we bias the emitter junction of an n-p-n transistor in the forward direction (the base positive with respect to the emitter), electrons will be injected into the base. These electrons will drift across the base. If the collector junction is biased is the reverse direction (with the collector positive with respect to the base) most of the electrons injected into the base will be collected. Under optimum conditions the base current, which represents those electrons which do not make it all the way to the collector, is only a few percent of the collector current. Thus the base is a high impedance control electrode and is analogous to the grid in a vacuum tube.

We may regard a transistor as a particular example of a general three-terminal network, which may be characterized by the currents and the potentials:

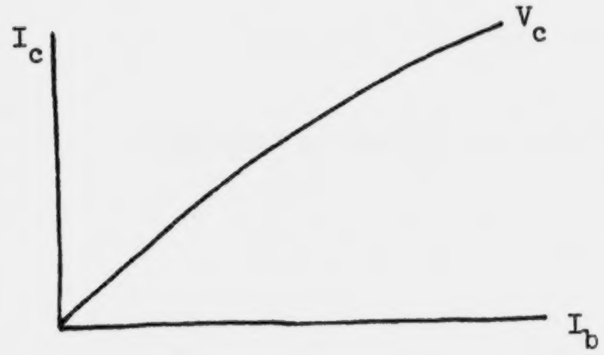


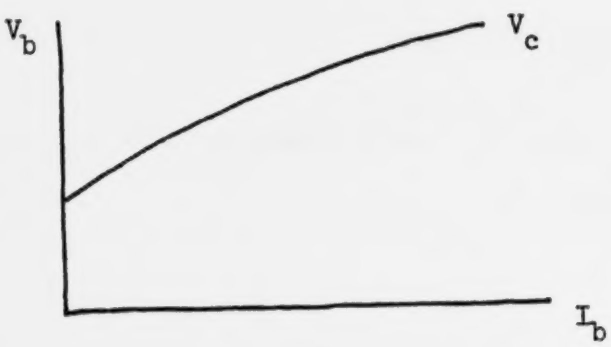
Since we are only interested in the relative potentials, and since we can be sure that $I_1 + I_2 + I_3 = 0$ we can characterize the network by two currents and two potentials:





For the transistor it will be convenient to take all potentials with respect to the emitter. Shown above right is the usual schematic representation of a transistor. If we establish the potentials of the base and collector, V_b and V_c, the currents must be determined. Thus of the four variables I_b, I_c, V_b, and V_c, only two of the variables need be considered as independent and the other two are dependent. Similarly for the vacuum tube the grid and plate currents are determined by the grid and plate potentials with respect to cathode. For the transistor it is usually convenient to think of the base current I_b and the collector potential V_c as independent variables and the collector current I_c and the base potential V_b as dependent variables. One may exhibit the characteristics of a transistor by plotting I_c or V_b as a function I_b for various values of V_c as shown below:





One normally establishes a central operating point and introduces a signal as a variation in base current, which we will write as i_b. Depending on the operating point and the external circuitry there will be associated changes in collector current i_c, in collector voltage v_c, and in base voltage v_b.

As regards the transistor characteristics there are two independent and two dependent variables so that we may expect two relations connecting the variation in currents and potentials. We may write these relations in the form:

$$i_c = \left(v_c/R_c\right) + \beta\, i_b$$

$$v_b = \left(v_c/\mu\right) + i_b\, R_b$$

The resistances R_b and R_c are the apparent internal resistances in the base and collector circuits. Since there will be a load resistance R_L is the collector circuit we have the additional relation:

$$v_c = -\, i_c\, R_L.$$

We can solve for i_c, v_c, and v_b in terms of i_b to obtain:

$$i_c = \beta[\, R_c/(R_c + R_L)]\ \, i_b$$

$$v_c = -\, \beta[R_c R_L/(R_c + R_L)]\ \, i_b$$

$$v_b = -\, [R_b + (\beta/\mu) R_c R_L/(R_c + R_L)]\ i_b$$

Normal values of the parameters are:

$R_b \approx 2700$ OHMS $\qquad \beta \approx 50 \qquad R_c \approx 70 \times 10^3$ OHMS $\qquad \mu \approx 3000$

If we take R_L small compared with R_c we may make the following simplifications:

$$i_c \approx \beta\, i_b$$

$$v_c \approx -\, \beta\, R_L\, i_b$$

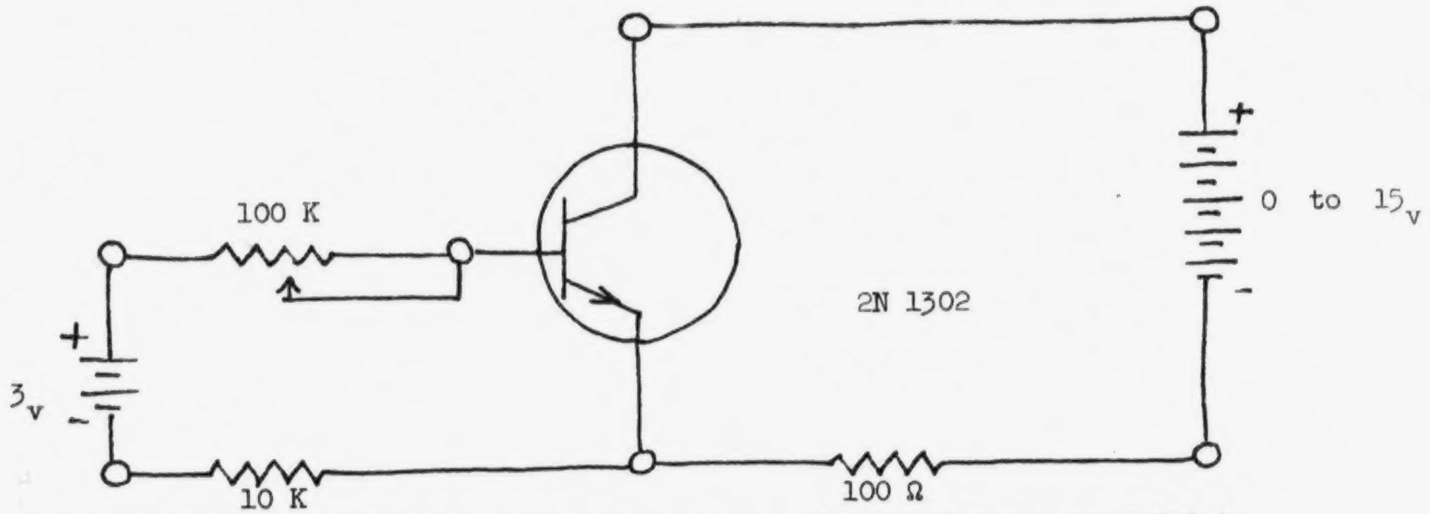
$$v_b \approx -\, R_b\, i_b.$$

The signal input power is $i_b v_b$ and the output power is $i_c v_c$. The power gain is thus given by:

$$P = i_c v_c / i_b v_b \approx \beta^2 R_L / R_b$$

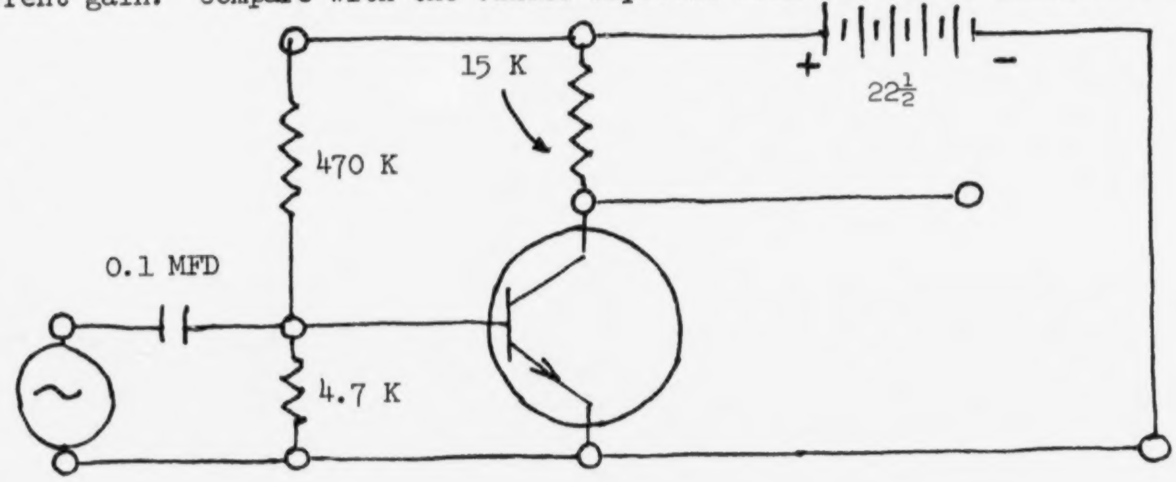
With R_L of the order of 15000 OHMS we may expect a power gain in excess of 10,000.

Using the circuit shown below obtain the static characteristics of the 2N-1302 n-p-n transistor, for several values of V_c. From the characteristics determine β, μ, R_b, and R_c for $\underline{I_c} = 1$ MILLIAMP and $\underline{V_c} = 7\frac{1}{2}$ VOLTS. Compute the expected current gain.



The potentials may be read directly with a vacuum tube voltmeter grounded to the emitter. The base and collector current may be determined from the voltage drop across the fixed resistors.

Assemble the transistor amplifier shown below and measure the voltage and current gain. Compare with the values expected from the static characteristics.

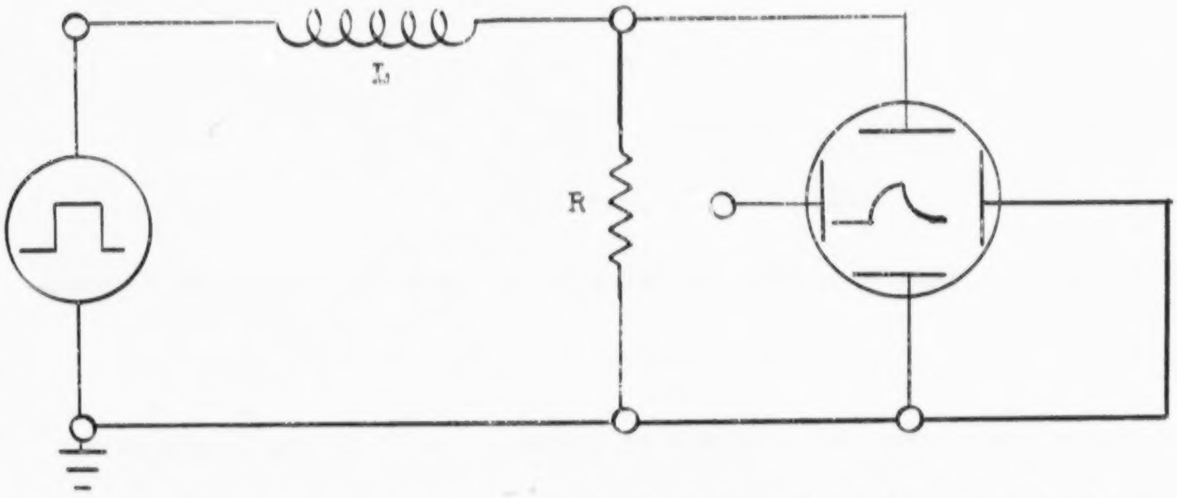


Measure the gain as a function of frequency. Note the frequency at which the gain begins to drop off. What do you think may be responsible for the drop in gain? How would you check your explanation?

Is a mechanical amplifier possible? Can you invent one?

In Experiment B-1 we measured the static characteristics of the 2N1302, which is a typical low power germanium transistor. In addition we observed the transistor as an amplifier in the audio range of frequencies.

If we apply an input signal at a frequency well above audio frequencies we will find that the apparent current gain drops with increasing frequency. This behavior is especially striking when a square wave of current is introduced into the base and the voltage at the collector is observed. The signal at the collector looks very much like the current in an LR circuit as observed in Experiment A-5.



We will first develop a simple phenomenological theory to describe the high frequency behavior of a transistor. Then we will investigate the way in which one can compensate for the drop in gain with frequency by feeding some of the output signal back into the input.

The analogy between the high frequency behavior of a transistor and the LR circuit suggests that something within the transistor inhibits the buildup in collector current. One can make a simple model of the high frequency behavior of a transistor by considering in addition to the currents I_b and I_c, the

Copyright © 1963 Educational Services Incorporated. No part of this manuscript may be reproduced without written permission of Educational Services Incorporated.

minority charge density within the base region Q_b. As we have seen, minority carriers are injected into the base across the emitter junction. Most of these carriers diffuse across the base and are received at the collector junction. However, some small fraction of the minority carriers (about 2 percent) recombine with majority carriers and never reach the collector. We can write an equation for the rate of change of minority charge within the base in the form:

$$dQ_b/dt = I_e + I_c - Q_b/\tau .$$

The first two terms on the right represent the flow of minority carriers into the base region. The third term represents the disappearance of minority carriers as a result of recombination. Remembering that the total current into the transistor is zero:

$$I_e + I_b + I_c = 0$$

we can write our equation in the form:

$$dQ_b/dt = - I_b - Q_b/\tau .$$

Finally we wish to make a connection between Q_b and the collector current:

$$i_c = - \beta q_b/\tau + v_c/R_c$$

where we have used lower case letters to indicate small signal currents and voltages. In steady state $dQ_b/dt = 0$ and

$$i_c = \beta i_b + v_c/R_c$$

which is the circuit equation used in Experiment B-1. But for frequencies of the order of the β cut-off frequency $\omega = 1/\tau$ or higher, transient effects will be important and we must work with the full equations. Eliminating Q_b in the above equations we obtain:

$$\tau\, d(i_c - v_c/R_c)/dt + (i_c - v_c/R_c) = \beta i_b$$

with a resistance R_L in the collector circuit we have the relation:

$$v_c = -i_c R_L$$

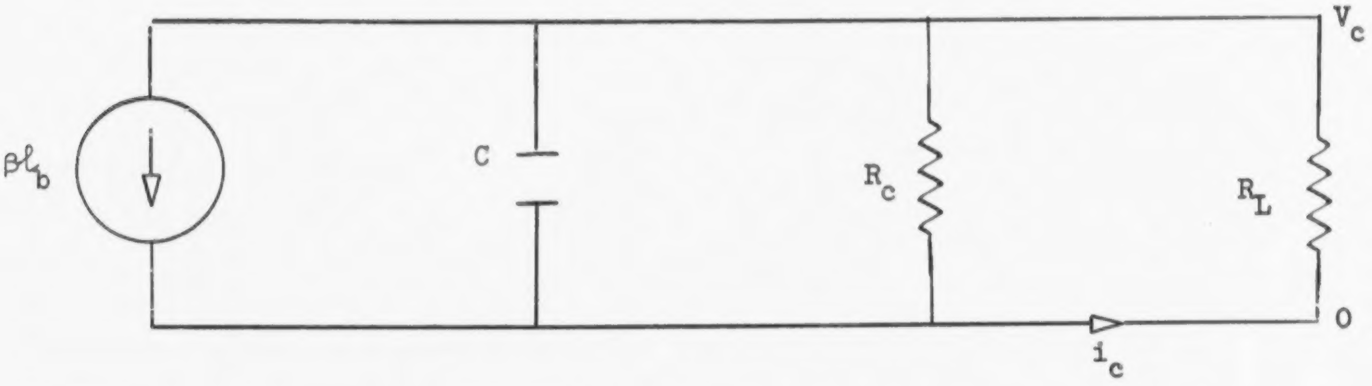
which leads to the equation:

$$\tau\, di_c/dt + i_c = \beta\, i_b R_c/(R_L + R_c)$$

or:

$$\tau\, dv_c/dt + v_c = -\beta\, i_b\, R_c R_L/(R_c + R_L)\,.$$

This equation has a simple circuit analog which is sketched below:

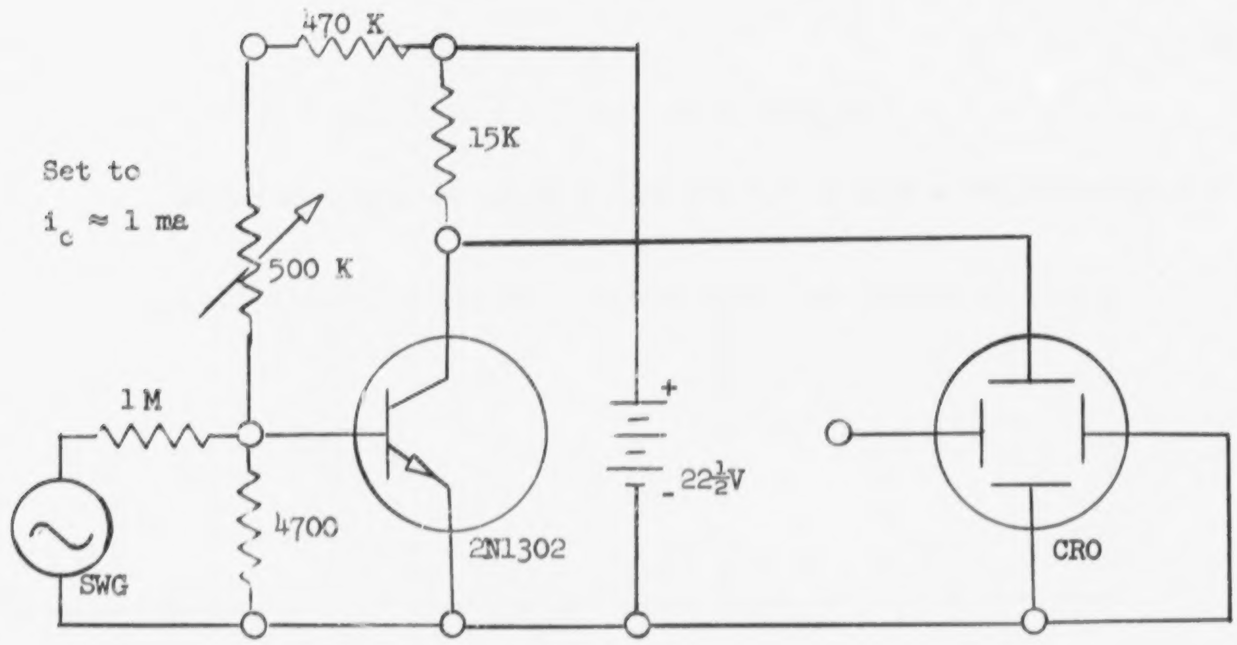


We imagine a current generator $\beta\, i_b$. The current from this generator flows into a capacitor of magnitude:

$$C = (1/R_L + 1/R_c)\tau$$

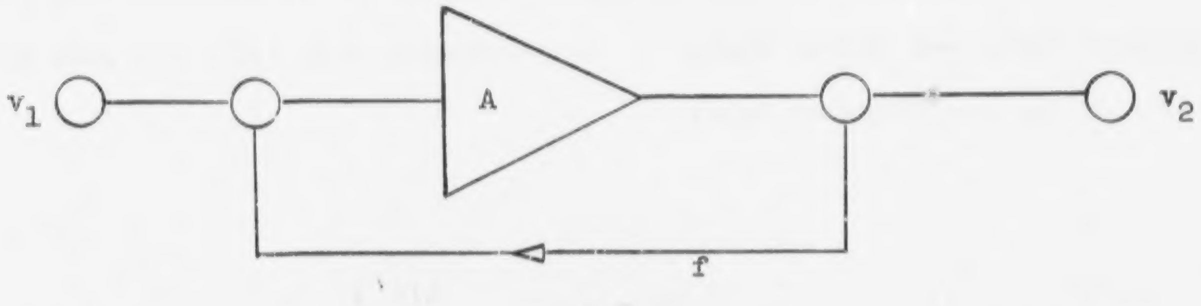
in parallel with the resistors R_L and R_c. Of course C is not a real physical capacitor. However it is often helpful to represent a physical phenomenon like recombination by a circuit element.

Assemble the transistor amplifier of Experiment B-1 as shown below and examine the transient response to a square wave. Using techniques similar to those described in Experiment A-5 measure the recombination time τ. Also introduce a sinusoidal signal and note the frequency dependence of the gain to 1Mc/sec. Plot your data on log-log paper. Compare τ as measured from the transient decay and $1/2\pi\nu$ where ν is the frequency at which the gain drops to 0.707 of the low frequency value.



One often wants a much higher frequency response than is available with the conventional amplifier as we have used it. If one is willing to accept a lower over-all gain per stage in order to achieve the higher frequency response, there is a very simple solution to the problem. One can introduce a fraction f of the output signal degeneratively into the input. As we see from the general analysis below this negative feedback tends to hold the gain constant at a value 1/f as long as the original gain is high compared with 1/f.

We first consider the following very simple problem: An amplifier has a voltage gain A, which may be a function of frequency. We feed back a fraction of the output signal fv_2 into the input:



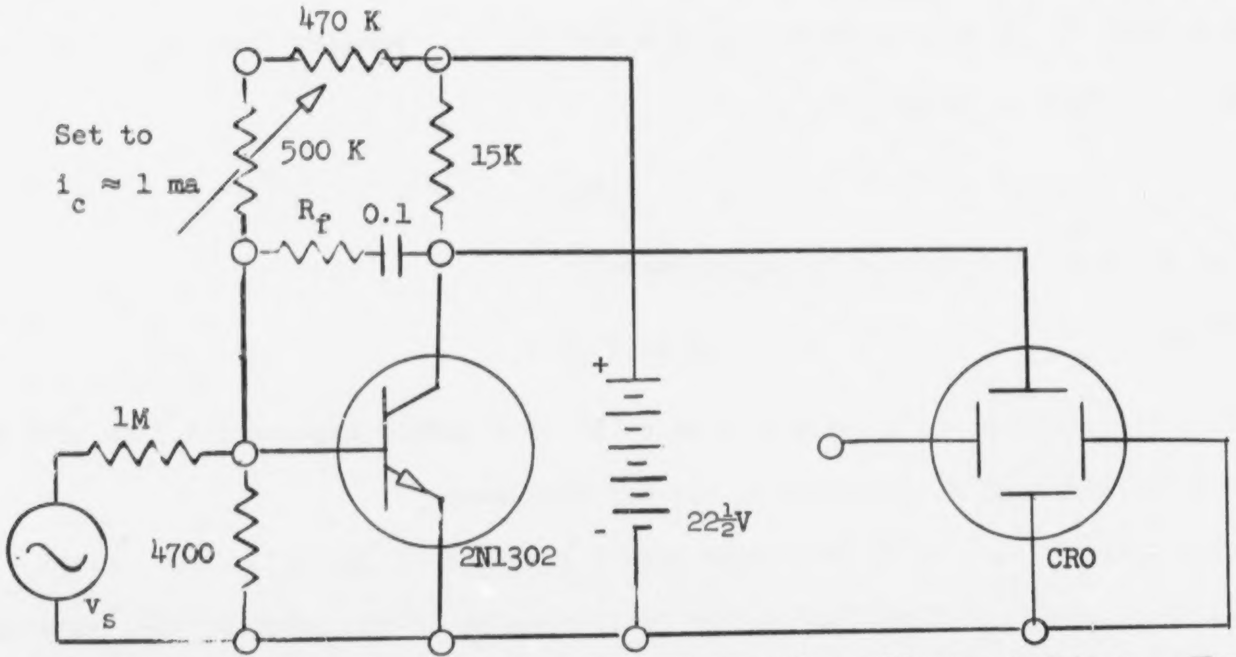
This gives us the expression:

$$v_2 = A(v_1 - f\,v_2)$$

or

$$v_2 = [A/(1 + fA)]\,v_1 \,.$$

Now as long as fA is much larger than one, the effective gain is simply $1/f$. A very simple way of introducing negative feedback into a transistor amplifier is shown in the circuit below:



We have simply introduced a resistor R_f between the collector and base. The series capacitor avoids any modification in the dc operating point. If this resistor is large compared with the base resistance $R_b \approx 2700\ \Omega$ we can consider that the resistor R_f injects a current $-f\,i_c$ into the base independent of any additional base current. We should now modify the equation:

$$\tau\, di_c/dt + i_c = \beta\, i_b R_c/(R_L + R_c)$$

by letting

$$i_b = i_s - f\,i_c$$

where i_s is the signal current and $- fi_c$ is the current fed back from the collector. The circuit equation now takes the form:

$$\frac{\tau}{1 + \beta\, fR_c/(R_L + R_c)}\, di_c/dt \;+\; i_c \;=\; \frac{\beta\, R_c/(R_L + R_c)}{1 + \beta\, fR_c/(R_L + R_c)}\, i_s \;.$$

The low frequency gain is given by:

$$v_c \;=\; - \frac{\beta\, R_L R_c/(R_L + R_c)}{1 + \beta\, fR_c/(R_L + R_c)}\, i_s$$

and the β cut-off frequency is given by

$$\omega \;=\; [1 + \beta\, fR_c/(R_L + R_c)]/\tau \;.$$

Assuming that β is much greater than one and for R_L smaller than R_c, the collector voltage is given by:

$$v_c \approx - i_s R_L/f$$

and the β cut-off frequency is approximately

$$\omega \;=\; (1 + \beta f)/\tau \;.$$

Note that the response to a transient is still of a simple exponential form with a reduction in gain and an increase in cut-off frequency.

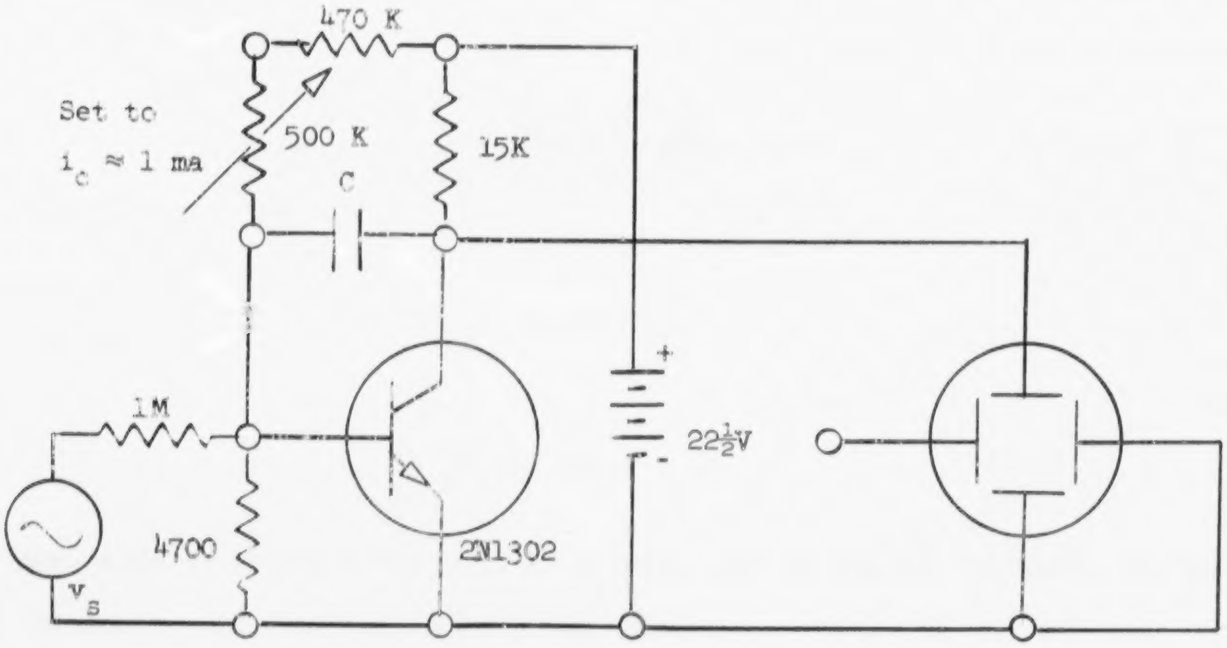
A resistor R_f = 68 K will feed back a fraction of the collector current $f \approx 1/5$ into the base. The factor βf will then be of the order of ten, raising the β cut-off frequency by an order of magnitude. The low frequency gain should correspondingly drop by about a factor of ten.

Measure and plot the gain as a function of frequency with a feedback resistor R_f = 68 K. Compare the reduction in low frequency gain and the increase in cut-off frequency. Observe the transient response to note that its exponential character has been preserved.

Appendix a. Operational Amplifiers

We have seen in Experiment B-2 that we can improve the frequency response of an amplifier by negative feedback through a resistive network. We can in a similar way simulate a non-linear response or develop a desired frequency characteristic through the use of frequency-dependent feedback. In particular we can make a network which differentiates or integrates an input signal by employing an appropriate feedback network. Amplifiers with this kind of structure are called operational amplifiers because they perform some specified mathematical operation on an input signal.

As a simple example of an operational amplifier consider the following circuit, which is the basic transistor amplifier supplemented by a capacitor from base to collector:



The capacitor C will introduce an additional current into the base proportional to the rate of change of collector voltage. To be more precise the base current will be given by:

$$i_b = i_s + Cd(v_c - v_b)/dt \ .$$

Since v_b is only about 1 percent of v_c we may safely ignore v_b and write:

$$i_b = i_s - R_L C \, di_c/dt \ .$$

Substituting into the expression:

$$\tau \, di_c/dt + i_c = \beta \, i_b R_c/(R_L + R_c)$$

we obtain:

$$[\tau + \beta \, C \, R_L R_c/(R_L + R_c)] \ \ di_c/dt + i_c = \beta \, i_b R_c/(R_L + R_c) \ .$$

The amplifier now shows an apparent cut-off frequency:

$$\omega = 1/[\tau + \beta \, C R_L R_c/(R_L + R_c)] \ .$$

For frequencies above this cut-off we can write:

$$i_c = \frac{\beta \, R_c/(R_L + R_c)}{\tau + \beta \, C R_L R_c/(R_L + R_c)} \int i_b \, dt \ .$$

For $\beta \, C R_L > \tau$ we have the simple relation

$$i_c = (1/R_L C) \int i_b \, dt$$

and

$$v_c = -\,(1/C) \int i_b \, dt \ .$$

Thus the collector voltage is the integral of the base current. With a capacitor C = 0.1 MFD we may expect to observe the integral of the base current for frequencies above 20 c/sec. Introduce a square wave into the base and note the triangular integral signal observed at the collector.

There is of course nothing remarkable about obtaining the integral of a current by introducing it into a capacitor. The function of the operational amplifier is to develop across a moderately low load resistance a signal that is to a large extent independent of the characteristics of the amplifier.

Electromagnetic Waves

In this experiment one measures the product $\mu_0\epsilon_0$. The theory of propagation of electromagnetic waves shows that the velocity of such waves if $1/\sqrt{\mu_0\epsilon_0}$; hence the title of the experiment.

A small magnetic dipole is attached to a suspended rotor assembly which includes the grounded plates of two condensers(Fig. 1). A second magnetic dipole is placed at right angles to the first and a distance L above it. The resulting torque is cancelled by applying a voltage V to the condensers. The magnetic dipole is now rotated near a coil of wire, and magnitude of the voltage induced in the coil is compared with the voltage V which was applied to the condenser. How this leads to the product can best be seen by carrying through a careful analysis.

THEORY

The expression for the electrostatic torque is the same as that derived in the instruction sheet for the wave impedance experiment. To recapitulate, there is an electrostatic force

$$F = -\frac{dU}{dy} = +\tfrac{1}{2} V^2 \frac{dC}{dy}$$

in the direction tending to increase the amount of overlap, y, of the condenser plates. The torque is the product of this force and the distance from the suspension axis to the middle of the plate. Since

$$\frac{dC}{dy} = \frac{2\epsilon_0(d-e)}{2f} \quad ,$$

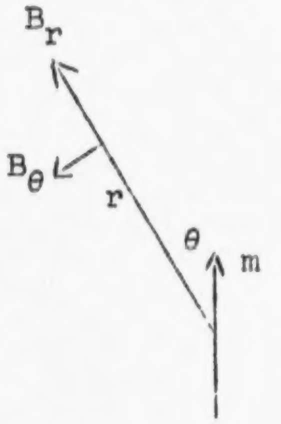
this means that the magnitude of the torque due to the two sets of condensers is

$$|\tau| = \frac{\epsilon_0(d-e)V^2}{2f} \cdot \frac{2(d+e)}{4} = \frac{\epsilon_0(d^2-e^2)V^2}{4f} \qquad (1)$$

It is now necessary to know the magnetic field of a dipole. This is discussed in sections 8.2 and 8.3 of Scott. The result is

$$B_r = \frac{2\mu_0 m \cos\theta}{4\pi r^3}$$

$$B_\theta = \frac{\mu_0 m \sin\theta}{4\pi r^3}$$

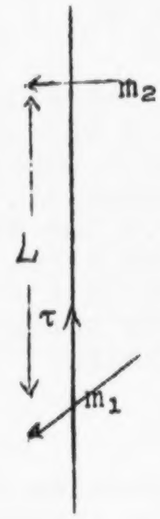


where m is the magnetic dipole moment.

To calculate the torque on the suspended dipole, one calculates the field at the first dipole due to the second. The torque is then given by (Scott, ibid.)

$$\tau = \vec{m} \times \vec{B}.$$

The dipoles are at right angles to one another and to the line joining them, and are separated by a distance L. At the position of the lower dipole, the field due to the upper is



$$B_r = 0$$

$$B_\theta = \frac{\mu_0 m_2}{4\pi L^3}$$

in a direction opposite to the direction of $\vec{m}_2$. The torque is then

$$\tau = \frac{\mu_0 m_1 m_2}{4\pi L^3} \qquad (2)$$

in the direction shown. If this torque is balanced by the electrostatic torque (1), then one obtains the relation

$$\frac{\mu_0 m_1 m_2}{4\pi L^3} = \frac{\epsilon_0}{4} V^2 \frac{(d^2-e^2)}{f} \quad . \qquad (3)$$

The dipole is now spun with a constant angular velocity near a pickup coil. The instantaneous induced e.m.f. is given by

$$E = \oint \vec{\mathcal{E}}\cdot d\vec{s} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} = -\frac{\partial}{\partial t}\iint \vec{B}\cdot d\vec{S} \ .$$

The apparatus used employs two coils connected in series. The dipole is attached to a rotating shaft which passes between the coils, in such a manner that its moment is always perpendicular to the axis of rotation (Fig. 2). This axis of rotation is perpendicular to the common axis of the coils. The calculation may be done for one coil, and the results doubled. The first step is to calculate the instantaneous flux linking the coil when the dipole is at an arbitrary angle (Fig. 3). Rather than calculate the flux through the surface I, it is somewhat simpler to calculate the flux through the spherical cap, II, bounded by the coil and with the dipole at the center. If the azimuthal angle, ϕ is measured from the plane in which the dipole rotates, and if a unit normal to the surface at the point (θ,ϕ) is denoted by $\hat{n}$, then

$$B_n = B_r = \frac{2\mu_0}{4\pi\rho^3}\vec{m}\cdot\hat{n} \ .$$

Let α represent the angle through which the dipole rotates. In rectangular coordinates

$$\vec{m} = m(\cos\alpha\,\hat{i} + \sin\alpha\,\hat{k}).$$

$$\hat{n} = \hat{i}\sin\theta\cos\phi + \hat{j}\sin\theta\sin\phi + \hat{k}\cos\theta,$$

so that

$$\vec{m}\cdot\hat{n} = m(\cos\alpha\sin\theta\cos\phi + \sin\alpha\cos\theta).$$

The flux is therefore given by

$$\vec{B}\cdot d\vec{S} = \frac{2\mu_0 m\rho^2}{4\pi}\int_0^{2\pi} d\phi \int_0^{\sin^{-1}(A/\rho)} d\theta \left\{\cos\alpha\sin^2\theta\cos\phi + \sin\alpha\cos\theta\sin\theta\right\}$$

$$= \frac{\mu_0 m\sin\alpha A^2}{2\rho^3} = \frac{\mu_0 m\sin\alpha A^2}{2(A^2+b^2)^{3/2}}.$$

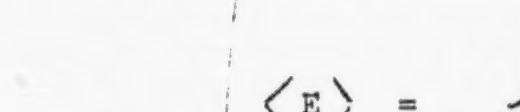
The e.m.f. induced in the Nturn coil is obtained by letting $\alpha = \omega t$, and taking N times the time derivative of the flux:

$$E = -\mu_0\frac{m}{2}N\frac{\omega A^2}{(A^2+b^2)^{3/2}}\cos\omega t.$$

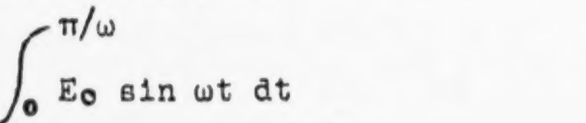
(Can you calculate the error introduced by not considering the length of the winding?) Finally, with the two coils connected so that the induced voltages add,

$$E = -\mu_0 mN\frac{\omega A^2}{(A^2+b^2)^{3/2}}\cos\omega t = E_0\cos\omega t.$$

This voltage is then fed through a commutator switch, so that the output voltage is always of the same sign:



before commutator



after commutator

The average of this voltage is compared with the condenser voltage V by using a galvanometer whose response time is longer than the period (Fig. 4).

The average of the rectified voltage is

$$\langle E\rangle = \frac{\int_0^{\pi/\omega} E_0\sin\omega t\,dt}{\int_0^{\pi/\omega} dt}$$

$$= \frac{2E_0}{\pi}.$$

The ratio F is

$$\frac{r_1}{R_1 + R_2} = \frac{d}{D}\frac{R_1}{R_1 + R_2} = F \doteq \frac{\langle E \rangle}{V} = \frac{2\mu_0 m N \omega A^2}{V(A^2+b^2)^{3/2}\pi}.$$

This ratio is measured separately for each magnetic dipole. Hence

$$m_1 m_2 = \frac{V^2\ (A^2 + b^2)^3 \pi^2}{4\mu_0^2 N^2 \omega^2\ A^4}\ F_1 F_2. \tag{4}$$

where

$$F_1 = \frac{1}{D}\frac{R_1}{R_1 + R_2}\, d_1. \tag{5}$$

Finally, combining eqs. (3) and (4) gives

$$c^2 = \frac{1}{\mu_0 \epsilon_0} = \frac{4N^2\ \omega^2\ A^4\ L^3\ (d^2 - e^2)}{\pi\,(A^2 + b^2)^3\ f\ F_1 F_2}\quad . \tag{6}$$

EXPERIMENTAL TECHNIQUES

Fig. 5 gives the circuit used to apply voltage to the condensers and to balance the rectified voltage from the spinner. The condensers are connected between the COND and COMMON terminals, and, with the dipole separation fixed, R_3 and S_2 are adjusted to give null. When this condition is obtained, the dial on R_3 is locked so that the value of R_3 will not be altered. The spinner and galvanometer leads are connected to the COMMON and GALV terminals as shown in Fig. 4. The resistance R_1 is adjusted for null, with each dipole in turn, to give the F_1 of eq. (5).

Connect the rubber drive belt from motor to spinner only when the apparatus is actually in use. This avoids undue stretching of the belt.

A strobotac is provided for measuring rotational speed.

The commutator is a switch arranged as shown in Fig. 6, directly coupled to the spinner shaft. One pair of contacts may be rotated to obtain the correct phasing, as shown in Fig. 7. (Use oscilloscope).

Caution: So as not to damage the galvanometer set the Ayrton shunt initially for minimum galvanometer sensitivity and adjust R_1 for null. Then progressively increase the sensitivity of the galvanometer and readjust R_1 until full sensitivity has been obtained.

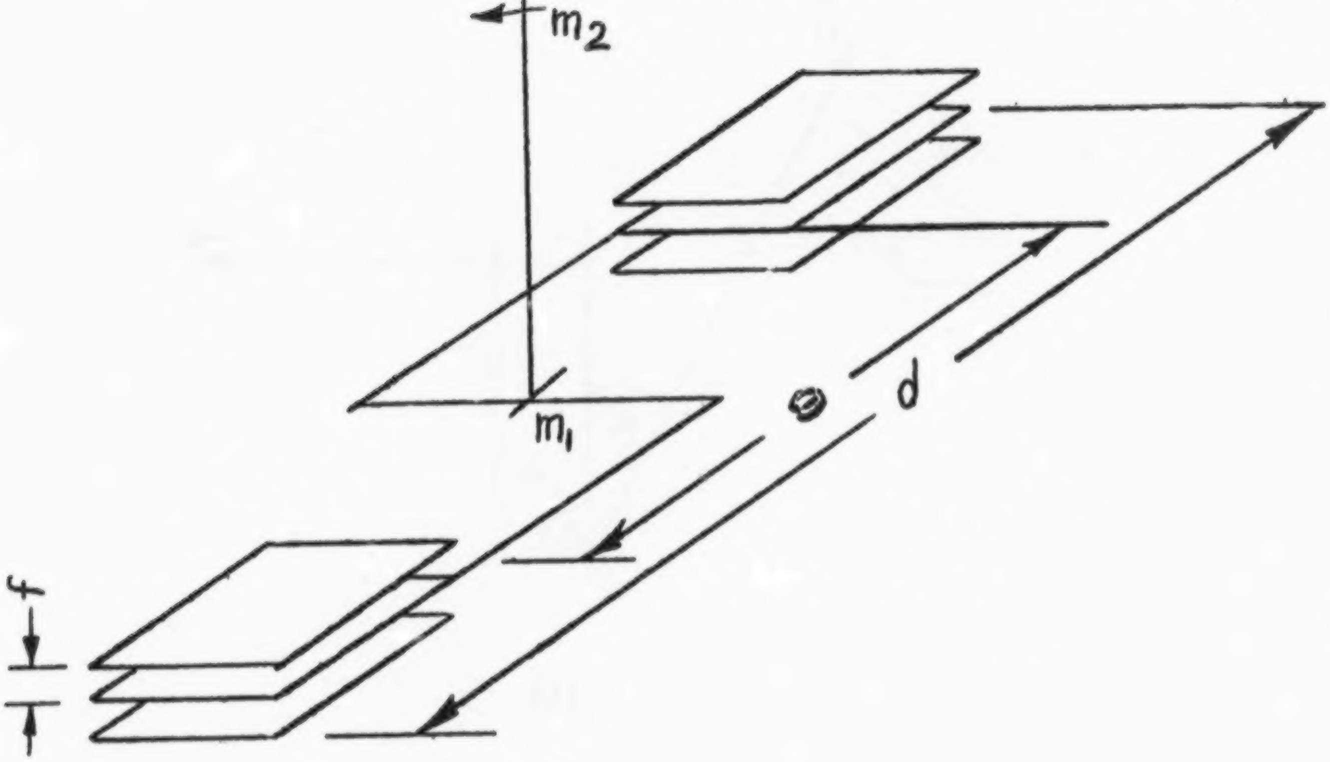


FIG. 1

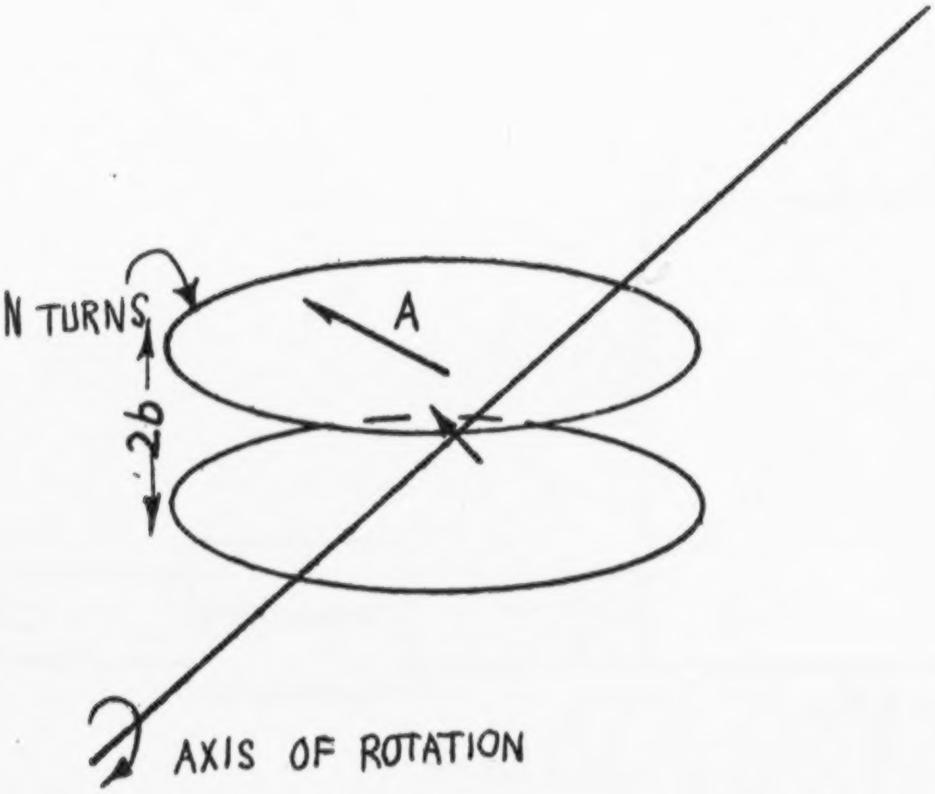

N TURNS
A
2b
AXIS OF ROTATION

FIG. 2

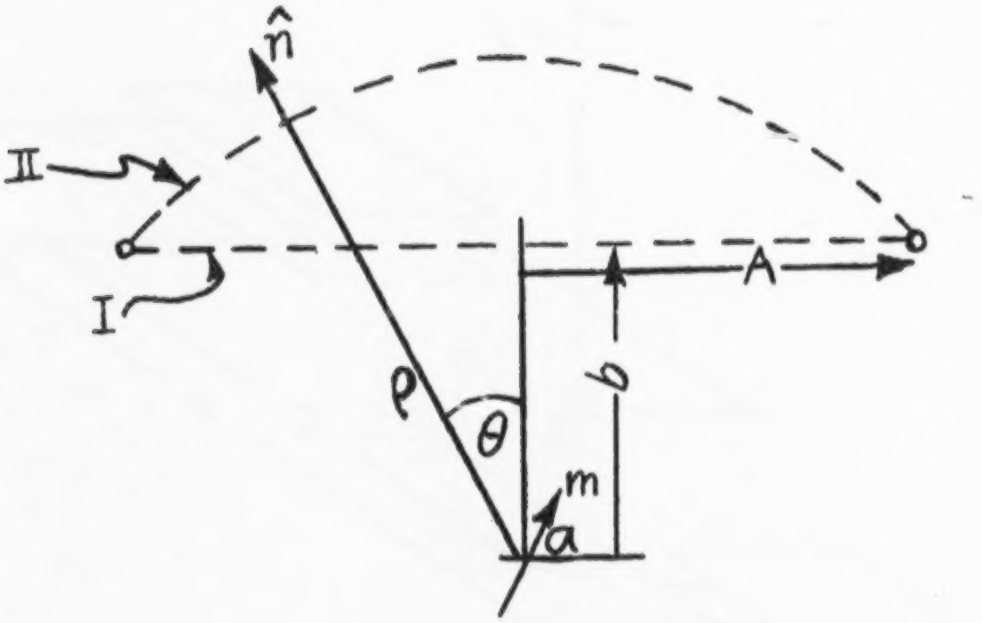

$\hat{n}$
II
I
A
b
ρ
θ
m
a

FIG 3

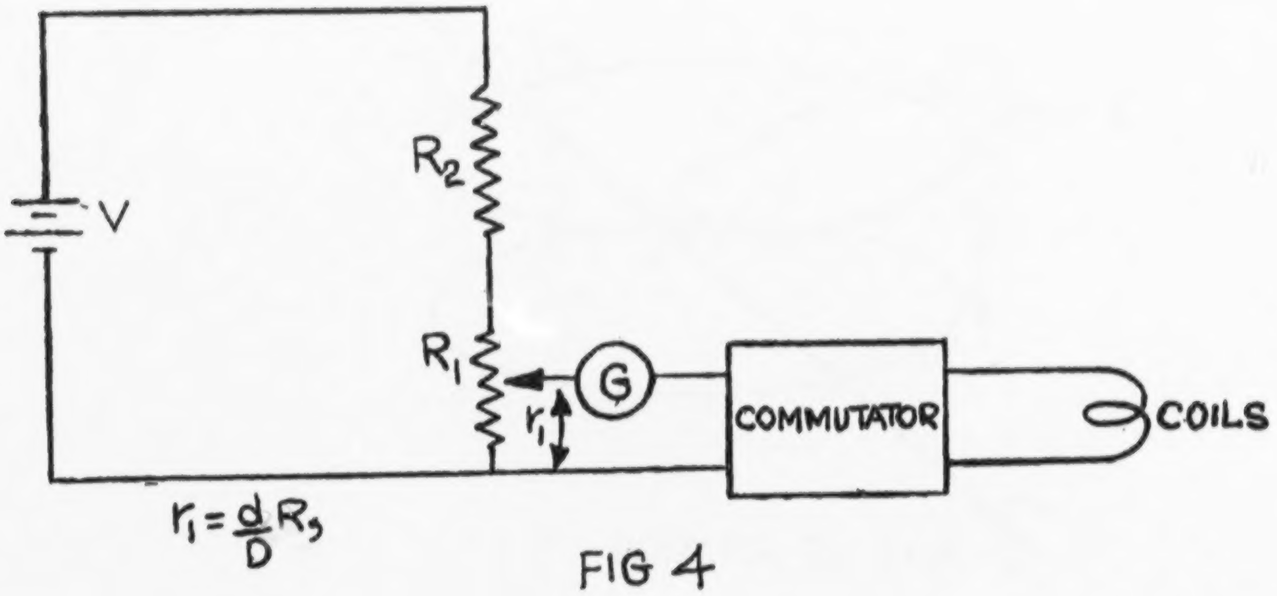

V
R_2
R_1
G
r_1
COMMUTATOR
COILS
$r_1 = \frac{d}{D} R_3$

FIG 4

175/76

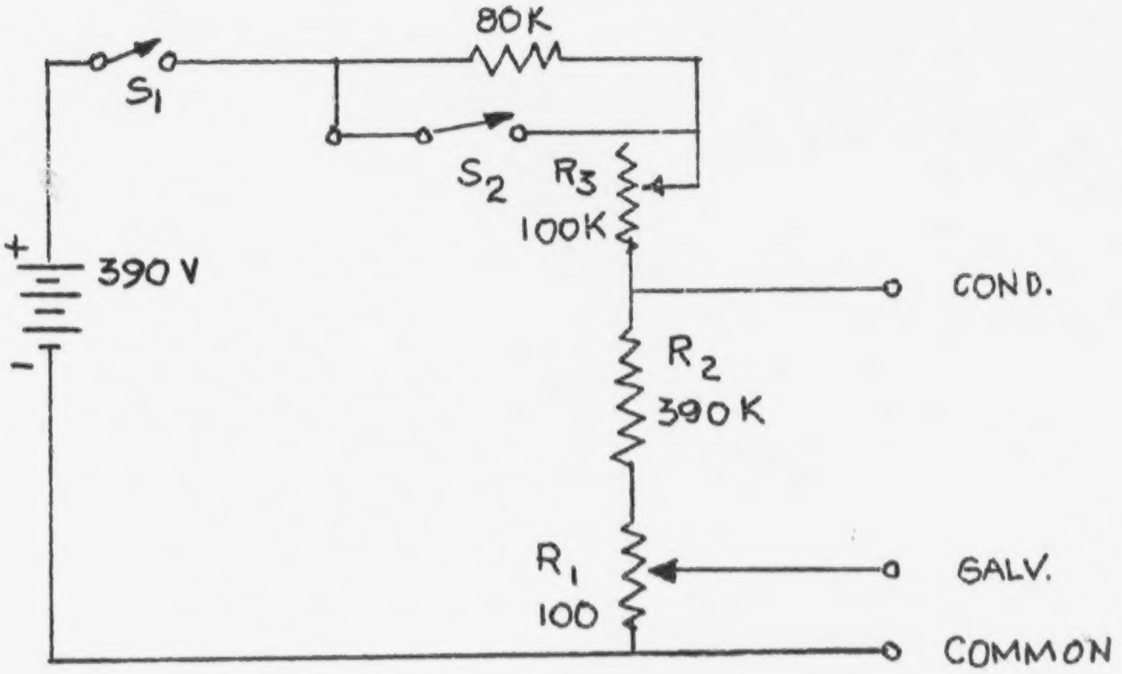

80K
S1
S2
R3
100K
390 V
+
-
COND.
R2
390 K
R1
100
GALV.
COMMON

Fig. 5

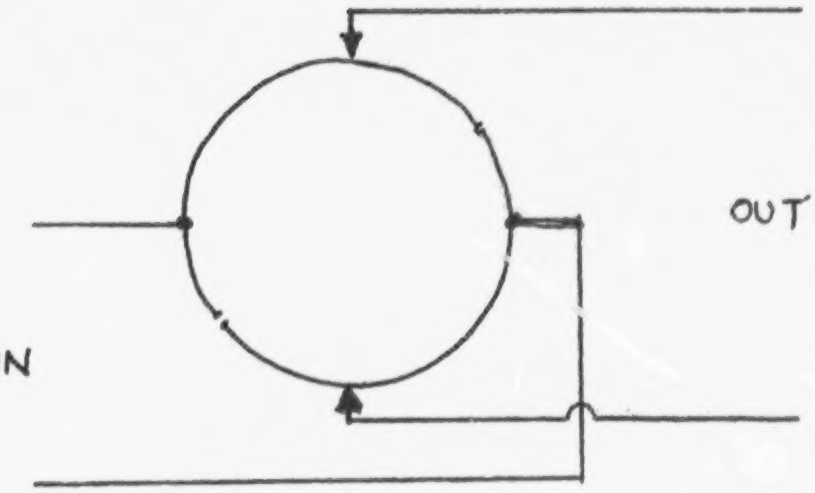


FIG 6

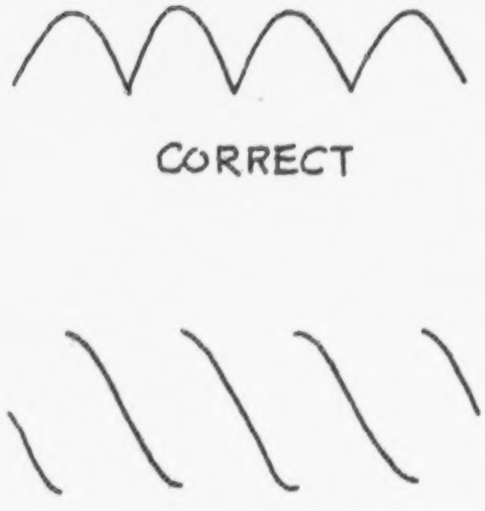
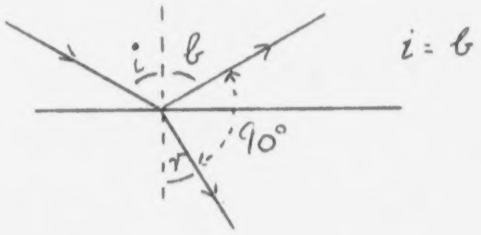

CORRECT
INCORRECT

FIG 7

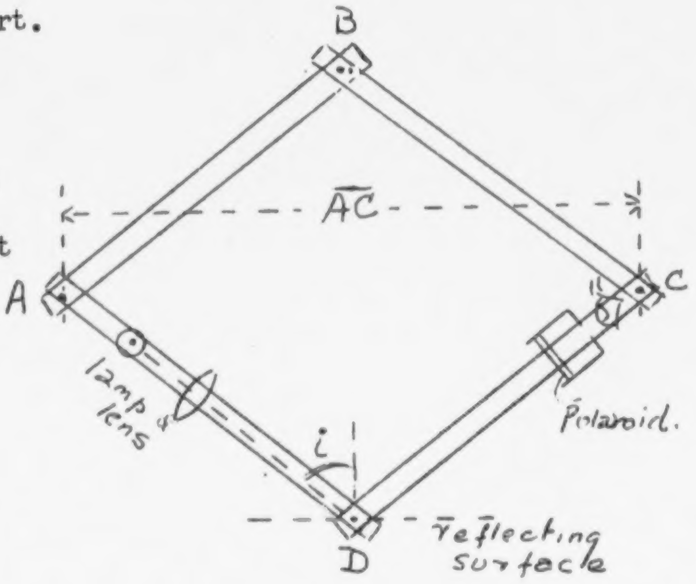
TO DETERMINE THE REFRACTIVE INDEX OF
A. GLASS, B. WATER, BY BREWSTER'S LAW

Iona College

Brewster was the first to discover that when light is reflected from a transparent substance the reflected beam is partially polarized parallel to the surface. The polarization is complete when the angle of reflection reaches a value known as the Brewster angle. Theory and experiment show that at the Brewster angle the reflected beam and the refracted beam are at right angles. This condition may be used to determine the refractive index of the material as seen from the diagrams. If the Brewster angle of reflection is b then the angle of refraction is $90^\circ - b$ Since the refractive index is sin i/sin r the value in terms of b will be tan b.



The apparatus consists of a parallelogram frame with pivots A,B,C,D exactly 50 cm apart. The points B, D are carried by sliding collars mounted on a vertical bar. The arm AD carries a small lamp and lens to produce a parallel beam of light to the point D. On the arm CD is mounted a metal holder carrying a polaroid which can be rotated about the axis CD. The arrangement is such that a reflecting surface mounted HORIZONTALLY with D lying in the plane of the surface will reflect light from the source to the polaroid viewing aperture so that angles i and r are automatically equal. This saves a lot of adjustment troubles.



To use the instrument set the glass block on the black velvet lying on the wooden block and level it carefully with a spirit level using plasticene balls under the block of wood. Lower or raise the point D so that D lies in the plane of the upper surface of the glass. Clamp it firmly. Look into the polaroid through the viewing aperture and rotate the polaroid to give minimum light. Raise or lower B to reduce the light through the polaroid and obtain the 'null' condition mentioned above as closely as possible. Carefully raise B until the null condition is just perceptibly off. Measure AC with a meter stick. LowerB until the null condition has been perceptibly passed and remeasure AC. Get three such pairs of readings and average them. This will give the best value for AC. Since sin i is $\frac{1}{2}$AC/50 it will also be AC/100. Look up the angle with this value as sine and this will be the Brewster angle. The tan of this angle will be n for glass.

Repaet using a small beaker of water adjusting the point D to lie in the plane of the water surface. Record all readings for AC and the calculations as before.

RESULT: Refractive index of water ....
,, ,, glass ....

Reprinted from AMERICAN JOURNAL OF PHYSICS, Vol. 30, No. 10, 724–726, October, 1962
Printed in U. S. A.

Construction and Use of a Fabry-Perot Interferometer

WALLACE A. HILTON*
William Jewell College, Liberty, Missouri

(Received May 14, 1962)

The construction of a student-laboratory, Fabry-Perot interferometer is described, together with its use in three experiments. The first is a basic experiment for making some optical measurements. The second uses the interferometer to measure the index of refraction of a gas, and the third uses it to observe the Zeeman effect.

AN interferometer consisting of an air film between two parallel plates was first developed by Fabry and Perot in 1897. In this type of interferometer, if monochromatic light from an extended source passes through the plates, a series of concentric circular fringes is formed at infinity.

Tolansky[1] has said that the Fabry-Perot interferometer is without doubt the most versatile of all interferometers. It has been used for absolute measurement of wavelength, determination of linewidths, determination of refractive indices of gases, hyperfine structure studies, and more recently, to measure fusion temperatures by measuring the Doppler broadening of spectral lines emitted by hydrogen plasmas.

CONSTRUCTION OF THE INTERFEROMETER

The shop-constructed Fabry-Perot interferometer (which is of the type known as an etalon) is shown in Fig. 1. The housing is made of brass and consists of two glass plates, 1½ in. in diameter, whose interference faces are spaced parallel by three Invar spacers contained in a brass ring. The three spacers must be of the same length to at least a half wavelength of the light used. Retaining rings hold each plate firmly against the Invar spacers. Final adjustment of the plates is obtained by turning three brass screws which are in contact with one of the retaining rings. The plates were purchased,[2] and then coated with aluminum by an evaporation process[3] using equipment in the laboratory of Professor R. H. Hughes at the University of Arkansas. The thickness of the aluminum film is such that about 90% of the incident light is reflected, and about 10% is transmitted or absorbed. When assembled and used with a light source, filter, achromatic lens, and eyepiece, as shown in Fig. 2, sharp circular fringes are observed.

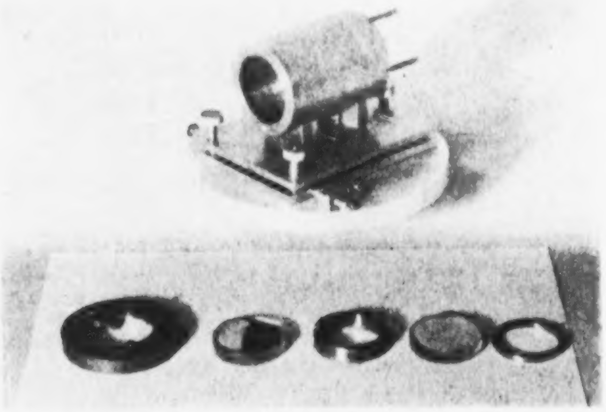


FIG. 1. Shop-constructed Fabry-Perot interferometer showing retaining rings, plates, and Invar spacers.

The total cost includes $140 for the glass plates, about $15 for brass stock, the shop time for construction of the housing, retaining rings, spacers, etc., and aluminizing the plates.

THEORY

If M_n is the order of interference of the nth ring, numbered from the center of the pattern, t is the thickness between the plates, λ is the wavelength of light between the plates, ϕ is the angular

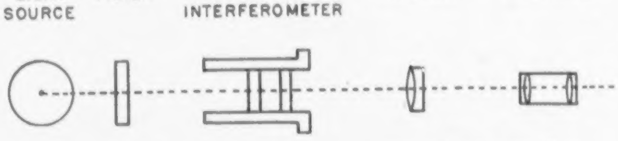


FIG. 2. Arrangement of apparatus for observing Fabry-Perot fringes.

---

* National Science Foundation Science Faculty Fellow, University of Arkansas, 2nd Semester, 1959–60.

[1] S. Tolansky, *An Introduction to Interferometry* (Longmans Green and Company, New York, 1955), p. 128.

[2] Unertl Optical Company, Pittsburgh 14, Pennsylvania.

[3] S. Tolansky, *High Resolution Spectroscopy* (Pitman Publishing Corporation, New York, 1947), pp. 128–168.

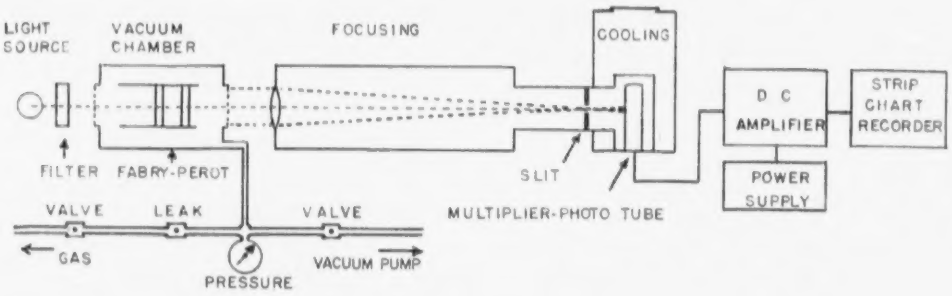


FIG. 3. Arrangement of equipment for measuring the index of refraction of a gas.

radius of the nth ring focused on a screen in the focal plane of an achromatic lens,[4] and θ_n is the angular diameter of the nth ring and equal to 2ϕ, then

$$2t\cos\phi = M_n\lambda \tag{1}$$

and

$$M_n = (2t/\lambda)\cos\phi.$$

If ϕ is small, $\cos\phi$ may be written as:

$$\cos\phi = [1-(\phi^2/2)] = [1-(\theta_n{}^2/8)]. \tag{2}$$

If d_n is the linear diameter of the nth ring and f is the focal length of the lens which forms the image on a photographic film or focal plane of a measuring microscope, then:

$$\theta_n = d_n/f$$

and

$$M_n = (2t/\lambda)[1-(d_n{}^2/8f^2)]. \tag{3}$$

Table I gives the ring diameters obtained with a measuring microscope when a lens of 61.8 cm focal length is used with a plate spacing of 1.04 cm. The constancy of $d_n{}^2 - d_{n-1}{}^2$ is also shown. The monochromatic 5461-Å line of mercury was used.

TABLE I. Ring diameters of Fabry-Perot fringes.

Ring number	d_n	$d_n{}^2$	$d_n{}^2 - d_{n-1}{}^2$
1	0.6275 cm	0.3937 cm²	0.78825 cm²
2	1.0872	1.1820	0.81174
3	1.4120	1.9937	0.77881
4	1.6651	2.7725	0.81392
5	1.8938	3.5864	0.78662
6	2.0911	4.3727	0.77748
7	2.2694	5.1502	0.80782
8	2.4409	5.9580	0.79122
9	2.5960	6.7392	0.78122
10	2.7483	7.5532	0.81394
		Average	0.795488

The change in wavelength corresponding to a shift of one fringe may now be determined.[5] If d_n and d_n' represent the linear diameter of the nth fringe of the wavelengths λ and $\lambda+\Delta\lambda$, the following equations may be obtained. Solving Eq. (3) for λ, we get:

$$\lambda = (2t/M_n)[1-d_n{}^2/8f^2] \tag{4}$$

$$\lambda+\Delta\lambda = (2t/M_n)[1-d'^2/8f^2]. \tag{5}$$

Subtract Eq. (4) from Eq. (5) and get:

$$\Delta\lambda = \lambda(d_n{}^2 - d_n{}'^2)/8f^2 \tag{6}$$

since from Eq. (1), $\lambda \approx 2t/M_n$. Therefore, the value of $\Delta\lambda$ for one fringe shift may be obtained by substituting the average value of $d_n{}^2 - d_{n-1}{}^2$, from Table I, into Eq. (6) as:

$$\Delta\lambda = (5460.74)(0.795488)/(8)(61.8)^2$$
$$= 0.142\ \text{Å/fringe},$$

where the focal length of the lens used was 61.8 cm and the monochromatic source was the green line of the mercury spectrum. If a line with a

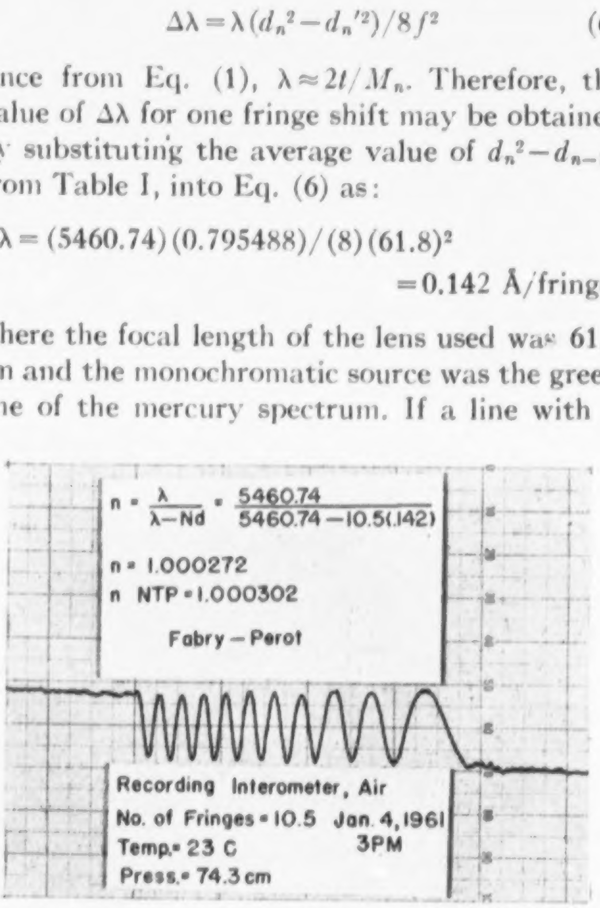


FIG. 4. Record of Fabry-Perot fringes moving across small circular slit as air slowly leaks into the vacuum chamber and between the plates of the interferometer.

---

[4] F. A. Jenkins and H. E. White, *Fundamentals of Optics* (McGraw-Hill Book Company, Inc., New York, 1957), 3rd ed., pp. 274–283.

[5] J. K. Robertson, J. Opt. Soc. Am. 9, 611 (1924).

satellite is photographed with a spectrograph in series with a Fabry-Perot interferometer, Eq. (6) permits the calculation of the difference in wavelength $\Delta\lambda$ of the satellite. This type of calculation is required in the experiment on the Zeeman effect.

MEASUREMENT OF THE INDEX OF REFRACTION OF A GAS

The apparatus for measuring refractive indices of gases is shown in Fig. 3. The Fabry-Perot interferometer is placed in a vacuum chamber which has optically flat glass plates at each end so that the monochromatic light may pass in and out of the chamber and through the Fabry-Perot interferometer. An achromatic lens focuses the fringes on a small circular slit. Air is pumped from the chamber and when a vacuum is obtained, a gas (air, CO_2, etc.) is allowed to leak slowly into the chamber. As the optical path between the Fabry-Perot plates changes, fringes move across the slit and are detected by the photomultiplier tube and recorded by a strip chart recorder as shown in Fig. 4.

The index of refraction of the gas may be calculated by using the following equation:

$$n = c/v = \nu\lambda/\nu\lambda' = \lambda/(\lambda - N\Delta\lambda), \qquad (7)$$

where c is the velocity of light in a vacuum, v is the velocity of light in the gas, ν is the frequency of the light, λ is the wavelength in a vacuum, λ' is the wavelength in the gas, N is the number of fringes moving past the small circular slit in front of the photomultiplier tube as the gas leaks into the chamber, and $\Delta\lambda$ is the change in wavelength per fringe passing the slit. If $\Delta\lambda$ is 0.142 Å/fringe and N is 10.5 fringes for air as shown in Fig. 4, then the index of refraction for air at a temperature of 23°C and 74.3 cm of Hg pressure was found to be;

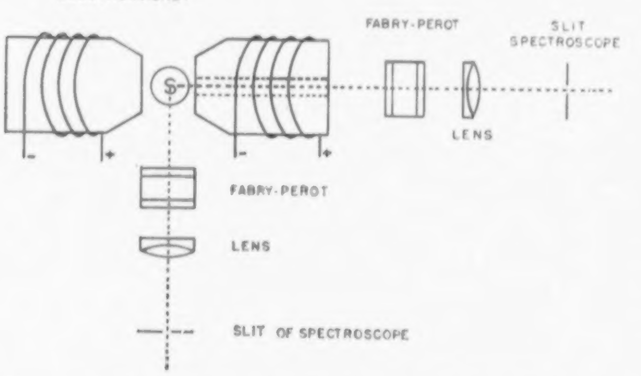


FIG. 5. Arrangement of magnet and Fabry-Perot interferometer for observing the Zeeman effect.

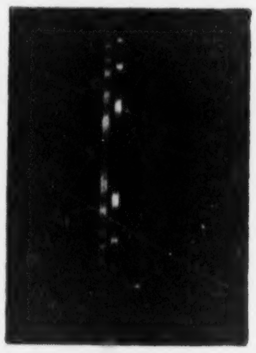


FIG. 6. Photograph of the 5400-Å line of neon showing the Zeeman effect. The line to the left is with magnetic field. The line to the right is without magnetic field.

$$n = 5460.74/[(5460.74) - (10.5)(0.142)] = 1.000302$$

which is in agreement with handbook values.

ZEEMAN EFFECT STUDIES

The Fabry-Perot interferometer has also been used to observe and record the Zeeman effect. The apparatus is arranged as in Fig. 5. A spectral tube containing neon is placed between the poles of a magnet and the Fabry-Perot fringes are focused on the slit of a spectrograph. Two exposures of the 5400-Å line of neon are shown in Fig. 6. The line to the right was photographed with no magnetic field and the line to the left with sufficient magnetic field to show the splitting of the 5400-Å line into two circularly polarized lines.

CONCLUSION

The construction and use of an economical but high-precision Fabry-Perot interferometer for instruction in an undergraduate optics laboratory has been a very interesting and worthwhile project.

Appreciation is expressed to the National Science Foundation for financial assistance through a Science Faculty Fellowship, to Professor P. C. Sharrah, University of Arkansas, who suggested the project, to Professor R. H. Hughes, whose direction and guidance made the project possible, and to Leo Takahashi, Art Cunningham, and Paul Todd, students at Willian Jewell College, who helped develop and test the experiments and equipment used with the Fabry-Perot interferometer.

References: Richtmeyer, Kennard and Lauritsen, Introduction to Modern Physics, pp. 285 - 286. (Larmor precession) Describes the basic cause of the Faraday effect by use of the rotating point-charge model of the atom.

Rossi, Optics, pp. 427 - 430. Uses rotating point-charge model to compute the polarization vector for an isotropic medium.

Jenkins and White, Fundamentals of Optics, pp. 596 - 598. The Faraday effect from an experimental viewpoint.

Wood, Physical Optics. Useful for information on experimental techniques with polarizers and quarter-wave plates.

If any transparent solid or liquid is placed in a uniform magnetic field and a beam of plane polarized light is passed through it in the direction parallel to the magnetic lines of force (through holes in the pole shoes of a strong electromagnet), it is found that the transmitted light is still plane polarized, but the plane of polarization is rotated by an angle proportional to the field intensity. This "optical rotation" differs in one important respect with the similar effect, called optical activity, occurring in sugar solutions. In a sugar solution, the optical rotation proceeds in the same direction, whichever way the light is directed. In particular, when a beam is reflected back through the solution it emerges with the same polarization as it entered before reflection. In the Faraday effect, however, the direction of the optical rotation, as viewed when looking into the beam, is reversed when the light traverses the substance opposite to the magnetic field direction; that is, the rotation can be reversed by either changing the field direction or the light direction. Reflected light, having passed twice through the medium, has its plane of polarization rotated by twice the angle observed for a single transmission.

By placing the sample between two polarizers (nicols), it can be arranged that no light is transmitted through the system in one direction, while it can pass, eventually with undiminished intensity, in the opposite direction. This effect is unique in this respect; it permits the construction of an irreversible optical instrument with which observer A can see observer B, while A cannot be seen by B.

Optical rotation is caused by circular birefringence. By the latter it is meant that the propagation velocity, or the refractive index, is different for right and for left circular polarized light. (For ordinary or linear birefringence, there is a difference of the refractive indexes for two plane polarized components of light which are normal to each other.) To show that optical rotation is equivalent to circular double refraction, we note that a plane polarized beam can always be considered as the coherent superposition of two circular polarized components of equal amplitude. For instance, a vibration $E_y = 2A \cos \omega t$ can be considered as the sum of

a right circular component	and	a left circular component
$E_x = A \sin \omega t$		$E_x = -A \sin \omega t$
$E_y = A \cos \omega t$		$E_y = A \cos \omega t$

If these equations represent the light incident on an active medium, the light emerging after passing through a distance D is given by similar equations that differ from the above only by the fact that the right circular components are shifted in phase by $2\pi n_r D/\lambda$, while the phase shift of the left circular component is $2\pi n_l D/\lambda$. n_r and n_l are the two indexes and $n_r - n_l = \Delta$ is the birefringence. The transmitted beam is therefore given by:

$$E_x = A \left[\sin(\omega t - 2\pi D n_r/\lambda) - \sin(\omega t - 2\pi D n_l/\lambda)\right] =$$

$$-2A \sin \left[(2\pi D/\lambda)\ 1/2(n_r - n_l)\right] \cos \left[\omega t - 2\pi D\ 1/2(n_r + n_l)/\lambda\right]$$

$$E_y = A \left[\cos(\omega t - 2\pi D n_r/\lambda) + \cos(\omega t - 2\pi D n_l/\lambda)\right] =$$

$$2A \cos \left[(2\pi D/\lambda)\ 1/2\ (n_r - n_l)\right] \cos \left[\omega t - 2\pi D\ 1/2\ (n_r + n_l)/\lambda\right]$$

These components are in phase; the transmitted light is therefore plane polarized, but the direction of vibration is changed from the y direction into a direction in the second quadrant which forms with the y direction an angle $-E_x/E_y = \tan\varphi$, whence a rotation

$$\varphi = \pi D(n_r - n_l)/\lambda$$

The rotation is counterclockwise when $n_r > n_l$.

Turning now to the Faraday effect, we remember that the refraction is the result of the interaction of the light with the electrons. Consider a right circular beam passing through a magnetized medium in the direction of the magnetic field B. For an observer looking into the beam, the light vector rotates clockwise, and the electronic structure rotates counterclockwise with the Larmor frequency. If ν is the frequency of light, it is apparent that to the rotating atom the light vector appears to be rotating at a higher frequency $\nu + \nu_L$ and it will act accordingly; that is the refractive index for this light will have the same value as the unmagnetized medium has for light of frequency $\nu + \nu_L$. Hence:

$$n_r(\nu) = n(\nu + \nu_L)$$

where n is the refractive index as ordinarily measured without magnetic field. In a similar way left circular light passing in the same direction as B appears to the atoms to have a lower frequency and, hence:

$$n_l(\nu) = n(\nu - \nu_L)$$

As ν_L is for visible light much smaller than ν, we can write

$$n(\nu \pm \nu_L) = n(\nu) \pm \frac{dn}{d\nu} \nu_L$$

or since,

$$\lambda = \frac{c}{\nu}, \frac{d\lambda}{d\nu} = - \frac{c}{\nu^2}, \frac{dn}{d\nu} = - \frac{dn}{d\lambda} \frac{c}{\nu^2} = - \frac{\lambda^2}{c} \frac{dn}{d\lambda}$$

and we get

$$n_r - n_l = 2 \frac{dn}{d\nu} L = - \frac{2\lambda^2}{c} \frac{dn}{d\lambda} \cdot \frac{e}{4\pi m_o} B$$

and finally for the optical rotation per unit length

$$\frac{\varphi}{D} = - 1/2 \frac{e}{m_o} \frac{\lambda}{c} \frac{dn}{d\lambda} \cdot B$$

The latter is called Becquerel's equation. $1/2\ (e/m_o)\ (\lambda/c)\ (dn/d\lambda) = C$ is called the Verdet Constant of the material and is given in degrees of rotation per gauss, per cm lightpath.

The Verdet Constant is seen to depend on the dispersion of the refractive index. Substances that are noted for their large dispersion; such as heavy flint glasses, and CS_2, also show a large Faraday effect, as predicted by the theory. In the visible range, the refractive index decreases with increasing wavelength (normal dispersion); hence, $dn/d\lambda$ is negative, and it follows that light travelling in the direction of B has its plane of polarization turned counterclockwise for an observer looking into the beam. The theory explains the reversal of the rotation when either the field or the light direction is reversed. The Faraday effect is particularly large near an absorption line and reverses its sign in the range of anomalous dispersion. All these theoretical conclusions are confirmed by the observations. The fact that the theory predicts the correct sign is a direct proof that the effect is related to the motion of a negative charge, the electrons.

With the exception of some paramagnetic materials, the quantitative observations are in excellent agreement with Becquerel's equation. Typical values are: From $\lambda = 6 \times 10^{-5}$ to 7×10^{-5} cm, the refractive index changes by about 10^{-2}; hence, $dn/d\lambda = 10^3$, $\lambda/c = 2.10^{-15}$, $e/m = 1.6 \times 10^7$ emu, and therefore C is about 10^{-5}, or of the order of 1/10 to 1/100 minutes or arc per gauss per cm. With good Nicol prisms, rotations of about one-half minute can be observed; and, since paths of several cm length and fields of a few thousand gauss can be used, the Faraday effect is quite easy to observe and is measurable with good accuracy. The experiment furnishes an outstanding example in showing that extremely small changes in the state of matter, which otherwise require the finest precision methods, can be studied with simple, cheap optical instruments.

Procedure:

PART I POLARIZED LIGHT

1. Examine the two images produced by a crystal of Iceland Spar. Study the polarization, the orientation, and direction of propagation of the two rays.

2. Determine the effects of a half-wave plate and a quarter-wave plate on plane and circularly polarized light. If no half-wave plate is given, make suitable use of two quarter-wave plates.

3. Produce elliptically polarized light by means of a polarizer and a quarter-wave plate. Then, by means of another quarter-wave plate and analyzer, determine the following: direction of rotation, position of the axes, and ratio of the axes of the elliptically polarized light. (If the plates are not provided with scales for measuring angles of rotation, only qualitative results will be expected.)

4. Place the piece of case hardened glass between two crossed polaroids and observe the effects of mechanical birefringence.

PART II FARADAY EFFECT

1. Set up the apparatus for measuring the Faraday effect. This consists of a monochromatic source (5890 A°), a polarizer, an analyzer, a water-cooled magnet, and the material in which the effect is to studied. The analyzer is a sheet of polaroid set in a graduated ring holder to allow measurement of the rotation of the plane of polarization. In this experiment, the material used is heavy flint glass in the form of blocks of varying thickness.

Place the glass between the pole pieces of the electromagnet. Polarized light is sent through the holes in the pole pieces and through the material parallel to the magnetic field.

The magnet is water-cooled, hence be certain that the water is running before using the magnet. It is best to send current through the magnet only when you are taking data.

2. Keeping the magnitude of the magnetic field constant, measure the rotation of the plane of polarization as a function of thickness of the material.

3. Keeping the thickness of material constant, measure the rotation as a function of magnetic field. Use the rotating coil gaussmeter to measure the magnetic field.

Report:

1. Describe, with a brief explanation, the results of Part I.

2. From the experimental data, develop an empirical relation between the angle of rotation φ, the magnetic field B, and the length of the block ℓ.

3. Calculate the Verdet constant for the glass.

4. Derive the relation:

$$\text{Verdet Constant} = \frac{\varphi}{B} = 1/2 \, \frac{e}{m_o} \, \frac{\lambda}{c} \, \frac{dn}{d\lambda}$$

5. From the experimental value of the Verdet Constant, and published values of $dn/d\lambda$, calculate e/m_o. What does this say about the relation between the Faraday effect and the motion of charge carriers in the glass used?

DISPERSION AND DOUBLE REFRACTION OF QUARTZ

University of Minnesota

The equilateral quartz prism used in this experiment has its optic axis normal to one refracting edge and parallel to one of the faces. All three faces are polished. It is mounted on a spectrometer so that light rays passing through at minimum deviation will make an angle of 60° with the optic axis. The unused third face may be temporarily "frosted" by drying a coating of sugar solution on it. This eliminates internal reflections that may be disturbing.

The spectrometer is adjusted in the standard manner and the refracting angle of the prism is measured. A mercury arc source is used behind the slit of the spectrometer. Two spectra will be observed due to the double refraction of quartz. The spectrum with the greater deviation is due to the extraordinary ray whose electric vibrations are in the plane of the optic axis and the wave-normal in the crystal. This is the socalled principal plane. With a Nicol prism or Polaroid filter one may check the vibration direction. The other spectrum, due to the ordinary ray, is found to contain vibrations at right angles to the principal plane.

The refractive index of the prism is measured by the minimum deviation method for three or four wave lengths for both the extraordinary and ordinary rays. A graph of the refractive index against the reciprocal of the square of the wave length gives a good approximation to a straight line and checks Cauchy's dispersion formula.

The index of refraction for the ordinary ray is the same as one of the two principal indices of refraction of quartz. (α) It does not vary with direction. The index of refraction obtained for the extraordinary ray is the value for 60° with the optic axis, μ_e. The corresponding principal index of refraction is the value for rays at 90° with the optic axis. This index, γ , is found from the index μ_e measured at 60° by the equation:

$$\frac{\cos^2 60^\circ}{\alpha^2} + \frac{\sin^2 60^\circ}{\gamma^2} = \frac{1}{\mu_e^2}$$

which is obtained from the index ellipsoid of quartz.

An optional exercise is to observe the circular double refraction of quartz when light travels along the optic axis. In this case one observes a closely spaced doubling of each spectrum line. Inserting a quarter wave plate compensator followed by a Nicol prism analyzer in the refracted light, one finds, by rotating the Nicol prism, that the two components of each spectral line are converted into plane polarized components vibrating at right angles to each other. This shows that the light in the two components is circularly polarized in opposite senses of rotation. If the fast axis of the compensator is known, one can find the sense of rotation for each component, as follows. If the resultant linear vibration is 45° counter clockwise of the fast axis of the compensator, the light is right-hand circularly polarized, while if it is 45° clockwise of the fast axis, the light is left-hand circularly polarized. In right-hand quartz, the clockwise rotating component has the lesser velocity and the greater refractive index and will hence be deviated more than the counterclockwise component.

STUDY OF MICROWAVES AS AN EXAMPLE OF WAVE PHENOMENA

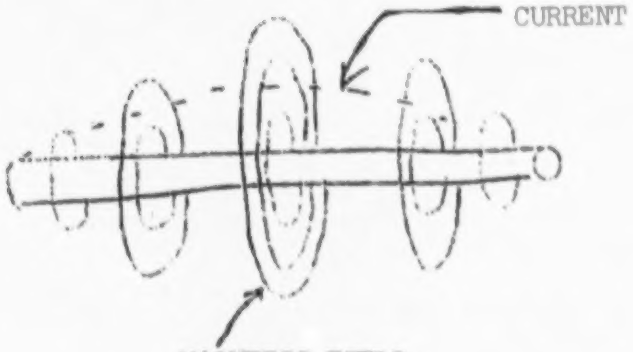
University of Colorado

OBJECTIVE: Electromagnetic waves are used to illustrate the properties of wave motion and some special examples of electromagnetic phenomena in wave motion are explored.
REFERENCE: Halliday and Resnick, Physics for Students of Science and Engineering, Chapters 38 and 39.

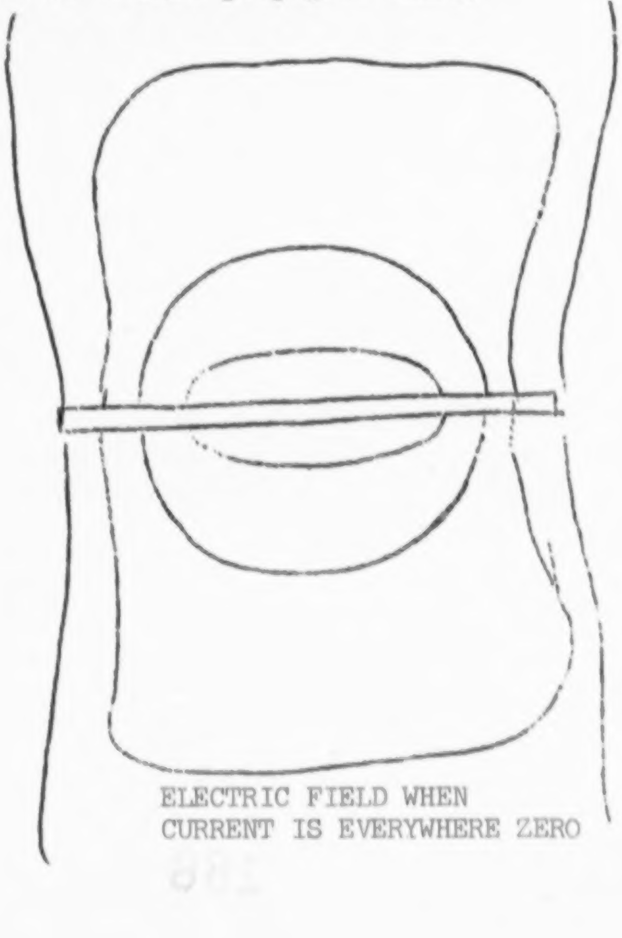
THEORY AND PROCEDURE:

In the mid 19th century Maxwell introduced a concept into the theory of electric and magnetic fields which provide a basis for understanding of electromagnetic phenomena and light in the sphere of classical physics (it must be modified in light of modern development for relativistic theories). The most significant outgrowth from this theory is the prediction that both electric and magnetic fields are propogated as waves in empty space. If an electric charge is suddenly displaced, the change in the electric field due to the charge does not change every where instantaneously but spreads out with a very high but finite speed. The same is true for magnetic fields. Furthermore as this changing electric field moves through space it is always accompanied by a magnetic field which is set up by the changing electric field. The two move through space together and form an electromagnetic wave. Maxwell predicted the speed of these electromagnetic waves to be 3×10^{10} cm/sec, which is the speed of light.

If an electrical oscillator is used to cause electric charges to move back and forth on a rod as pictured in Figure 1 below, oscillating electric and magnetic fields are set up around the charges. Figures 1a and 1b show only two extremes of the continually varying configuration, but you can work out for yourself a feeling for how the fields look at various stages of the oscillation. The electric and magnetic fields so created propogate energy away as an electromagnetic wave.



MAGNETIC FIELD
WHEN CHARGE IS
EVERYWHERE ZERO



ELECTRIC FIELD WHEN
CURRENT IS EVERYWHERE ZERO

If another such rod or antenna is set somewhere in space and oriented appropriately, these fields will induce changing currents in this remote antenna and may be used as a receiver of these electromagnetic waves.

> How should the receiver antenna be oriented with respect to the transmitting antenna in order that the maximum effect may be obtained, i.e. to get the maximum signal in the receiver?

Hertz used such a transmitter and receiver arrangement and the interference of such waves when reflected back on themselves by a metal reflector in order to substantiate the existence of the waves and to verify the speed predicted by Maxwell.

When an electromagnetic wave is reflected by a conducting sheet, one must consider the action of the electric and magnetic fields of the wave on the electrons of the conductor. From Figure 1 it follows that the electromagnetic wave has associated with it electric and magnetic fields. For the plane wave the two fields are mutually perpendicular to one another and to the direction of propagation. The electric field acts on the free electrons and causes them to oscillate at the same frequency but 180° out of phase so the wave which they generate cancel the incident wave and create the reflected wave. Considering the magnetic field action it is possible to arrive at the explanation for the recoil effect which takes place when light wave is reflected. Consider a plane wave with the E vector in the x direction at normal incident on a perfect conductor which lies in the xy plane. Such a polarized plane electromagnetic wave can be defined by

$$E_x i = E_m \cos \beta (vt - z) \tag{1}$$

Since the E field inside the perfect conductor must be zero (E - 0 in a perfect conductor) there must be a reflected wave so that the complete description of the waves above the plate must include a term for this reflected wave, which for the moment we will designate fy $f_r(vt + z)$ so that

$$E_x = E_x i + f_r(vt + z) = E_m \cos \beta (vt - z) + f_r(vt + z) \tag{2}$$

Because the reflecting plane is a perfect conductor E_x at $Z = 0$ must vanish so

$$f_r(vt + 0) = -E_m \cos \beta (vt - 0) \tag{3}$$

Thus changing the argument of f_r from vt to vt + z we have

$$E_x = E_m \cos (vt - z) - E_m \cos (vt + z). \tag{4}$$

From the definition of the wave length of such waves we can see that for the wave with frequency ω to repeat with a wavelength λ we must set β equal to $\frac{2\pi}{\lambda} = \frac{\omega}{v}$. Then the wave above the reflector is defined by

$$E_x = E_m \cos (\omega t - \beta z) - E_m \cos (\omega t + \beta z)$$

$$E_y = E_z = 0$$

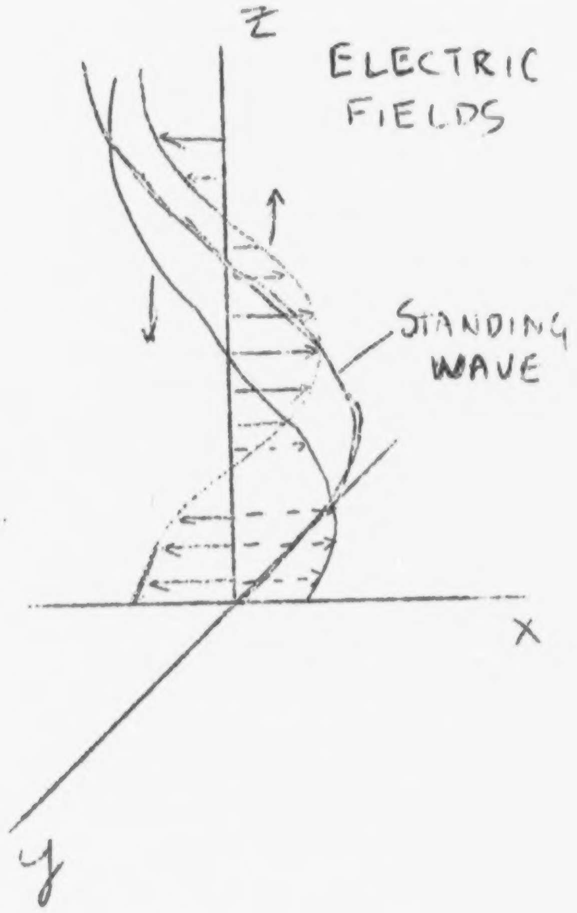
From Maxwell's equations the magnetic field associated with these E fields will be

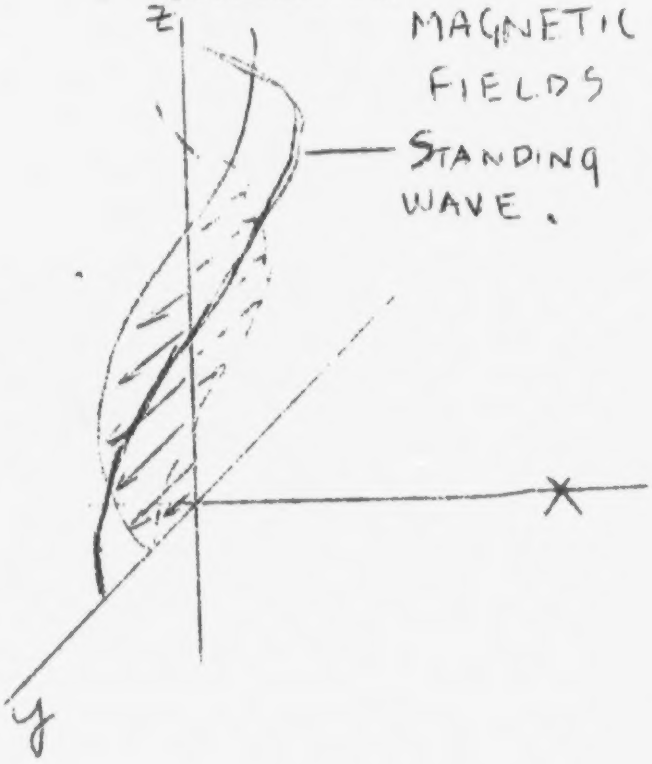
$$H_y = \frac{E_m}{\eta} \cos(\omega t - \beta z) + \cos(\omega t + \beta z) \text{ where } \eta = \mu/\varepsilon$$

$$H_x = H_z = 0.$$

μ = is the permeability of the space in which the wave travels.

ε = permittivity





We have in the laboratory for this experiment the necessary equipment to work with electromagnetic waves of a wavelength about 10 cm. This consists of a transmitter which is no more than a high frequency electric oscillator connected across a small gap in the middle of short conducting rod, forming a dipole antenna driven from the center. There is also a double dipole antenna which is just two such short split rods connected by cables to the same oscillator so that they can both radiate waves in phase with each other while free to be separated in space by up to one meter. The receiver consists of a short rod which is grounded in the center and has a contact connected just a little bit off center. This contact is connected through a diode and a shielded cable to a DC meter which reads the signal level induced in the antenna by the passing electromagnetic waves from the transmitter. There are also available various sheet metal reflectors, wire grids and metal sheets with appropriate slits for interference and diffraction measurements.

First set up the transmitter with the single antenna and use the receiver to plot the field strength of the waves transmitted. How do the fields vary around the transmitter assuming that the receiver meterreading is proportional to the magnitude of the E field?

Determine the variation of field strength with distance from the source. Try reflecting the waves with various objects. Explain why each reflector behaves as it does. Are there any special ways any of the reflectors tried must be oriented?

Explain.

Is the strength of the reflected wave reasonable compared to the incident wave in terms of your observations and explanations?

Can you form standing waves with this apparatus? Why? What is the wavelength of these waves (measure this as accurately as possible and then check with the instructor)?

What should be the shape of the interference pattern from two dipole antennae which are driven in phase from the same source? Give a derivation for this shape. Set up the two dipole antennae and measure the interference pattern and plot it for several nodal lines. Compare this pattern with that of one antenna and its image in a mirror (so called Lloyd's mirror setup.) Does this agree with what you expect?

From the dimensions of the slits in one of the metal plates, predict quantitatively what the space plot of the maximum magnitude of the E field should be if the metal plate were between the transmitter and the receiver as shown below in Figure 3. Check experimentally.

As mentioned before the fields from a dipole antenna are polarized. Determine experimentally how the E field vector is oriented at a point in the horizontal plane through the transmitter dipole which is itself vertical. How could you do this if you had no knowledge about how the receiver antenna was constructed.

REFERENCES:

Optics, C. L. Andrews
Instruction Manual, Cenco
Physics for Students of Science and Engineering, Halliday and Resnick, Pps. 827-834.
Physics for Engineers & Scientists, Fowler and Meyer, Chapter 25.

References:
1. Mimeographed Notes
2. Slater, *Microwave Electronics*
3. Ginzton, *Microwave Measurement*

Experiment Procedure:

I. Microwaves in free space

1. *Circuit:* Power supply - klystron - attenuator - wavemeter slotted section (probe in slotted section not used) - horn pickup probe (or loop) - galvanometer.

2. Connect voltmeter across terminals marked "reflector" on klystron power supply. Hold pickup probe in ring stand in front of horn. Adjust repeller (reflection) voltage on klystron to value giving good deflection on galvanometer. (It may be necessary to adjust the "double stub tuner" attached to the probe). Record the repeller voltage.

3. Use the pickup probe to find the radiation pattern from the horn. Do this *qualitatively*. (Aim horn away from reflecting surfaces).

4. Use probe and loop to find polarization of E and H fields, near horn and two feet away from horn.

5. How does radiation from horn differ from plane wave radiation?

6. Using the metal reflector, set up standing waves in free space. By moving the *reflector*, and observing the location of nodes with the probe determine the free space wavelength.

II. The Wavemeter

1. *Circuit:* Same as in I.

2. Leave klystron repeller voltage at value in I. Adjust probe position to give full scale deflection on galvanometer. *Very slowly* and *smoothly* turn wavemeter knob until a *sharp* decrease in deflection occurs. (This happens when wavemeter resonant frequency = klystron frequency). The decrease should be 15-20 galvanometer scale divisions - you should see the needle "kick" when you are at the resonance point. Use calibration curve on laboratory bench to obtain corrected wavemeter reading. Add 3.00 cm to find free space wavelength, λ_0. Compare with your first result for λ_0.

3. Calculate the klystron frequency.

III. Microwaves in wave guide

1. Circuit:
Power supply - klystron - attenuator - wavemeter slotted section (prove in slotted section connected to galvanometer) various terminating impedances.

2. Measurement of guide wavelength (λ_g).

(a) The guide operates in the $TE_{0.1}$ mode. By terminating the guide with a short, set up standing waves, and measure the difference in probe position between several minima to find guide wavelength (λ_g). (You should use same klystron frequency, i.e., same setting of repeller voltage and mechanical tuner, as in Parts I and II of this experiment).

(b) If klystron frequency has been changed, remeasure with the wavemeter.

(c) Measure the inner guide dimensions and calculate cutoff wavelength, λ_C.

(d) Use (b) and (c) to compute λ_g. Compare with experimental result in (a).

3. Reflection coefficient measurements:

Measure the voltage standing wave ratio (VSWR) and calculate reflection coefficient for waveguide terminated in short circuit, open circuit, matched load, horn. Interpret results.

IV. Klystron voltage modes:

1. Circuit: Power supply - klystron - attenuator - wave meter - slotted section - matched load.

2. Connect probe in slotted section to galvanometer (with shunt crross it). Connect voltmeter across terminals marked "reflector" (repeller) on klystron power supply. Plot relative power output vs reflector voltage. Why do we use a matched load here?

3. Connect probe in slotted section to vertical plates of oscilloscope. (Make sure the coax ground is connected to the scope ground). Connect scope sweep voltage (proper terminal on back of scope) to terminal marked "sweep" on klystron power supply. Vary the repeller D.C. voltage and observe the klystron "modes" on the scope. (Screw probe all the way in to get maximum power in the scope.

4. Adjust the wavemeter so that a dip appears on each of the mode curves. Measure the frequency of the klystron as a function of repeller voltage, establishing the voltage scale on the scope by comparing the mode curves on the scope with the mode curves plotted with the galvanometer.(Do this only for the strongest mode).

5. Change the setting of the mechanical tuner on the klystron. What happens to the central frequency of the mode?

V. Measurements on cavity

1. Circuit: Power supply - klystron - wavemeter - cavity slotted section (with probe attached to scope) - short.

2. Turn on vacuum system and evacuate cavity. Measure pressure.

3. Simultaneously adjust klystron tuner, repeller voltage, and probe position until you observe on the scope a large klystron mode with a dip in it. (This dip represents absorption of microwave power by the cavity). Turn wavemeter dial to ascertain that this dip is the cavity resonance, not a wavemeter resonance.

4. Adjust so that cavity dip is at center of voltage mode. (You will probably find two cavity dips; choose one of them for all your subsequent measurements.

5. Use wavemeter dip to measure resonant frequency and approximate "Q" of cavity.

6. Turn off pump and slowly readmit air into cavity. From the change in the cavity resonant frequency, calculate index of refraction of air at this frequency. (see below)

7. Evacuate cavity again, isolate it from pump, and admit NH_3 into cavity slowly (do this by opening and closing tank valve, then slowly opening the needle valve). Measure the index of refraction on NH_3 at 1/4, 1/2, 3/4 and 1 atmosphere.

8, Measure Q of cavity with NH_3 at 1 atmosphere. How do you account for the large absorption by NH_3? (See C.H.Townes, Phy.Rev. 70, 665, 1946; D.M. Dennison, Rev. Mod. Phys., 175, 1940.

MEASURING INDEX OF REFRACTION

The basic waveguide relation between free space wavelength, guide wavelength (λ_g) and cutoff wavelength is:

$$\frac{1}{\lambda_o^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$$

Since a cavity is a section of guide shorted at both ends, (ex. cavity used in Part V above, wavemeter, etc. this equation still applies. However, while in a non-shorted guide, λ_g may have any value under the cutoff wavelength, this is not true for a cavity. For cavities, λ_g has only certain allowed values determined by the cavity length, L, from: $\frac{n\lambda_g}{2} = L$ where n = 1, 2, 3, ... If the cavity operates in the lowest mode, n=1. Thus, since λ_g and λ_c are both fixed in a cavity, λ_o is also fixed.

Consider a wave of frequency =F being propagated in an unbounded region of empty space, with velocity = c (the velocity of light). The length of this wave is then: $\lambda = c/f$. If the space is now filled with dielectric, of index of refraction, n, the velocity of propagation is changed to $v = c/n$. Therefore, the new relation between λ and f is: $\lambda = c/nf$.

We apply this to a cavity. Here, $\lambda = \lambda_o$ and is fixed by the cavity dimensions (assuming cavity is operated in lowest mode). Thus we have:

$$\lambda_o = \frac{c}{f_1} \qquad \text{without dielectric}$$

$$\lambda_o = \frac{c}{nf_2} \qquad \text{with dielectric}$$

where f_1 = resonant frequency of cavity without dielectric, and f_2 = resonant frequency of cavity with dielectric. Therefore,

$$nf_2 = f_1 \qquad \text{or,} \quad n = f_1 / f_2$$

Hence, from the change in the resonant frequency of the cavity, we may determine the index of refraction.

Note on the wavemeter: The wavemeter is a cavity of variable length, so that λ_g in the wavemeter may be varied. λ_C is fixed by the wavemeter cross section. Therefore the wavemeter may be calibrated to read free space wavelength, λ_o directly. This calibration curve is on the laboratory bench.

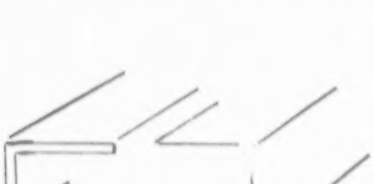
SPECIAL QUESTIONS:

1. (a) Generation of microwaves: Discuss briefly the operation of the klystron, including description of the klystron construction, modes, etc. What are the fine and course frequency controls on the klystron? How can the klystron frequency be varied periodically in time?

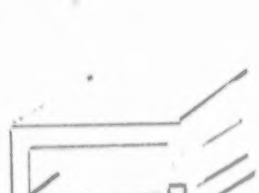
(b) Detection of microwaves: Describe operation of crystal detector. What does the crystal measure when used as a microwave detector?

2. (a) Using your knowledge of current distributions in a rectangular waveguide, discuss qualitatively the effects on TE modes of a thin slot in the following wave guides (as shown below)

a)

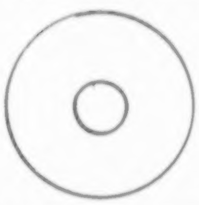


b)

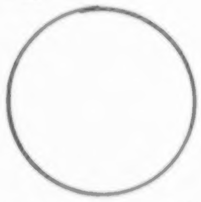


3. Microwaves of wavelength λ_o are to be transmitted through a rectangular waveguide of dimensions "a" and "b" $(a > b)$. Determine the allowed dimensions of the guide (i.e., find the permissible values of "a" and "b" such that the guide transmits only the TE_{10} mode.

4. Consider the following two cylindrical waveguides:



a) concentric cylinders



b) hollow cylinder

On the basis of physical reasoning only, which guide will have the lower cutoff frequency?

5. Group and Phase velocity

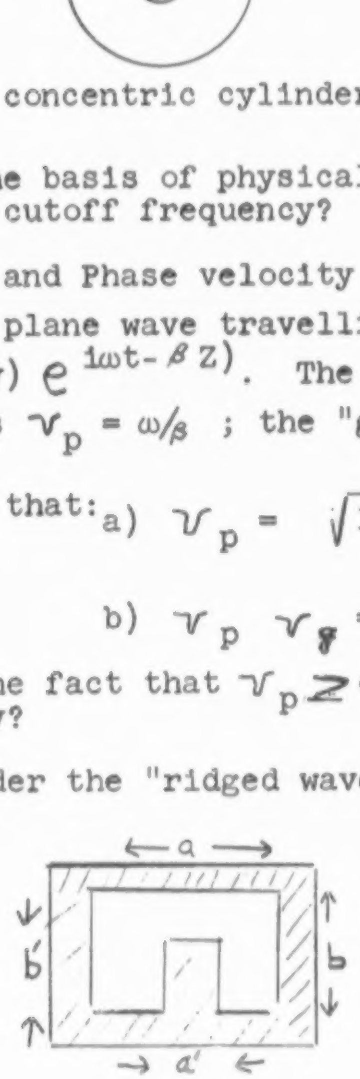
For a plane wave travelling in the positive Z direction, $\vec{E} = \vec{E}_o(x,y)\, e^{i\omega t-\beta Z)}$. The "phase velocity" of this wave is defined as $v_p = \omega/\beta$; the "group velocity" is given by $v_g = 1/\frac{d\beta}{d\omega}$.

Show that: a) $v_p = \frac{C}{\sqrt{1-(\frac{\omega_c}{\omega})^2}}$ ω_c = cutoff frequency

b) $v_p\, v_g = C^2$

Is the fact that $v_p \gtrsim c$ consistent with the Special Theory of Relativity?

6, Consider the "ridged waveguide shown below:



with no ridge

the cutoff wavelength is: λ_c.

From physical reasoning only, determine whether the cutoff wave length for the ridged guide, λ'_c is larger than, equal to, or less than λ_c.

7. What is a wavemeter? Does a wavemeter measure free space wavelength or guide wavelength? What is the relation between wavemeter reading and resonant frequency of wavemeter?

MICROWAVE PROPAGATION

University of California, Berkeley

In this experiment we will investigate the propagation of electromagnetic fields in free space and in bounded media. Such studies are performed most simply in the microwave frequency range where the available power in a narrow frequency range is high and where the wavelength is of the order of convenient laboratory distances.

The most satisfactory way to develop the theory of electromagnetic propagation is from Maxwell's equations. For this development we refer you to your text. We will present an alternate theoretical treatment here, which follows the results of Experiment B-8. In B-8 we computed the inductance per unit length and the capacitance per unit length for a coaxial line and obtained the results:

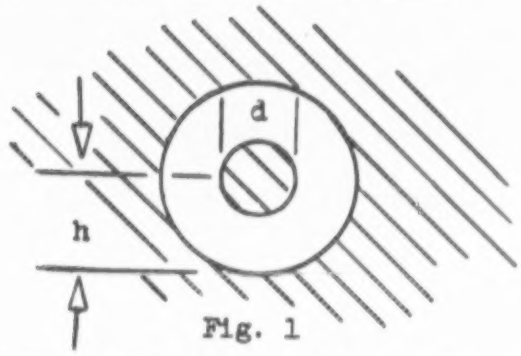


Fig. 1

$$L' = (2/c^2) \ln (2h/d) \qquad (1)$$

$$C' = [2 \ln(2h/d)^{-1}] \qquad (2)$$

In the experiment we studied the propagation of a sinusoidal wave on a single line near ground. We chose the single line because of the ease in varying the position of the line and the accessibility of the central conductor. The inductance and capacitance per unit length for the single wire near ground are:

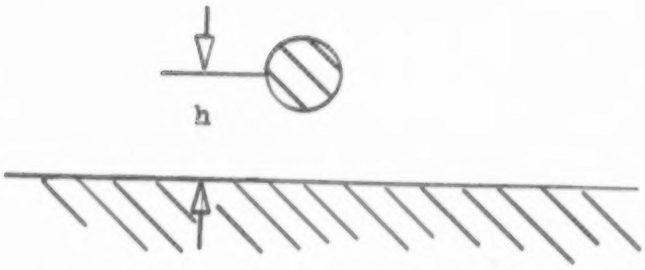


Fig. 2

$$L' = (2/c^2) \cosh^{-1} (2h/d) \qquad (3)$$

$$C' = [2 \cosh^{-1} (2h/d)]^{-1} \qquad (4)$$

We also developed the theory of propagation on a distributed line as a

---

Copyright © 1963 Educational Services Incorporated. No part of this manuscript may be reproduced without written permission of Educational Services Incorporated.

limiting case of the lumped line of Experiments B-6 and B-7. We found an equation for a voltage wave of the form:

$$d^2V/dz^2 = (L'C')d^2V/dt^2 \qquad (5)$$

This equation has the same form as the familiar wave equation:

$$d^2y/dz^2 = (1/u)^2\, d^2y/dt^2 \,. \qquad (6)$$

By comparing Eqs. (5) and (6) we obtain the result for the velocity of propagation:

$$u = (1/L'C')^{\frac{1}{2}} \,. \qquad (7)$$

Substituting from Eqs. (1) and (2) or (3) and (4) we see that the velocity of propagation in both cases is given by

$$u = c \,. \qquad (8)$$

Before discussing propagation in space we will consider one final example of propagation on a transmission line. Consider a parallel line formed from two conducting strips separated by a distance h short compared with their width a. The capacitance per unit length can be computed directly to be:

$$C' = A'/4\pi h' = a/4\pi h \qquad (9)$$

where A' is the area per unit length and is equal to the width a.

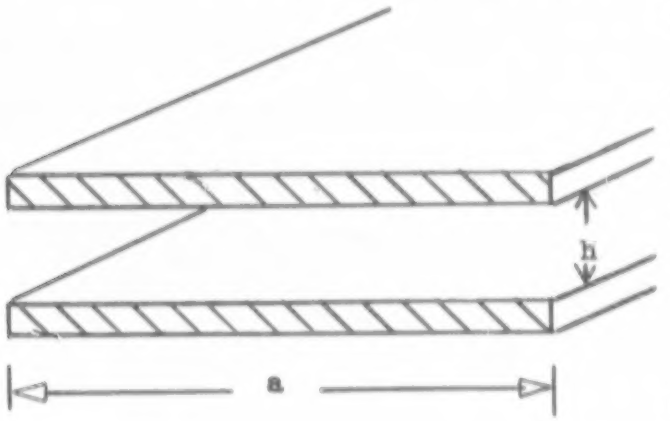


Fig. 3

The inductance per unit length may be computed once we know the included flux. From Stoke's Law the integral of the induction around a closed path is given by:

$$\oint B \cdot dl = 4\pi I/c \,. \qquad (10)$$

The induction is largely concentrated between the strips so that we can write

$$\oint B \cdot dl = Ba = 4\pi I/c \qquad (11)$$

where B is the value of the induction between the strips. The inductance per unit length is given by

$$L' = (1/c)\,\Phi'/I = Bh/Ic = (1/c)^2 4\pi h/a\,. \qquad (12)$$

By substituting Eqs. (9) and (12) into Eq. (7), we see that the velocity of propagation is again equal to c. In fact we can prove that for any uniform transmission line the velocity of propagation in vacuum is c. For the strip line the electric field is everywhere uniform and is simply equal to V/h. If we divide Eq. (5) by h on both sides we can write a wave equation for the electric field:

$$d^2E/dz^2 = (1/c)^2\, d^2\, E/dt^2 \qquad (13)$$

where E is normal to the plane of the strips.

Let us now imagine that we increase the width a of the strips and their separation h, keeping the ratio h/a small so that the electric field is uniform in the transverse plane. Eq. (13) is unaffected by the absolute magnitudes of a and h so long as the field is uniform. In the limit that h is infinite we are left with a description of the propagation of an electric field in a conductor-free space. Thus Eq. (13) provides a description of the propagation of a plane electromagnetic wave in free space.

We begin our experimental investigation by studying the multiple reflection of a plane electromagnetic wave by a pair of parallel plates. In order to couple the wave into and out of the interior space we may use metal screening or hardware cloth. Consider the arrangement shown in Fig. (1). A microwave transmitting horn injects an electromagnetic wave into the space between the plates. The wave is multiply reflected by

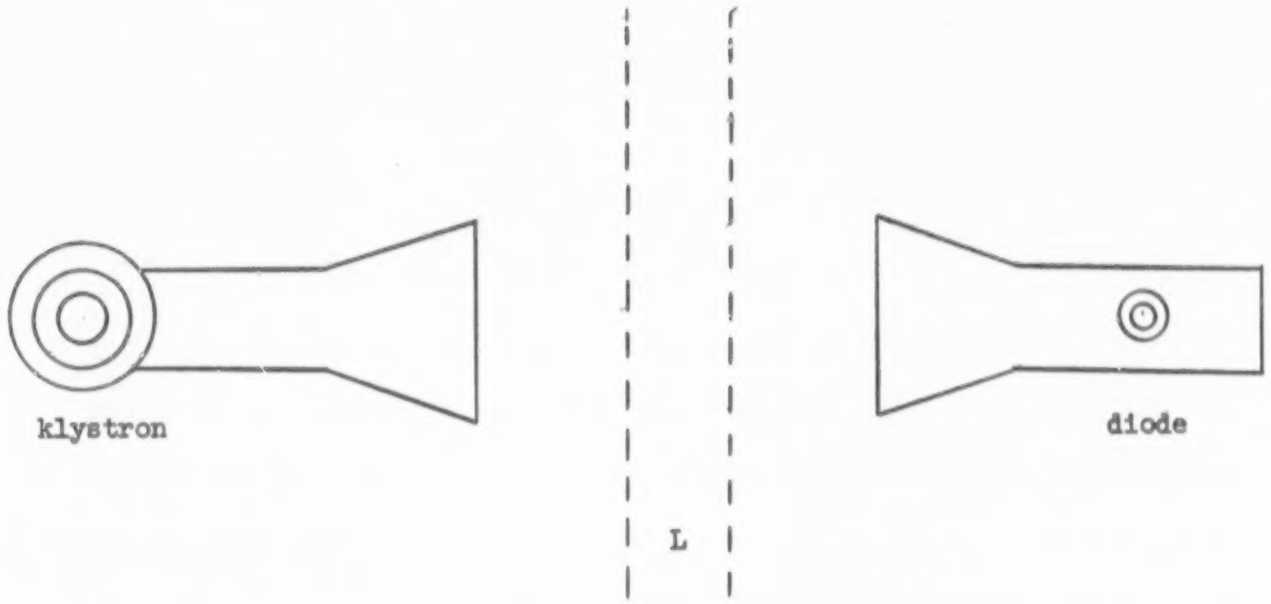


Fig. 4.

the plates. The rate at which microwaves are radiated into the receiver horn depends on the microwave level between the plates. We can expect this level to be appreciable only if the incident microwave field is reinforced by the field which has been reflected twice:

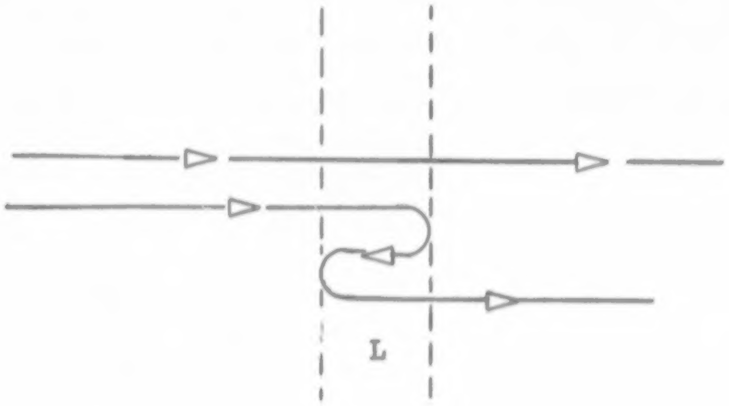


Fig. 5.

The two waves will reinforce each other only if the path difference is equal to an integral number of wave lengths:

$$2L = n\lambda_o \quad \text{or} \quad L = n(\lambda_o/2) . \qquad (14)$$

Adjust the klystron oscillator to the middle of its mechanical tuning range and find those values of L for maximum intensity. You should be able to produce a plot of the kind shown below:

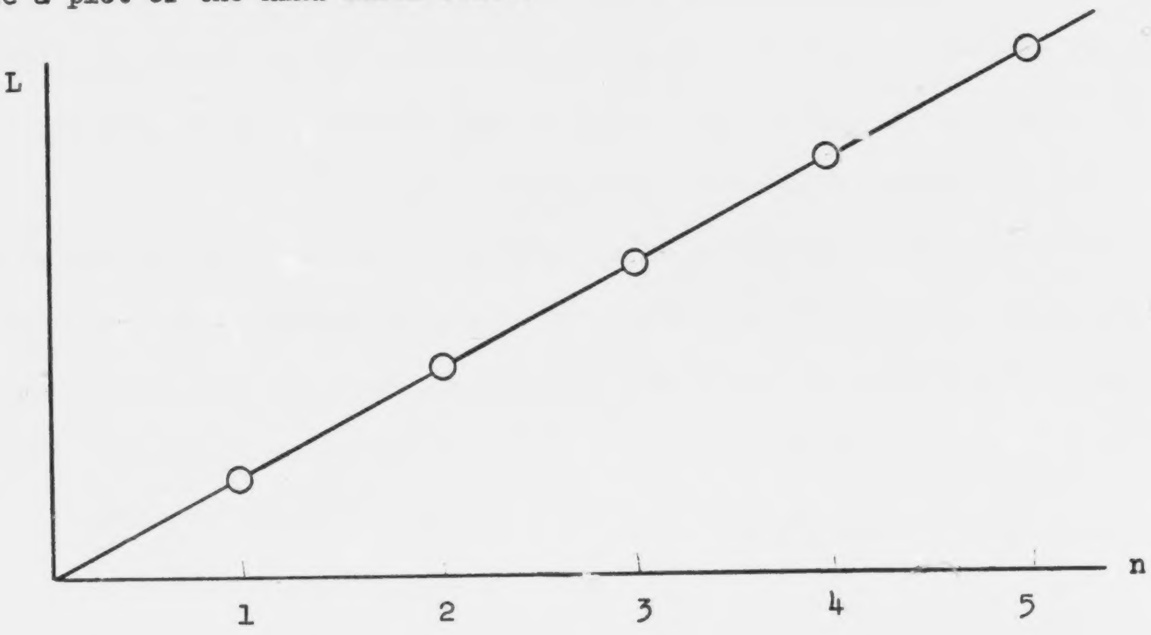


Fig. 6.

From your data determine the wave length λ_o and compute the klystron frequency $\nu = c/\lambda_o$.

We next consider the propagation in the z direction of a wave bounded by a pair of planes parallel to the electric field E as shown below. What is the wave equation for the electric field under these conditions? We must remember that a wave equation is a general equation which has as its solutions all physically possible waves. Although we are interested in a wave propagating along z our

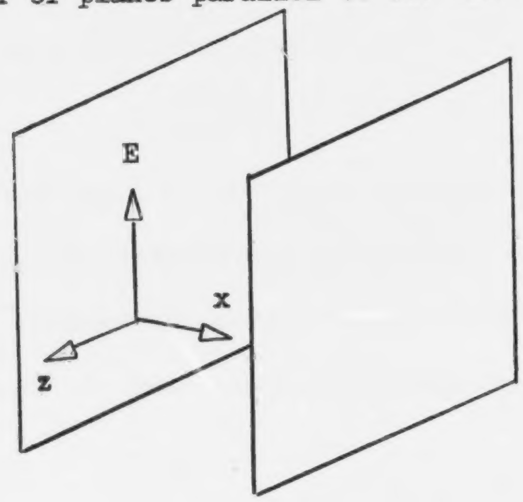


Fig. 7.

equation must reduce to Eq. (13) for a wave propagating along x. In this way we are led to an equation of the form:

$$d^2E/dx^2 + d^2E/dz^2 = (1/c)^2 d^2E/dt^2 . \tag{15}$$

For a plane wave propagating along x, E is independent of z so that the second term on the left side of Eq. (15) vanishes and we obtain Eq. (13) as required. Whereas Eq. (13) is the analog of the wave equation on a string, Eq. (15) is the analog of the wave equation of a diaphragm.

We could have developed Eq. (15) by starting from the delay net rather than the delay line. A delay net is most simply formed by arranging the inductors on a square net as shown in Fig. 8 and connecting capacitors from the points of intersection to ground. The rate of change of voltage across a capacitor is now related to four currents rather than two. We can consider a pair of parallel plates as the limiting case of the delay net. Finally we may let the separation between the plates increase to infinity, giving us Eq. (15).

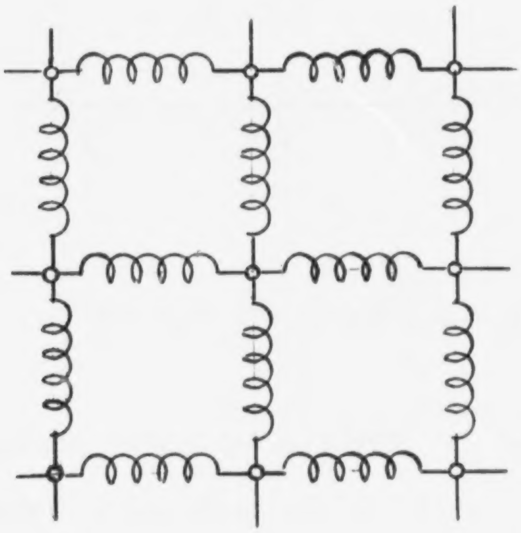


Fig. 8.

What is the phase velocity of a wave traveling along the z direction as shown in Fig. 7? Since the side plates are conducting, the electric field must vanish at x = ± a/2. In order to satisfy the wave equation we look for a form of E such that d^2E/dx^2 has the same dependence on x and z as does E. Consider the form:

$$E(x,z,t) = E(z,t) \cos \pi x/a . \tag{16}$$

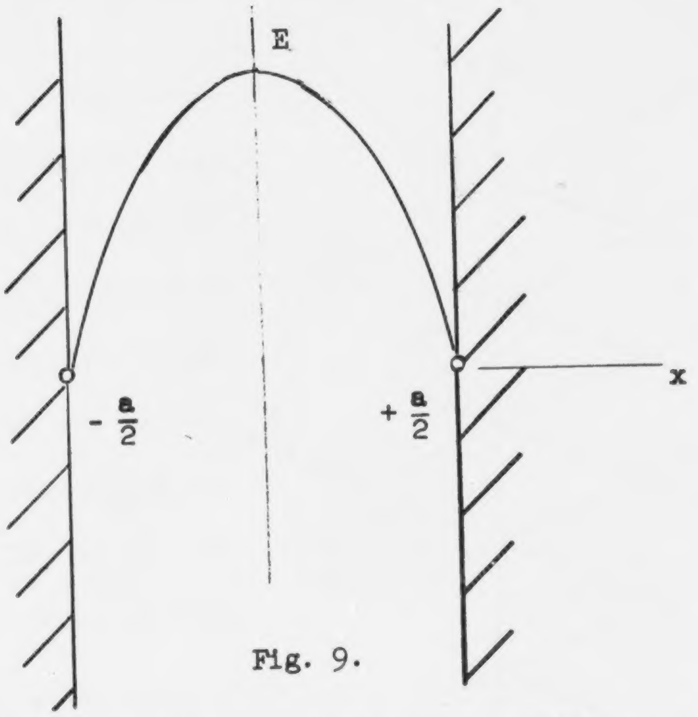


Fig. 9.

Taking the second derivative with respect to x we obtain:

$$d^2E(x,z,t)/dx^2 = -(\pi/a)^2\ E(z,t)\cos \pi x/a \ . \quad (17)$$

Substituting into Eq. (15) we obtain:

$$d^2E(z,t)/dz^2 - (\pi/a)^2E(z,t) = (1/c)^2d^2E(z,t)/dt^2 \ . \quad (18)$$

Now let us imagine that $E(z,t)$ is a sinusoidal traveling wave of the form:

$$E(z,t) = E_o \cos \omega(t - z/u) \ . \quad (19)$$

Substituting into Eq. (18) we obtain

$$(\omega/u)^2 - (\pi/a)^2 = (\omega/c)^2 \ . \quad (20)$$

Rearranging terms we obtain for the phase velocity

$$u = c/[1 - (\lambda_o/2a)^2]^{\frac{1}{2}} \quad (21)$$

or for the wave length

$$\lambda = \lambda_o/[1 - (\lambda_o/2a)^2]^{\frac{1}{2}} \quad (22)$$

where $\lambda_o = c/\nu$ is the free space wave length. There is a simple geometrical construction which also gives the wave equation solution. We

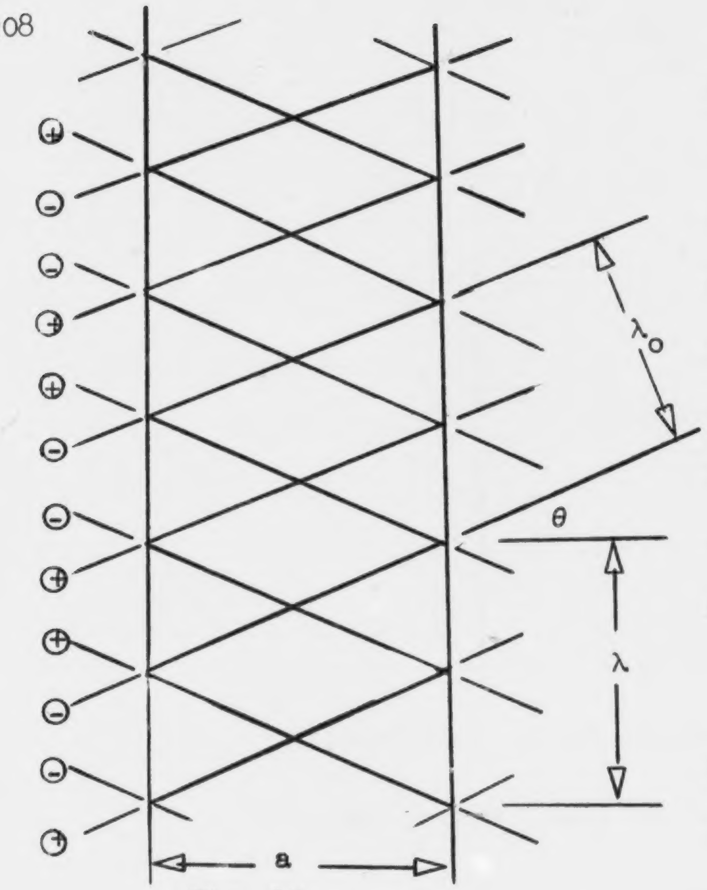


Fig. 10.

can obtain a transverse electric field of the form given by Eq. (16) from a pair of plane waves oriented as shown. In order for the net electric field to be zero at the walls the sine of the angle between the wavefront and the normal to the plates must be given by

$$\sin\ \theta\ =\ \lambda_o/2a\ . \qquad (23)$$

Now the wavelength is given from the figure by

$$\lambda\ =\ \lambda_o/\cos\ \theta\ =\ \lambda_o/[1\ -\ (\lambda_c/2a)^2]^{\frac{1}{2}} \qquad (24)$$

which is the result of Eq. (22).

In order to study the propagation of bounded electromagnetic waves we return to the experimental arrangement of Fig. 4 except that we add side baffles to restrict the waves.

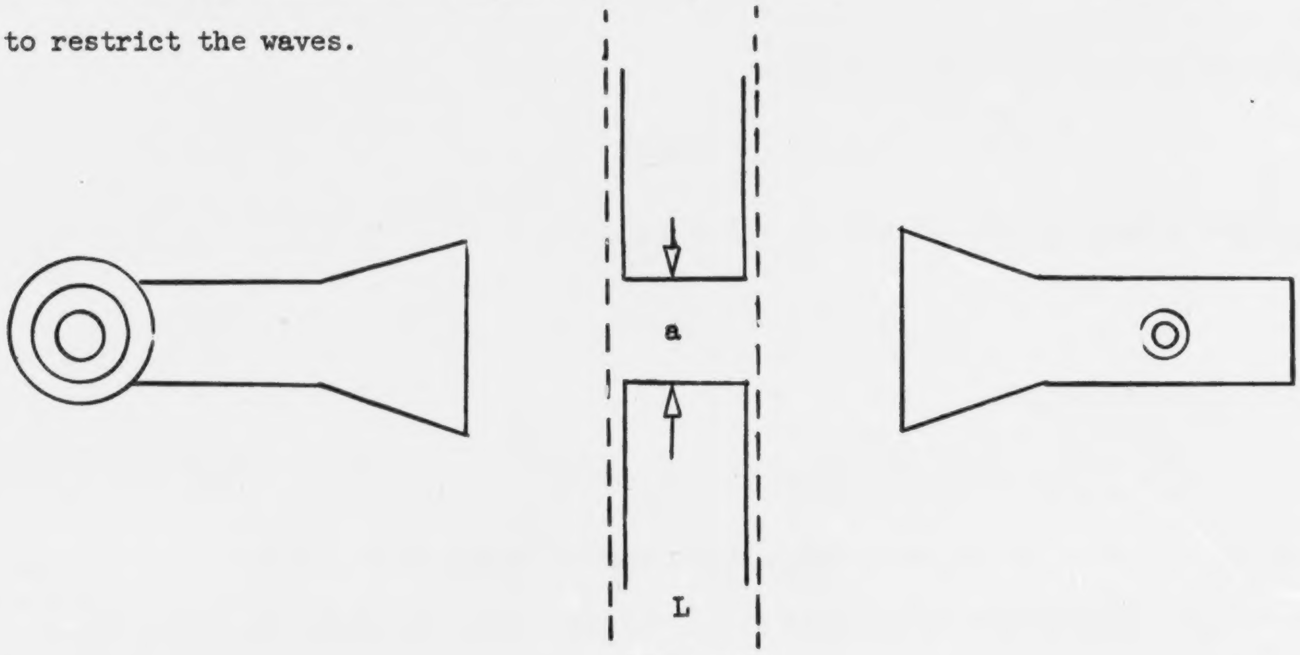


Fig. 11.

Again, we will expect maximum transmission under the condition:

$$L = \lambda/2 = (\lambda_o/2)/[1 - (\lambda_o/2a)^2]^{\frac{1}{2}} . \qquad (25)$$

Find several combinations of a and L which give maximum transmission. By rearranging Eq. (25) we can obtain the expression

$$(1/2L)^2 + (1/2a)^2 = (1/\lambda_o)^2 . \qquad (26)$$

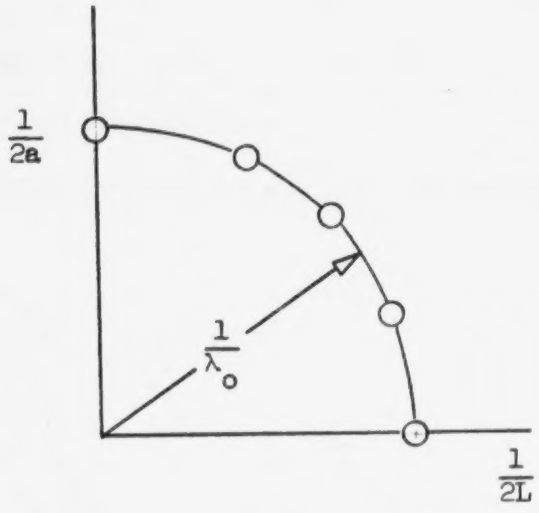


Fig. 12.

This form suggests a simple way of plotting your experimental data. If (1/2L) and (1/2a) are taken to be the x and y coordinates of a point then all points must lie on a circle of radius $(1/\lambda_o)$. Record your data on a plot of this kind.

We finally consider the character of the propagation as a approaches $\lambda_o/2$. From Eqs. (16) and (19) we have for the electric field:

$$E(x,z,t) = E_o \cos \pi x/a \cos \omega(t - z/u) \qquad (27)$$

where the phase velocity is given by:

$$u = c/[1 - (\lambda_o/2a)^2]^{\frac{1}{2}} .$$

Note that when 2a is smaller than λ_o the velocity becomes imaginary:

$$u = ic/[(\lambda_o/2a)^2 - 1]^{\frac{1}{2}} = iw . \qquad (28)$$

We can find the form of E(x,z,t) very simply by recalling that

$$\cos \omega(t - z/u) = \text{Real part } [e^{i\,\omega(t - z/u)}] .$$

Then we have

$$\cos \omega(t + iz/w) = \text{Real part } [e^{i\,\omega(t + iz/w)}]$$

$$= e^{-\omega\, z/w} \cos \omega t \quad . \qquad (29)$$

Thus we see that when $2a$ is smaller than λ_o we have an exponential decrease in the field along the guide. The critical frequency is given by

$$\nu = c/\lambda_o = c/2a \quad . \qquad (30)$$

This frequency is called the cut-off frequency for a guided wave. It is the lowest frequency at which electromagnetic waves can propagate down the guide.

In order to study the cut-off of a guided wave arrange a pair of baffles as shown below:

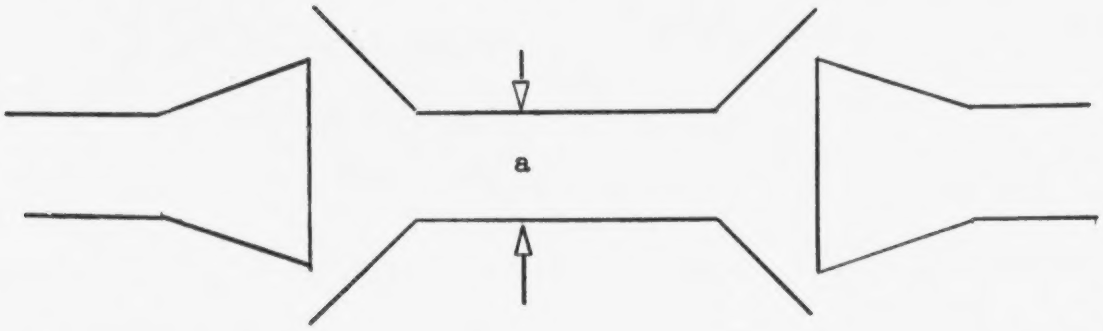


Fig. 13.

Gradually reduce the separation between the plates keeping them parallel. You may note an oscillatory variation in transmitted signal because of multiple reflections from the ends of the plates. You will find a critical separation at which the trensmitted signal drops abruptly to zero. This is the cut-off condition. Enter the measured separation on Fig. 12 at $(1/2L) = 0$.

As we have seen from Eqs. (21) and (22) the phase velocity u and wavelength λ both go to infinity as $2a$ approaches λ_o. Does this mean that we can signal with infinite speed under these conditions? We would be very surprised if special relativity were so easily violated. This problem is most simply discussed in terms of Fig. 10 where we have used a pair of plane waves to represent the propagation of a guided wave. From this figure we see that the guide wave wavelength:

$$\lambda = \lambda_o/\cos\theta \tag{31}$$

is purely a geometrical quantity and neither it nor the phase velocity

$$u = c/\cos\theta = c/[1 - (\lambda_o/2a)^2]^{\frac{1}{2}} \tag{32}$$

has any physical significance. When we speak of signaling we are in fact concerned with energy transfer. From Fig. 10 we see that energy travels down the guide with a velocity which is equal to the component of c on the guide direction:

$$v = c\cdot\cos\theta = c[1 - (\lambda_o/2a)^2]^{\frac{1}{2}} . \tag{33}$$

This velocity v is usually called the group velocity since it is the velocity with which the envelope of a group of waves moves down the guide. As the frequency ν approaches the cut-off frequency c/2a the phase velocity u goes to infinity. However the group velocity, which is the velocity with which we can signal, goes to zero. Since any wave can be made up out of plane waves moving in various directions we can easily see that c must be an upper limit for the rate of energy transfer, which is equivalent to the velocity of signaling.

MICROWAVE DIFFRACTION

University of California, Berkeley

In Experiment B-10 we examined two kinds of simple solutions of the two-dimensional wave equation:

$$d^2E/dx^2 + d^2E/dz^2 = (1/c^2)d^2E/dt^2 \qquad (1)$$

where E is along the y direction and is independent of y. One solution is the familiar plane wave:

$$E = E_0 \cos \omega(t - z/c) \qquad (2)$$

which is shown in Fig. 1. A second solution is that of a guided wave, which is constrained to be zero at the conducting surfaces, whose planes are designated by $x = \pm a/2$. This wave has the form:

$$E = E_0[\cos \pi x/a] \cos \omega(t - z/u) \qquad (3)$$

where the phase velocity is given by:

$$u = c/[1 - (\lambda_0/2a)^2]^{\frac{1}{2}} . \qquad (4)$$

There is another simple solution of the two-dimensional wave equation, which has cylindrical symmetry:

$$E = E_0(c/\omega r)^{\frac{1}{2}} \cos \omega(t - r/c) \qquad (5)$$

and is shown in Fig. 2.

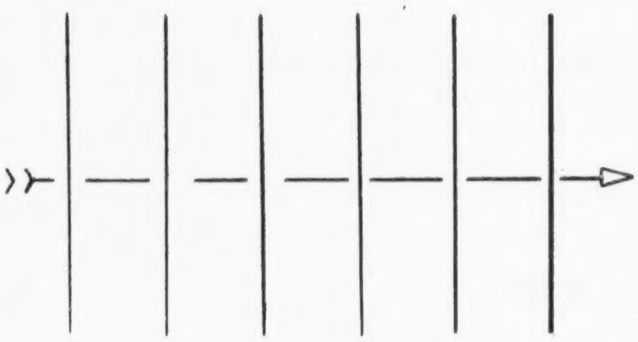


Fig. 1

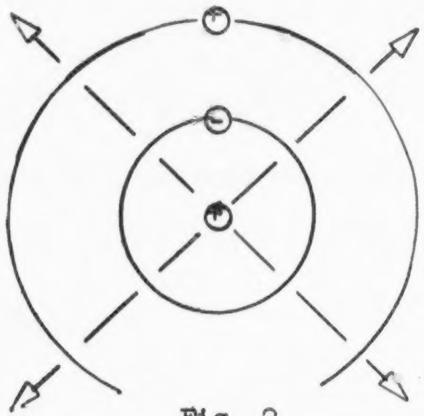


Fig. 2.

Copyright © 1963 Educational Services Incorporated. No part of this manuscript may be reproduced without written permission of Educational Services Incorporated.

How can we determine whether Eq. (5) is a solution of the wave equation as given By Eq. (1)? We examine this question by direct substitution of Eq. (5) into the wave equation. In order to simplify the differentiation we note the following relations:

If we have a function $f(r)$ we can write the derivative of $f(r)$ with respect to z in the form:

$$(d/dz)\, f(r) = (z/r)(d/dr)\, f(r) \qquad (6)$$

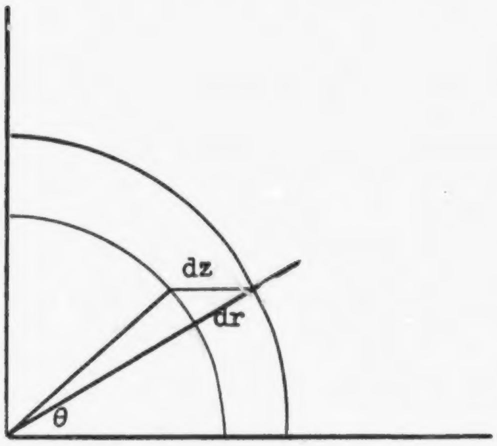


Fig. 3.

This relation follows simply from geometry as shown to the left in Fig. 3. The change in $f(r)$ in a distance dz is simply given by the change in $dr = (\cos\theta)dz$. But $\cos\theta$ is equal to z/r, which gives Eq. (6).

The second derivative may be obtained from the relation:

$$\begin{aligned}(d^2/dz^2)\, f(r) &= (d/dz)[(z/r)\,(d/dr)\, f(r)] \\ &= (1/r)\,(d/dr)\, f(r) + (z^2/r)(d/dr)[(1/r)(d/dr)\, f(r)]\end{aligned} \qquad (7)$$

Similarly we can write:

$$(d^2/dx^2)\, f(r) = (1/r)\,(d/dr)\, f(r) + (x^2/r)\,(d/dr)[(1/r)\,(d/dr)\, f(r)]\;. \qquad (8)$$

Adding these two equations we obtain:

$$\begin{aligned}(d^2/dx^2 + d^2/dz^2)\, f(r) &= (2/r)\,(d/dr)\, f(r) \\ &\quad + r(d/dr)[(1/r)\,(d/dr)\, f(r)] \\ &= (1/r)\,(d/dr)\, f(r) + (d^2/dr^2)\, f(r)\;. \end{aligned} \qquad (9)$$

Letting $f(r)$ be $E(r)$ we obtain:

$$(d/dr)\ E(r) = -\tfrac{1}{2}\,(\omega/c)\ E_0\ (c/\omega r)^{3/2} \cos \omega(t - r/c) + (\omega/c)\ E_0\ (c/\omega r)^{\frac{1}{2}} \sin \omega(t - r/c) \qquad (10)$$

$$(d/d^2r)\ E(r) = (3/4)\ (\omega/c)^2\ E_0\ (c/\omega r)^{5/2} \cos \omega(t - r/c) - (\omega/c)^2\ E_0\ (c/\omega r)^{3/2} \sin \omega(t - r/c) - (\omega/c)^2\ E_0\ (c/\omega r)^{\frac{1}{2}} \cos \omega(t - r/c)\ . \qquad (11)$$

Substituting Eqs. (10) and (11) into Eq. (9) we obtain:

$$(d^2/dx^2 + d^2/dy^2)\ E(r) = -(\omega/c)^2\ E_0\ (c/\omega r)^{\frac{1}{2}} \cos \omega(t - r/c) + \tfrac{1}{2}\,(\omega/c)^2\ E_0\ (c/\omega r)^{5/2} \cos \omega(t - r/c)\ . \qquad (12)$$

For distances r sufficiently great that

$$(c/\omega r)^2 = (1/2\pi)^2\ (\lambda_0/r)^2$$

is small compared with one, we can neglect the second term in Eq. (12). If we are interested in making measurements of the transmitted power to 10 percent accuracy this means r greater than one wave length. Now the right side of Eq. (1) is simply given by:

$$(1/c^2)(d/dt^2)\ E(r) = -(\omega/c)^2\ E_0\ (c/\omega r)^{\frac{1}{2}} \cos \omega(t - r/c) \qquad (13)$$

which is the dominant term in Eq. (12) for $r > \lambda_0$.

Our conclusion is that a cylindrical wave of the form given by Eq. (5) is an approximate solution of the wave equation which is accurate to 10 percent for $r > \lambda_0$ and becomes increasingly more accurate for larger values of r. We may expect a radiation pattern of the form of Eq. (5) to be produced by a simple vertical oscillatory current as shown in Fig. 4.

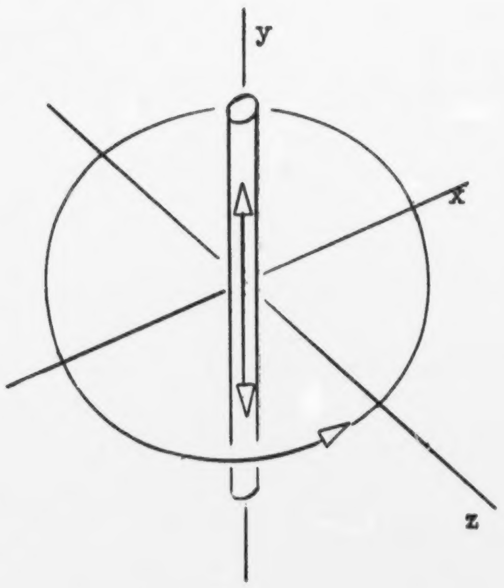


Fig. 4.

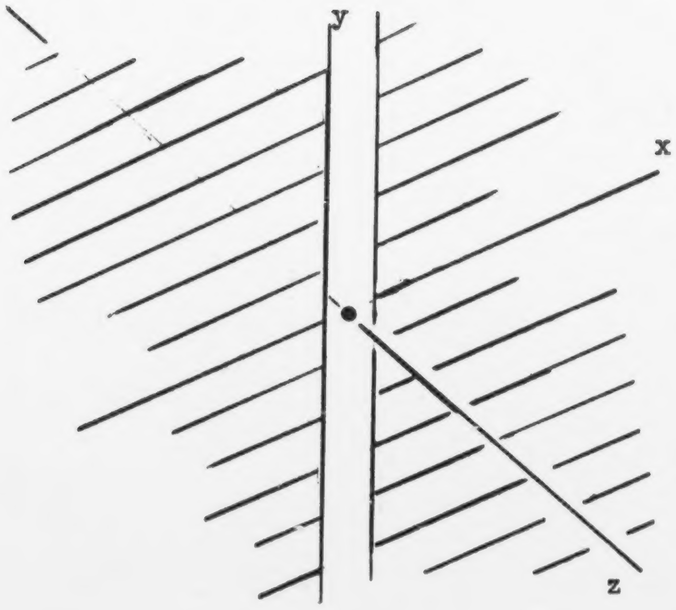


Fig. 5.

In the present experiment we will be interested in the radiation pattern from a slit in a conducting plane of the form shown in Fig. 5. It is clear that Eq. (5) can not describe the radiation field from a slit. Since the slot is in a conducting plane at $z = 0$ the electric field must be identically zero at $z = 0$ for all r. We can simulate such a solution by writing a wave which is (c/ω) times the derivative of Eq. (5) with respect to z:

$$\begin{aligned}(c/\omega)\ (d/dz)[(c/\omega r)^{\frac{1}{2}} E_0 \cos \omega(t - r/c)] &= \\ &(c/\omega)\ E_0\ (\cos\theta)\ (d/dr)[(c/\omega r)^{\frac{1}{2}} \cos \omega(t - r/c)] \\ &= E_0\ (\cos\theta)\ (c/\omega r)^{\frac{1}{2}}\ [\sin \omega(t - r/c)\ -\ \tfrac{1}{2}(c/\omega r) \cos \omega(t - r/c)]\ . \end{aligned} \qquad (14)$$

Again for $r > \lambda_0$ the second term is small and may be neglected. Thus we obtain for the radiation field of a single slit in a conducting plane:

$$E(r,\theta,t) = E_0(\cos\theta)(c/\omega r)^{\frac{1}{2}} \cos \omega(t - r/c) \qquad (15)$$

where the angle θ is as shown in Fig. 3. This radiation pattern may be generated by a pair of out of phase currents displaced from each other along the z direction as shown in Fig. 6.

As an experimental study we will wish to measure the detected power at a constant distance r from a single slit with microwaves incident from the left as shown in Fig. 7. We will expect that the received power, which is

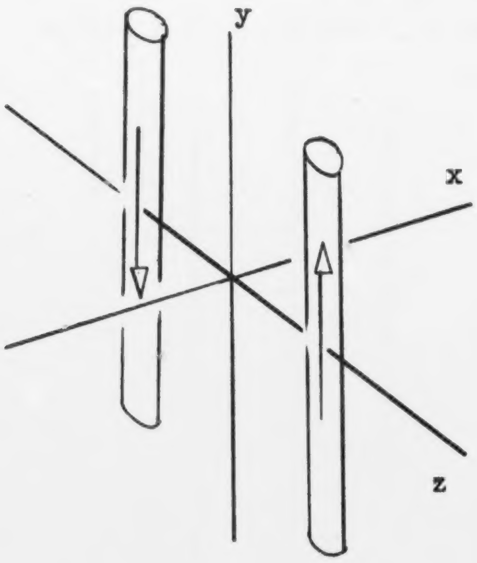


Fig. 6.

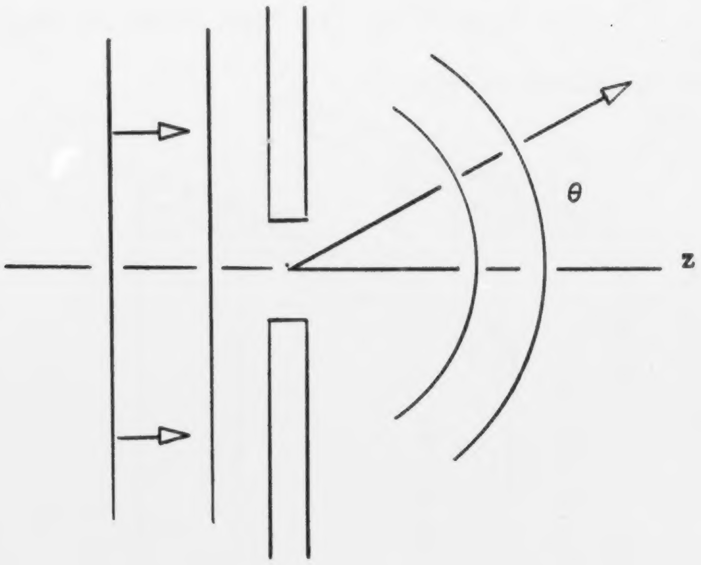
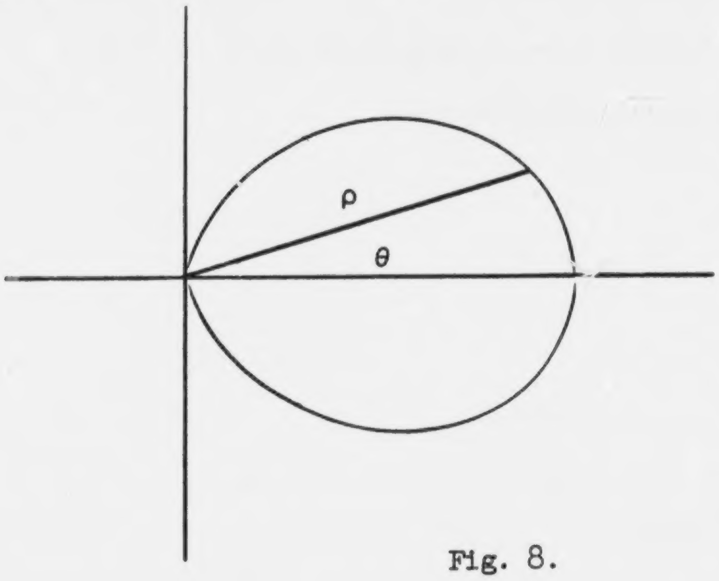


Fig. 7.

proportional to $\overline{E^2}$ should vary as $(\cos^2 \theta)/r$.

The radiation pattern from an antenna is conventionally shown in terms of a polar plot with the radius ρ proportional to the received power. The radiation pattern of a single slit is shown plotted in this way in Fig. 8.



$\rho = \cos^2 \theta$

Fig. 8.

This pattern has a single lobe directed along the normal to the plane. We have assumed in writing Eq. (15) a slit that is narrow compared with λ_o.

For a slit comparable to λ_o or larger than λ_o we must consider a composite of line sources of the form shown in Fig. 8 so as to correspond to a wide slit as shown in Fig. 9.

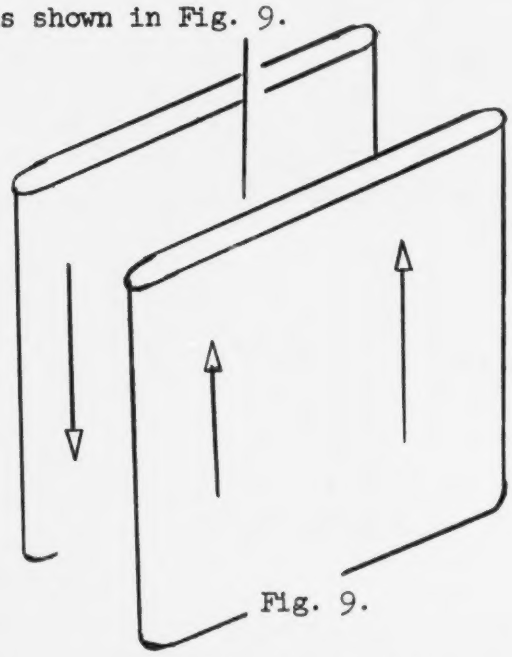


Fig. 9.

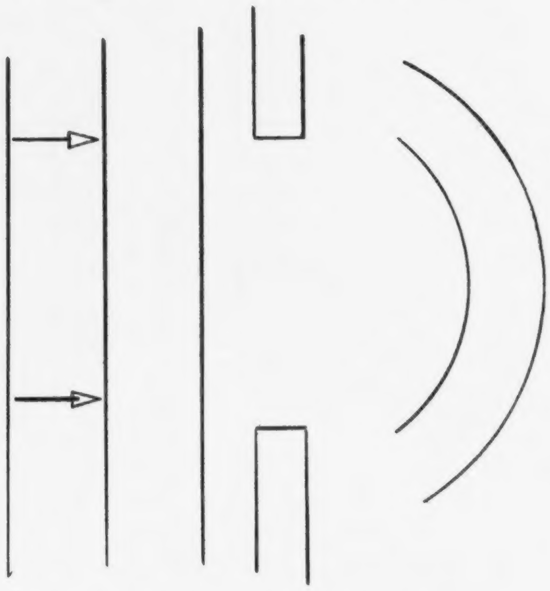


Fig. 10

What is the radiation field from a wide slit? We can compute the pattern by integrating Eq. (15). We will not carry out the mathematics here although the procedure is reasonably straight forward. What we would find is that as the slit opens, the forward lobe becomes more pronounced. For slits wider that a wavelength a pair of secondary lobes appear as shown in Fig. 11. As the slit becomes still wider additional lobes develop.

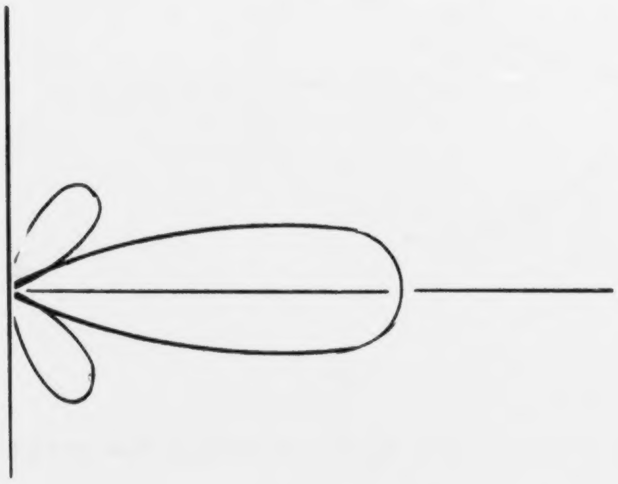


Fig. 11.

To study the lobe pattern of a wide slit with a horn one must make r sufficiently large that the angle subtended by the horn is smaller than the angular width of the lobe.

We will examine next the radiation pattern expected from a pair of narrow slits of variable separation as shown in Fig. 12.

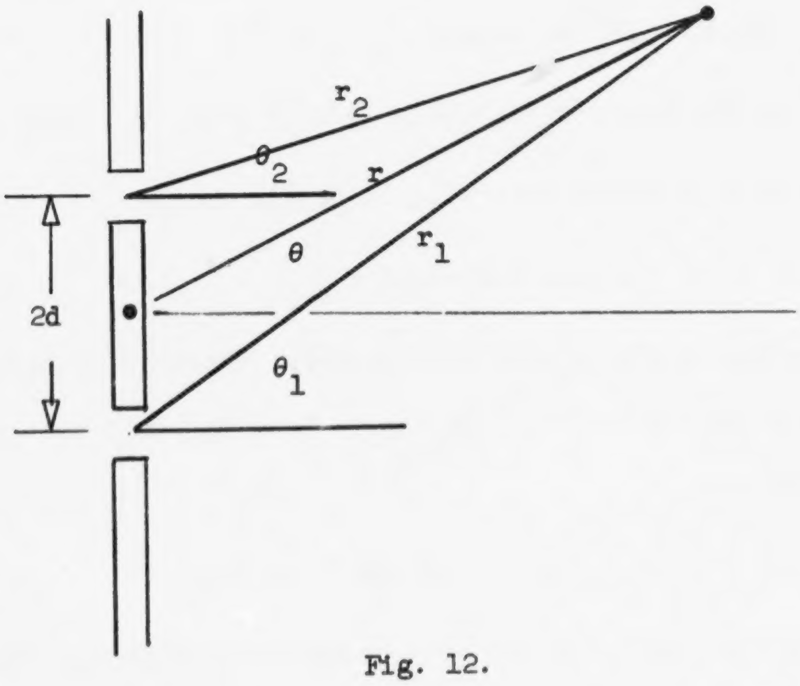


Fig. 12.

From Fig. 12 for r large compared with d we have the following approximate relations:

$$r_2 \approx r - d \sin \theta \qquad\qquad r_1 \approx r + d \sin \theta \ . \qquad (16)$$

We wish to compute a total $E(r,\theta,t)$ which is the sum of the electric fields from the separate slits:

$$E(r,\theta,t) = E_1(r_1,\theta_1,t) + E_2(r_2,\theta_2, t) \qquad (17)$$

As long as d is small compared with r the dominant effect of the displacement of the two slits comes in the phase of the waves. Substituting Eq. (15) into Eq. (17) we obtain for the net field:

$$E(r,\theta,t) \approx E_o(\cos \theta)(c/\omega r)^{\frac{1}{2}} [\cos \omega(t - r_1/c) + \cos \omega(t - r_2/c)] \ . \quad (18)$$

From the trigonometric relation:

$$\cos (A + B) + \cos (A - B) = 2 \cos A \cos B \qquad (19)$$

substituting from Eq. (16) we obtain:

$$\cos \omega(t - r_1/c) + \cos \omega(t - r_2/c) = 2[\cos(\omega d \sin \theta/c)] \cos \omega(t - r/c) . \qquad (20)$$

Finally substituting into Eq. (18) we obtain

$$E(r,\theta,t) = 2E_o (\cos \theta) [\cos (\omega d \sin \theta/c)] (c/\omega r)^{\frac{1}{2}} \cos \omega(t - r/c) . \qquad (21)$$

The radiation pattern from a double slit will be of the form:

$$\rho \approx (\cos^2 \theta) [\cos^2(\omega d \sin \theta/c)] . \qquad (22)$$

The first term in Eq. (22) is the single slit pattern, which is multiplied by the interference between the two slits. We expect from Eq. (22) that there will be secondary lobes for:

$$\cos (\omega d \sin \theta/c) = \pm 1 \qquad \text{or} \qquad 2d \sin \theta = n\lambda_o . \qquad (2*)$$

This is simply the angle at which the two waves interfere constructively as shown in Fig. 13. For example for $2d = (2)^{\frac{1}{2}} \lambda_o$ we obtain on substitution:

$$\sin \theta = (2)^{-\frac{1}{2}} n . \qquad (24)$$

Since $\sin \theta$ must be smaller than one, there are three allowed values of n:

$$n = 0, \quad +1, \quad -1 .$$

The first is the central lobe and the next two are the side lobes at $+\pi/4$ and $-\pi/4$.

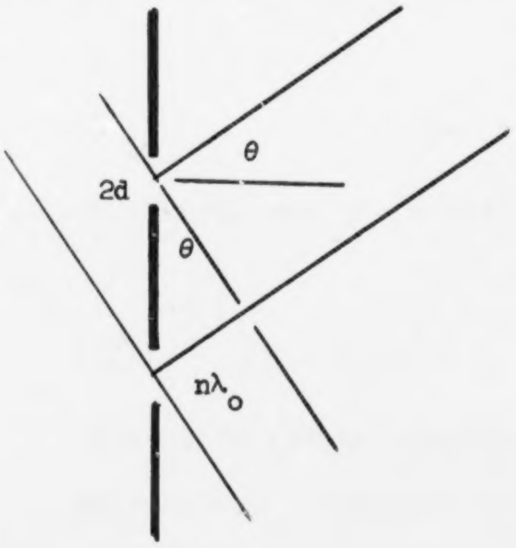


Fig. 13.

In this experiment we will measure the radiation pattern for a single slit and for a pair of narrow slits separated by a distance 2d which is larger than a wavelength. A suggested experimental arrangement for the pattern of a single slit is shown in Fig. 14. The baffles are designed to concentrate the incident radiation onto the slit.

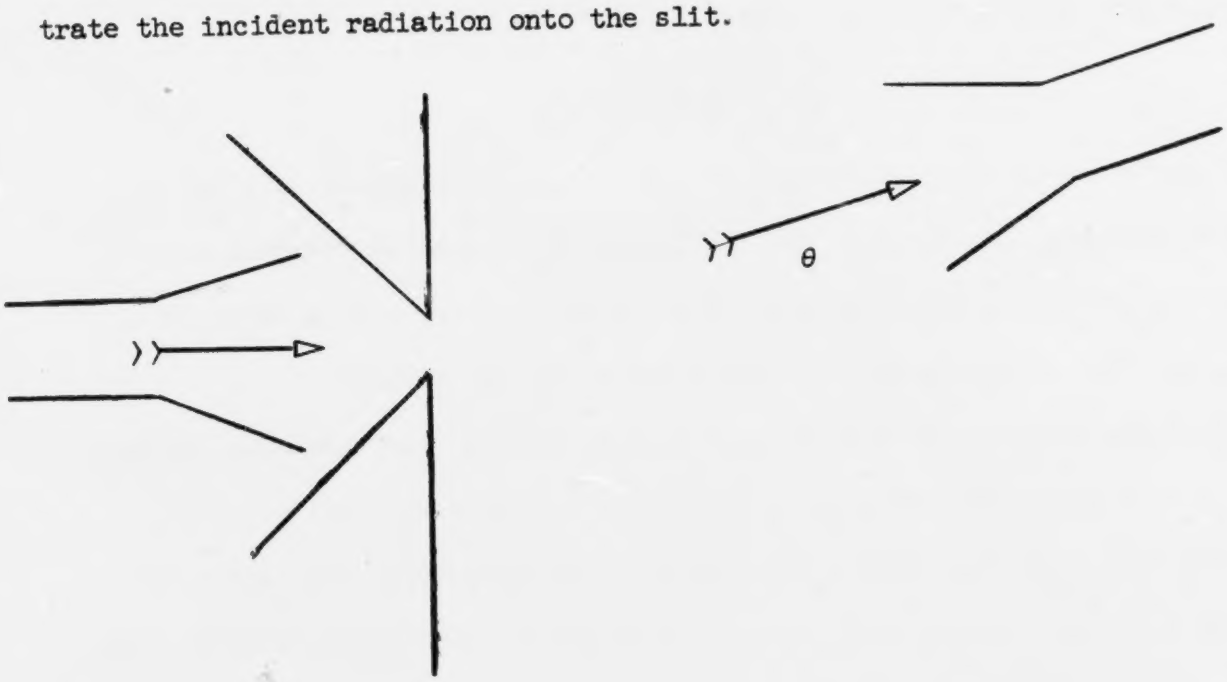


Fig. 14.

A suggested experimental arrangement for a pair of slits is shown in Fig. 15. Here the baffles are designed to split the microwave radiation and concentrate it on the two slits.

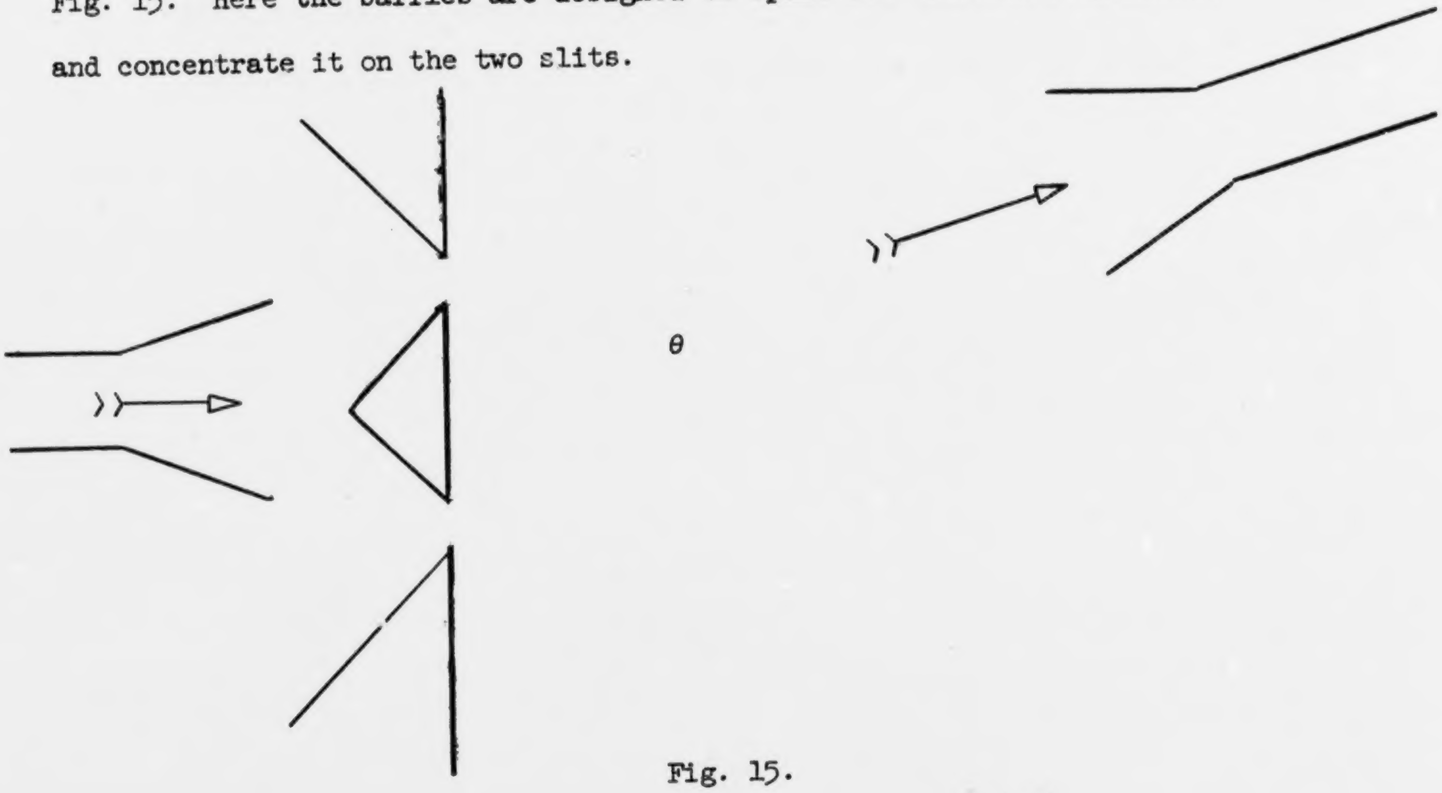


Fig. 15.

Measure and plot the detected power from the single and double slits as a function of θ at constant r. Made a plot of the radiation patterns. Compare the single slit pattern with the theoretical pattern:

$$\rho = \cos^2 \theta \ .$$

From the positions of the minor lobes of the double slit pattern and the measured separation 2d compute the wavelength λ_o. Make an independent determination of wavelength from the cut-off condition for guided waves as shown in Fig. 13 of Experiment B-10 and compare the two values.

If you have sufficient time you may wish to examine the radiation pattern for more widely separated slits or possibly for a single wide slit.

If the two slits were driven by separate klystrons would you expect to observe an interference pattern? What kind of pattern would you expect? What does the observation of interference tell you about the electric fields at the two slits?

If you were to interchange the receiving and the transmitting horns -- that is, to place the slits in front of the receiver and rotate the transmitter -- would you expect to observe an interference pattern? Explain.

Atomic Physics

THE MILLIKAN OIL DROP EXPERIMENT

California Institute of Technology

PRINCIPLES*: A small droplet having a mass of 3×10^{-15} kg., when carrying an excess charge of 1 electron, can be suspended against gravity in a vertical field of about 1.8×10^{5} Vm^{-1} , as may easily be calculated. If the field is between parallel plates with a spacing of 0.5 cm, the applied potential would be 900 volts. Hence the charge of the electron could thus be determined if the mass of the droplet were known.

The mass of the droplet is known if its density and radius are known. The rate of fall of a small drop in a viscous fluid follows Stokes' "Law", which states that the viscous force F acting on the drop is given by $F = 6\pi \eta a v_1$, where η is the coefficient of viscosity of the fluid, "a" the radius of the drop and v_1 its velocity**. The force due to gravity acting on the drop is $F = 4/3 \pi a^3 g(\rho - \rho')$, where ρ is the density of the oil of which the drop is composed and ρ is the density of the fluid (air). These two forces are equal when

$$v_1 = \frac{2}{9} \frac{ga^2}{\eta} (\rho - \rho'), \tag{1}$$

Since η , v_1, g, ρ and ρ' can be measured, the radius, a, of the droplet may thus be found and its mass calculated. This, then, is the fundamental principle behind Millikan's Oil Drop Experiment.

In practice it is found expedient to apply an excess electric field and actually cause the particle to move upward. Just as the viscous drag limits the velocity of fall, so also it limits the velocity of rise. Since the velocities are proportional to the forces, we have

$$\frac{v_1}{v_2} = \frac{mg}{\mathcal{E}q - mg}, \quad q = \frac{mg}{\mathcal{E}} \frac{v_1 + v_2}{v_1} \tag{2}$$

Where the magnitudes of the quantities only are considered. In these expressions v_1 is the downward velocity, v_2 the upward velocity, $\mathcal{E}$ the electric field, and q the charge on the droplet. This charge is always found to be an integral number of elementary charges, the charge on the electron. Thus, $q = e_1 n$.

Millikan found, however, that when accurate measurements were made, the charge e_1 found by using Eqs. (1) and (2) depended on the size of the drop, being larger for the smaller drops. He soon realized that the trouble was with Stokes' law of fall. When the size of the drops become comparable with the distances between molecules of the air, they tend to fall between the molecules and hence fall faster than in an ideal continuous fluid. Millikan reasoned that since the effect was small, a linear correction to the velocity was probably sufficient. Such a linear correction applied to Eq. (1) would be

---

*The student should read at least some of the following pages of Millikan's book, "Electrons + and - etc.", Univ. of Chicago Press, 1947, pp. 54 to 124, for a more complete description. Millikan was one of the great experimental physicists of modern times. His description of how he stumbled onto the single drop method together with the experimental results achieved is an excellent example of how an experimentalist works.

**This "law" is actually not a fundamental law like Newton's laws, but is only a name given to the relationship between retarding force and droplet radius. The relationship can be derived by considering the flow of the viscous fluid around the drop.

$$v_1 = \frac{2}{9}\,\frac{ga^2}{\eta}\,(\rho - \rho')(1 + \frac{b}{pa})$$

where p is the pressure of the air, "a" the radius of the drop and b a constant $\ll$ pa. Hence the correction applied to the calculated mass m, of the drop would be

$$m_o = \frac{m}{(1 + \frac{b}{pa})^{3/2}} .$$

Or, if no corrections for failure of Stokes' law are applied but m is calculated by using Eq. (1), then when the apparent charge of the electron, e_1, is raised to the 2/3 power and is plotted against 1/pa, a straight line should result. The intercept on the $e_1^{2/3}$ axis when 1/pa = 0 gives the correct value, $e^{2/3}$, for the continuous fluid.[1] (See Millikan's book for further details and plots of his data.)

APPARATUS: Oil drop apparatus; light source giving an approximately parallel beam; atomizer with oil; high voltage power supply; voltmeter; one reversing switch; one DPDT switch; stop clock.

PROCEDURES: (1) Carefully disassemble the oil-drop apparatus to determine the plate spacing. The calibration of the scale in the eyepiece may also be determined at this time by using the calibrated screw provided. After reassembling, the telescope may be focused on the region through which the droplets will come by inserting a small wire through the small hole in the piece inset into the top plate, and focusing on the wire.

(2) The procedure for obtaining drops is to spray the inside of a glass bottle with oil from the atomizer, then place this over the piece in the top plate that contains the small hole. Wiggling the bottle will tend to drive the droplets into the space between the plates.

If all adjustments have been made properly the droplets will appear like bright stars on a dark background.

Note the Brownian motion in the clusters of small drops. Note also the diffraction pattern around the drops. Physics is where you find it!

(3) Finding a suitable drop: A drop should contain not more than a few unit charges to arrive at a unique value of the charge on the electron. A suitable drop for this apparatus is one that requires from 10 to 30 seconds to move, by gravity only, across half the field of view and requires a similar time to rise when a potential of 1000 to 2500 volts is applied between the plates. Why do the drops have an electrical charge?

(4) Measurements and Calculations: One should first find the mass of the droplet without correcting for departures from Stokes' law of fall. The "apparent" charge* e_1 should then be found. Now locate your point on the graph in the laboratory which gives Millikan's values of $e_1^{2/3}$ vs 1/pa, as stated under principles (this experiment). You need not rely on this graph if you use two or more drops of different sizes.

Discussion of Errors: In a measurement of this kind, it is important to realize where errors creep in. If only statistical errors are entering, the error in your final value for the charge of the electron should be consistent with the known errors in the individual quantities involved. Is this true?

From the fluctuations in the times of rise and fall of your drop, how many determinations of each would you need to make to reduce the statistical error to 1 percent? to 0.1 percent?

Coefficient of viscosity of air (20°C) = 1.8325 x 10^{-5}newton sec per m^2.

*Charge calculated using mass value uncorrected for departures from Stokes' law.

University of California, Berkeley

References: W.B. Kunkel and J.W. Hansen, Rev. Scientific Instruments, 21, 308-314 (Apr. 1950)

Read this before coming to class.

E.E. Dodd, Ph.D. Thesis QC 3.6 D 639

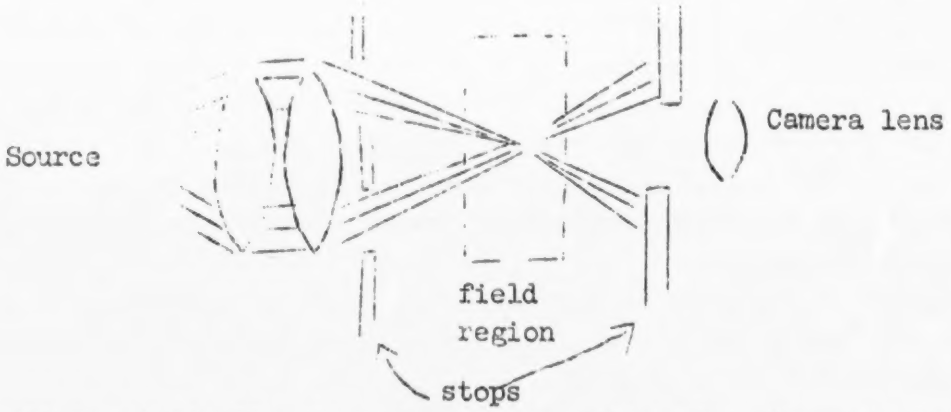
V.D. Hopper and T.H. Laby, "The electron charge," Proc. Roy. Soc. A178 243 (July 1941)

E.Cunningham, Proc. Roy. Soc. (London)A83 357 (1910)(Main Library)

Apparatus: See reference Kunkel and Hansen.

Purpose: To measure the charge of the electron.

Method: Paths of charged oil drops are observed by photographing them with dark field illumination. Oil drops are sprayed into the top of the settling column and allowed to settle between two electric field plates which produce a field in the horizontal direction. The drops are illuminated by a system of lenses which prevents all light but that scattered from the drops from entering the camera lens.



The oil drops will be statically charged upon separation from the main body of oil. Charging is of the nature of a statistical variation and charges of both signs will be present equally.

Particles of the size observed here approach terminal velocity in less than a milli second. Diameter and charge of particles are evaluated through direct application of Stokes's Law from measurement of vertical and horizontal components of velocity.

Stokes's Law $F = 3\pi \eta v d$

where η is the coefficient of viscosity, v is velocity, and d diameter. From this we can find the diameter by

$$d = \left[\frac{18 \eta v_y}{(\rho_1 - \rho_2) g}\right]^{1/2}$$

where v_y is the vertical component of the velocity, ρ_1, the density of the drops, and ρ_2 the density of the supporting medium.

The charge can be found by

$$q_{(e.s.u.)} = \frac{3\pi\eta v_x d}{E_{(e.s.u.)}} = \frac{900\pi v_x \eta d \ell}{V_{(volts)}}$$

and the preceding formula. Here v_x is the horizontal component of velocity, ℓ is the separation of the field plates, and V the potential measured in volts.
In the present apparatus v_x and v_y are determined by

$$v_x = \frac{\nu X}{MN} \qquad v_y = \frac{\nu Y}{MN}$$

where ν is the number of flashes per second, M the magnification, N the number of intervals measured, X and Y the horizontal and vertical measurements of the magnified traces.
The separation of the field plates, ℓ, is maintained by quartz spacers and was measured as 7.742 ± .004 mm at 23.4° C.
The chopper is run by a synchronous motor at 450 rpm.
ρ_1 , the density of mineral oil is 0.8805 g/ml.

Procedure:

WARNING - Do not turn on the mercury arc lamp until the cooling water is flowing at the rate of 4 liters per minute. If the lamp does not operate after turning on the required flow of water call the instructor.
Rate of change of the apparatus temperature greatly affects the experimental results. Therefore the experiment should be run as early in the lab. period as possible. Afternoon temperatures are usually rising rapidly. The insulation cannot prevent internal temperature change, but only causes a time lag and damping of variations.
Good conditions require an apparatus temperature constant to 0.01°C. Conditions in the lab. may cause a temperature rise of 0.05°C between 1:00 and 2:00 p.m. and more than double this later on.
Therefore students should completely familiarize themselves with the apparatus and the preliminary darkroom procedure before the lab. period. For this reason you will not expect to get the accepted value of e, but rather a value consistent with the amount of temperature rise.
Begin by loading the magazine with 2 feet of "Tri-X" film in total darkness.
Mount the camera, open four slots in the chopper and disconnect the field source.
Turn on and check the required water flow of 4 liters per minute.
Check to see that the shutter is in place and start the arc. Start the chopper.
Carefully take the instulation from the top of the settling column, and with the atomizer spray about 10 strong, fast squirts of oil drops into the column. Replace everything and take a 2 sec exposure immediately to determine direction of gravity.
Do not let light into the analyzing chamber any longer than necessary for taking the picture. Record the time, room temperature, and apparatus temperature and keep a record of these throughout the entire experiment to be handed in with your report.
Record the potential of the field source.
Connect the field source and take the following expqsures remembering to reverse the switch between frames.

After the 2nd frame you should have time to slide the focaslide up to observe the drops on the ground glass plate. Throw the shutter no more than 5 sec for each partner. You should be able to see 3 to 6 drops crossing the field all the time. Don't forget to replace the camera.

Elapsed time	Frame	Exposure
1/2 min	1	2 sec -for vertical
connect field		
2	2	2 sec
4	3	2 sec
6	4	3 sec
close chopper to one opening		
9	5	3 sec
12	6	4 sec
15	7	4 sec
18	8	5 sec
21	9	6 sec
24	10^*	7 sec
27	11	8 sec

Continue with increasing exposures every three minutes to end of film.* If you would like to check the direction of the field lines for frame 10 throw the 4 PDT switch one way for 3 sec and then very quickly the other way for 3 sec. After finishing the photography you may observe the drops as long as you wish.

Turn off the chopper and the arc and after a few minutes the water, and disconnect the field source.

35 mm roll film should be developed in the cylindrical stainless steel can with the spiraled wire film holder.

Practice loading this in total darkness with exposed film. To load this cut the corners off the tail end of the film and insert this tongue in the spring clamp in the center of the spool squeezing the film to make it narrow enough. Then roll the film around the spool, put the spool in the can and cover it. The lights may now be turned on for developing. Beakers in the darkroom should be marked with tape at the right level to just cover film in this can.

The magnification M has been determined by photographing a standard reticle placed in the focal plane of the optical system. This film is available and should be placed in the negative holder of the enlarger together with a track film. Make a 8 x 10 enlargment of both films simultaneously.

Measure traces in good focus, making an angle of 20° - 70° , having a length of 20 dots or more for best accuracy. 20 dot traces with Y less than 1.3 cm long should be discounted as too small ($M \approx 50$). 30 to 50 traces should give you a fair statistical value for e. To speed calculations, it is suggested that you carry through the formulas with say N = 10. Then plug in $X\,Y^{1/2}$ equivalent to 10 intervals. Scratch the frame numbers on the shiny side of the film.

Plot intervals of charge (in e.s.u.) versus number of particles falling within each interval. Mark the squares with the diameter of the drop and hand this in with your report. From any groupings which may occur pick a value for the unit charge which will give you the minimum average deviation.
Plot percent deviation from your value of e versus drop diameter and hand this in with your report. Here you will see the effect of the small temperature change.
Hand in the film with your report.

1. Derive the formulas for d and q from Stokes's Law.

2. What is the upper limit of drop diameters which can be calculated by Stokes's Law?

3. Why is it unnecessary to apply the Cunningham correction?

4. Plot temperature vs time on one graph showing points of beginning and ending frames.

5. Plot distribution of charges measured in e.s.u.

6. Plot percent deviation vs. diameter. Explain the cause of the trend of the deviation in view of the temperature change.

7. If in frame 10 you measured the angle the field makes with the vertical, include the values and the average value. If not, tell how one would determine this angle.

8. Estimate how much your potential measuring device may have changed the potential and why.

CHARGE-MASS RATIO OF ELECTRONS

Oberlin College

While the idea of electrons as small negatively charged particles is commonplace today, it is quite a recent idea, only a little more than sixty years old. In 1897 J.J. Thomson conducted some investigations on what were then called cathode rays; the nature of these rays was unknown, and Thomson adopted purely as a working hypothesis the assumption that the rays were streams of charged particles. His experiments consisted of allowing the particles to be acted upon by electric and magnetic fields and observing the subsequent motions of the particles. The simple manner in which the particle hypothesis allowed the results of Thomson's experiments to be interpreted was one of the first pieces of evidence that gave support to the idea of small negatively charged particles as constituents of cathode rays; later the term electron was applied to these small particles. In this experiment you will perform observations very similar to those of Thomson, but the details of the apparatus that you will use are different from the details of Thomson's original apparatus. One piece of apparatus you will employ was designed by the German investigator, Lenard; another piece by the German investigator, Classen. As you will see, the principles involved in the operation of both of these pieces of apparatus are exactly the same; it is only the details of making the observations that are different. The result of your investigations will be an experimentally determined value for the ratio of charge to mass for these small negatively charged particles called electrons.

THEORY

The basic technique of this experiment is the sending of a stream of fast moving electrons through a magnetic field in such a manner that the velocity of the stream is at right angles to the direction of the magnetic field. The resulting force which acts on the moving charged particles is directed at right angles to both the velocity and the magnetic field; since it is always directed at right angles to the velocity, it acts as a centripetal force causing the particles to move in a circle without changing the speed of the particles. If we apply Newton's second law to the motion of an individual particle and use the usual expression for the magnetic force, we have

$$f = ma$$
$$Bev = m\frac{v^2}{R} .$$

This equation can be rewritten as

$$\frac{e}{m} = \frac{v}{BR} .$$

In these equations e = the charge on the electron, m = the mass of the electron, v = speed of the electron, B = the magnetic induction of the magnetic field, R = the radius of the circle in which the electron moves in the magnetic field. It is clear from an examination of this expression how the charge-mass ratio may be measured. For if the magnetic field is produced by an electromagnet, B can be determined from the geometry of the apparatus and the current in the magnet coils. R and v are quantities that depend upon the motion of the particles, so that if the motion can be observed these quantities can be measured.

In the apparatus of Lenard and that of Classen the electrons are obtained by heating a filament of metal to incandescence, so that electrons are given off. These electrons are then brought to a high speed by placing near the filament a positively charged metal electrode so that there is a strong electric field between the filament and the electrode. By arranging a suitable hole in the metal electrode some electrons can be accelerated in the electric field but still not be captured by the electrode, since some of them will pass through the hole in the electrode itself. If we call the potential difference between filament and accelerating electrode V, then the amount of work done on the electrons in accelerating them is eV, and this is equal to the gain in kinetic energy of the electron during the acceleration; if the final speed of the electron is made very large, the initial kinetic energy may be negligible in comparison with the final kinetic energy so that we can write

$$eV = \tfrac{1}{2}mv^2$$

or

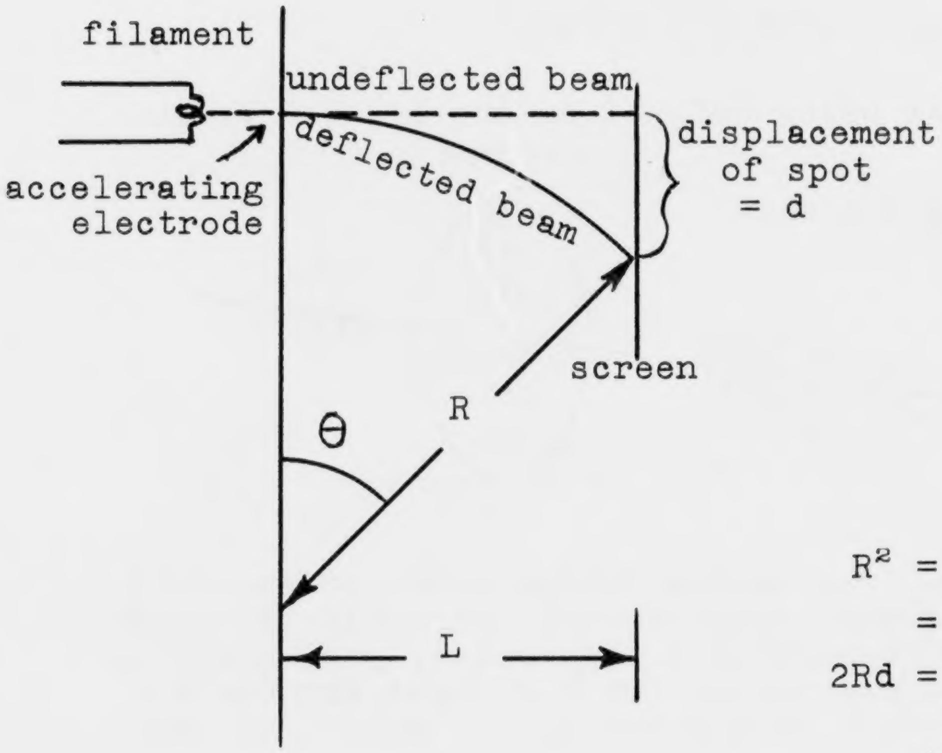
$$v = \sqrt{\frac{2eV}{m}}$$

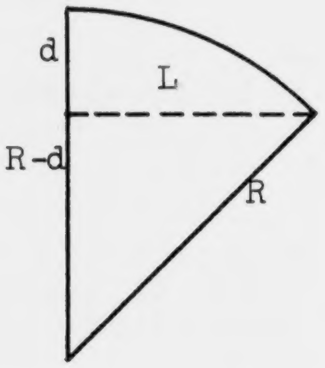
If we put this expression for v back in the other e/m expressions, we have

$$\frac{e}{m} = \frac{2V}{B^2R^2} .$$

The path followed by the electrons can be made visible because the electron stream will cause flashes of light if it strikes a suitable phosphor. In the apparatus of Lenard the filament and the accelerating electrodes are located at the end of a rather long glass envelope. The opposite end of the envelope is coated on the inside with a phosphor so that a spot of light is produced where the electron beam strikes this "screen." R is then determined by first allowing the beam to pass through the tube with the magnetic field turned off. The magnetic field is then turned on and the distance the beam is displaced is observed by noticing the displacement of the spot of light on the phosphor. The diagram will make this clearer. In the apparatus of Classen the electrons after passing through the hole in the accelerating electrode are bent through 180° by the action of the magnetic field and so fall back onto the accelerating electrode. The back surface of the electrode has some phosphorescent material on it so that the spot of light is clearly visible, and the diameter of the circle can be directly measured by making use of lines ruled directly onto the metal electrode.

Lenard Apparatus


filament
undeflected beam
deflected beam
displacement of spot = d
accelerating electrode
screen
R
θ
L



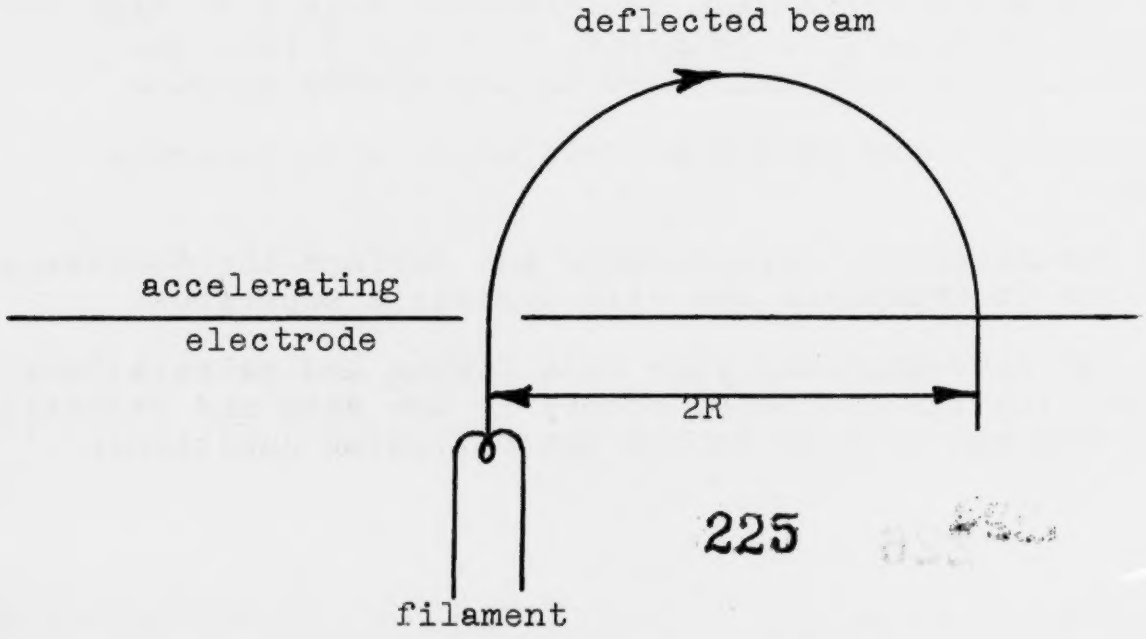
$$R^2 = L^2 + (R - d)^2$$

$$= L^2 + R^2 - 2Rd + d^2$$

$$2Rd = L^2 + d^2 \approx L^2 \text{ if } d \ll L$$

$$\therefore R = \frac{L^2}{2d}$$

Classen Apparatus


deflected beam
accelerating
electrode
2R
filament

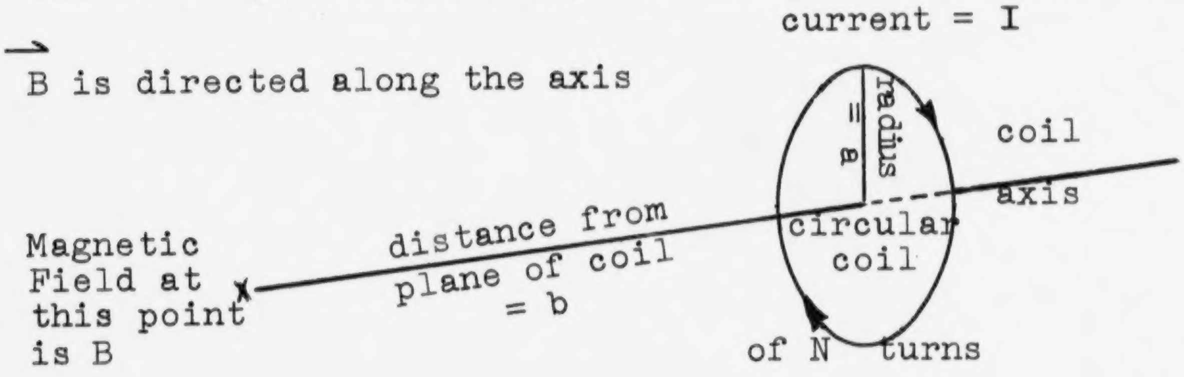
In both the apparatus of Lenard and that of Classen the magnetic field is produced by current carrying coils. The magnitude of the magnetic induction on the axis of a current carrying coil is given by

$$B = \frac{\mu_o N I a^2}{2(a^2 + b^2)^{3/2}} ,$$

where $\mu_o = 4\pi \times 10^{-7}$ nt/amp^2.

The other quantities being defined on the diagram below.

$\vec{B}$ is directed along the axis


current = I
radius = a
coil
axis
circular coil
of N turns
distance from plane of coil = b
Magnetic Field at this point is B

If the magnetic field is needed in the plane of the coil (where b = 0) or if more than one coil produces the total magnetic field in the particular apparatus, you should be able to employ this expression for B without any difficulty once you are able to see the actual apparatus you will use.

Procedure

1. Become familiar with the apparatus, making sure that you know how all the quantities of interest in the experiment can be measured with the instruments available.

2. Make out data sheets for the determination of e/m using Lenard's apparatus and using Classen's apparatus.

3. With the assistant's help use Lenard's method to take the measurements necessary to calculate P, V and R from the appropriate expressions mentioned in the theory section.

4. Calculate a value of e/m as determined with Lenard's apparatus.

5. Take the necessary measurements and perform the necessary calculations to determine e/m with Classen's apparatus.

6. When you have concluded your data taking and calculations (not forgetting the estimated errors in the data and results) answer at the end of your report the following questions:

6. continued-

   a. Compare the advantages and disadvantages of Lenard's and Classen's methods for the determination of e/m.

   b. How could the precision of the results you have obtained by these methods be improved?

   c. What justification is there for neglecting the kinetic energy of the electrons as they leave the hot filament?

   d. Calculate the speed of the electrons after they have passed through the accelerating field in both sets of apparatus you have used. Express the results in mi/hr.

   e. Calculate the energy gained by an electron in accelerating from rest to the speed calculated in 4. Compare this amount of energy with that gained by a 1 kg body when it is lifted 1 m from the floor to a table top. Does this calculation give you any hint of the mass of the electron as compared with the masses of ordinary objects?

   f. The oil drop experiment gives a value 1.60×10^{-19} coulcomb for the charge on the electron. Using this value and your values of e/m calculate a value of the mass of the electron.

MEASUREMENT OF e/m FOR THE ELECTRON

Dartmouth College

Introduction

One of the important properties of a charged particle is the ratio of its charge to its mass. This can be determined by measuring the deflection of a particle of known energy in a magnetic field.[1] The ratio of e/m can then be computed from the following equation:

$$e/m = 2V/B^2R^2, \tag{1}$$

where V is the potential through which the particle falls, B the magnetic field, and R the radius of curvature. In this experiment, this ratio will be determined for the electron.

Apparatus

The equipment has been designed after the experiment of Bainbridge.[2] The apparatus consists of a large vacuum tube supported at the center of a pair of "Helmholtz coils." An electron gun, composed of a filament parallel to the axis of the coils and surrounded by a coaxial anode containing a single slit, produces a narrow beam of electrons whose paths are rendered visible by a trace of mercury vapor in the tube.

The particular arrangement of two co-axial, circular coils of radius "a" with their planes separated by a distance "a" is known as a pair of Helmholtz Coils. This particular coil arrangement is useful because it gives an almost uniform field over a fairly large region. The magnetic field at the center is parallel to the axis of the coils and its magnitude is given by the equation below.

$$B = \frac{32\pi N I}{\sqrt{125}\, a} \qquad B = \frac{8\mu_0 N I}{\sqrt{125}\, a} \tag{2}$$

Electromagnetic System Unrationalized — MKS System Rationalized

N is the number of turns in each coil, I the current, a the coil radius, and μ_0 the permeability of free space. The above equations hold for the region directly between the coils and under the condition that the thickness of the windings is negligible compared to a. The derivation of the above equations is not difficult if one begins with the expression for the magnetic field on the axis of a circular loop of wire.[3] The field off the axis is considered in texts on electricity. The effect of the finite thickness of the windings is discussed in the article by Bainbridge.

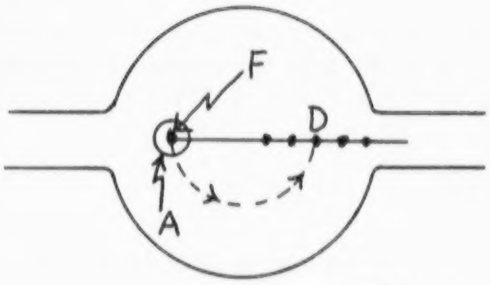
When current flows through the coils, the resulting magnetic field causes the electron beam in the tube to move in a circular path whose radius decreases as the magnetic field increases. The magnetic field can be adjusted until the sharp edge of the beam coincides with the edge of one of the five posts spaced at different distances from the filament.

---

[1] Sears and Zemansky, University Physics (Addison-Wesley, 1955) Chap. 31

[2] K. T. Bainbridge, The American Physics Teacher, 6, 35 (1938)

[3] Sears and Zemansky, op. cit., Ch. 33.



F filament
A anode
D one of five posts

B is perpendicular to paper

Fig. 1

For several reasons, the electron beam spreads as it goes around the tube, but fortunately, because all of this spreading is toward the inside of the circle, none of it affects the measurement of e/m if the outside edge of the beam is used. The effects which cause the spreading are discussed below:

a. Not all the electrons travel perpendicular to B. An electron traveling at an angle θ to the direction of the field will have a component of its velocity parallel to B equal to v cos θ. This component is unaffected by the magnetic field. The radius of curvature for electrons moving at an angle θ with respect to the magnetic field is

$$R = \frac{m}{e}\,\frac{v\ \sin\theta}{B} \qquad (3)$$

and is therefore a maximum when θ is 90°. Hence the electrons at the outside edge of the beam are moving perpendicular to the field as assumed in equation (1).

b. Those electrons leaving the filament perpendicular to line FD which joins the filament and the post in the figure above will be bent into a circle whose center lies on the line FD. Other electrons leaving at an angle to the perpendicular will move in a circle of the same radius, but with center not on the line FD. One can show that electrons leaving at any angle, α, positive or negative, will cross FD slightly inside those which left at $\alpha = 0$, i. e., perpendicular to FD.

c. Those electrons which leave the negative end of the filament fall through the greatest potential difference between filament and anode and thus have the greatest velocity. The accelerating voltage is referred to the negative end of the filament.

d. The electrons collide with mercury atoms in the tube thus making their path visible. Such a collision is inelastic and results in a decrease in electron energy and thus a spreading of the beam toward the inside.

In summary, the electrons which are found at the outside edge of the beam lost no energy by ionization, were emitted from the negative end of the filament, departed perpendicular to the line joining filament and posts, and have no component of velocity parallel to the magnetic field.

A block diagram of the apparatus is shown in Fig. 2. The accelerating voltage V is provided by a variable B+ power supply. The filament current is taken from the current supply (green plugs). It is half-wave rectified, and is controlled by a variable transformer control on the front of the cabinet. The polarity can be reversed by a switch on the front of the cabinet. The negative terminal is available externally (black plug) for either polarity. The current for the Helmholtz coils is obtained from a separate circuit in the same cabinet (white plug). This is full-wave rectified and smoothed. It is controlled by a separate external Variac. A 0-5 amp meter on the cabinet can be connected in either the filament or the Helmholtz coil circuit by means of a switch.

The number of turns N in each of the Helmholtz coils is 72.

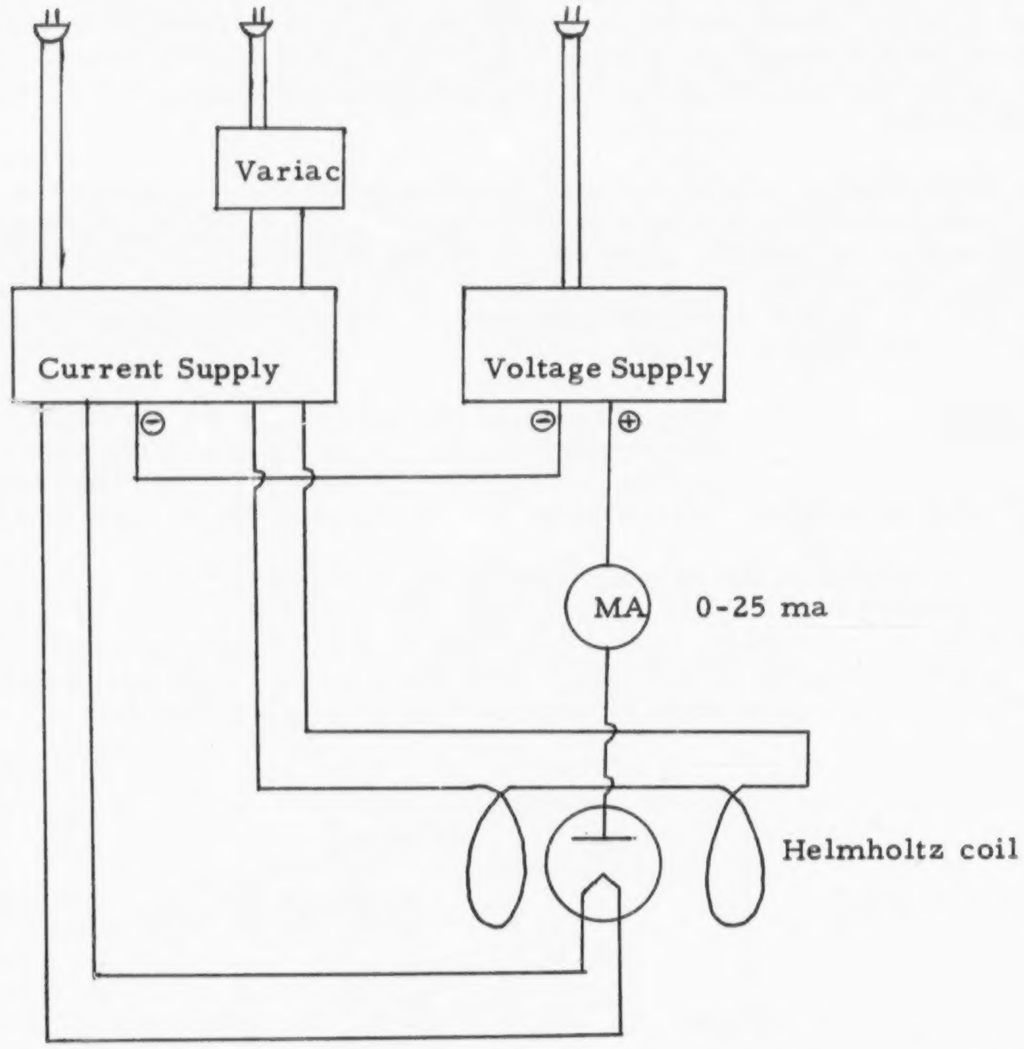


Fig. 2

Procedure

The electron beam is produced by setting the anode potential at about 50 v and then slowly increasing the filament current until the anode current is about 6 ma. CONSULT THE INSTRUCTOR BEFORE TURNING ON THE FILAMENT SUPPLY.

CAUTION: Never run the filament current higher than necessary. You may burn out the filament, which is very expensive.

Having set the proper filament current, the 0-5 amp ammeter may be switched into the field circuit.

The magnetic field of the earth is not negligible in comparison to the field of the Helmholtz coils. Therefore the axis of the coils must be set parallel to earth's field, and its magnitude must be measured and included in the computation. (i). The direction of the earth's field is measured with a special compass. Then the support brackets of the coils are adjusted so their axis is parallel to it. (ii). In the absence of current in the coils, the electron path is not perfectly straight, because of the earth's field. The magnitude of the earth's field is found by balancing it out by the field of the coils. That is, measure the applied field which is necessary to make the electron path straight. It is essential to make this measurement, since local fields often differ greatly from the handbook value (look this up for comparison).

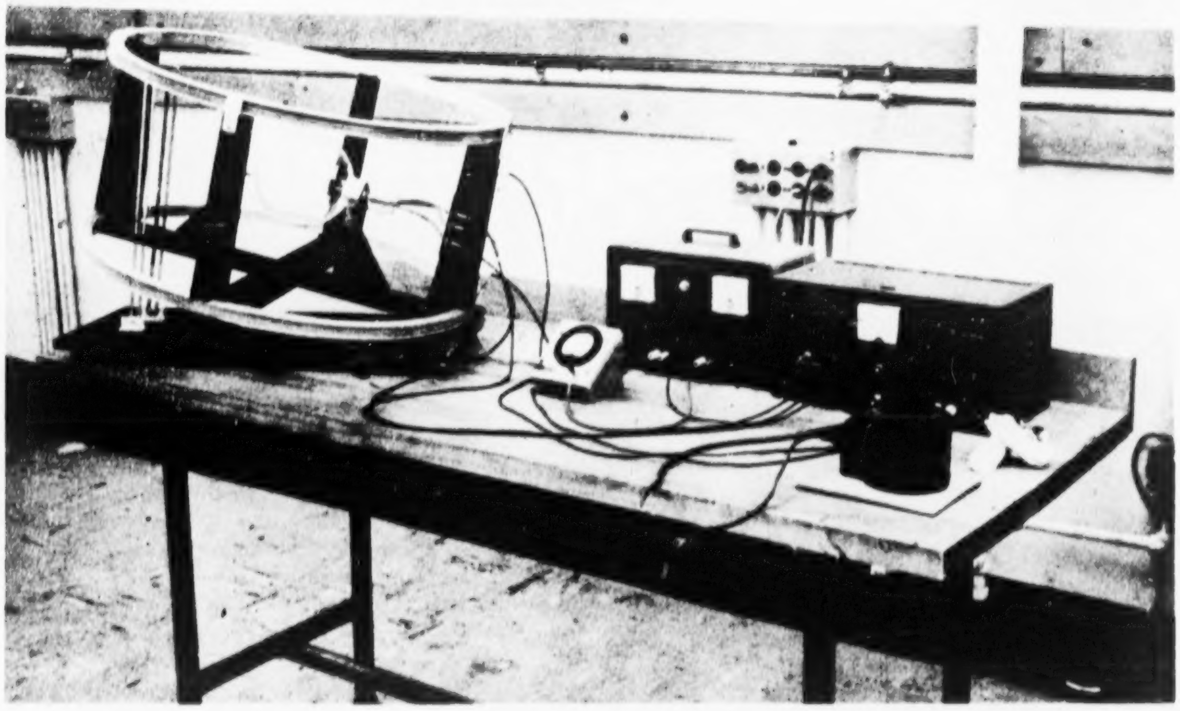
Next send a current through the Helmholtz coils large enough to bend the electron beam in a small circle. Reverse the filament current and notice the change in trajectory. This effect can be minimized by rotating the tube about its axis so that the electron beam will strike the back of the anode cylinder. It is best to rotate it downward, so that light from the filament is not in your eyes.

CAUTION: Always turn off the current in the Helmholtz coils except when actually making a measurement. Do not let the current remain above 2 amp. for more than one minute at a time. Otherwise you may burn out the current supply.

Now to obtain the data necessary for the computation of e/m, choose several different values of voltage between 20 and 100 v, and for each voltage measure the current in the coils which causes the electrons to strike each of the five posts in the tube. The distance in centimeters between the filament and the outer edge of each post is given in Table 1.

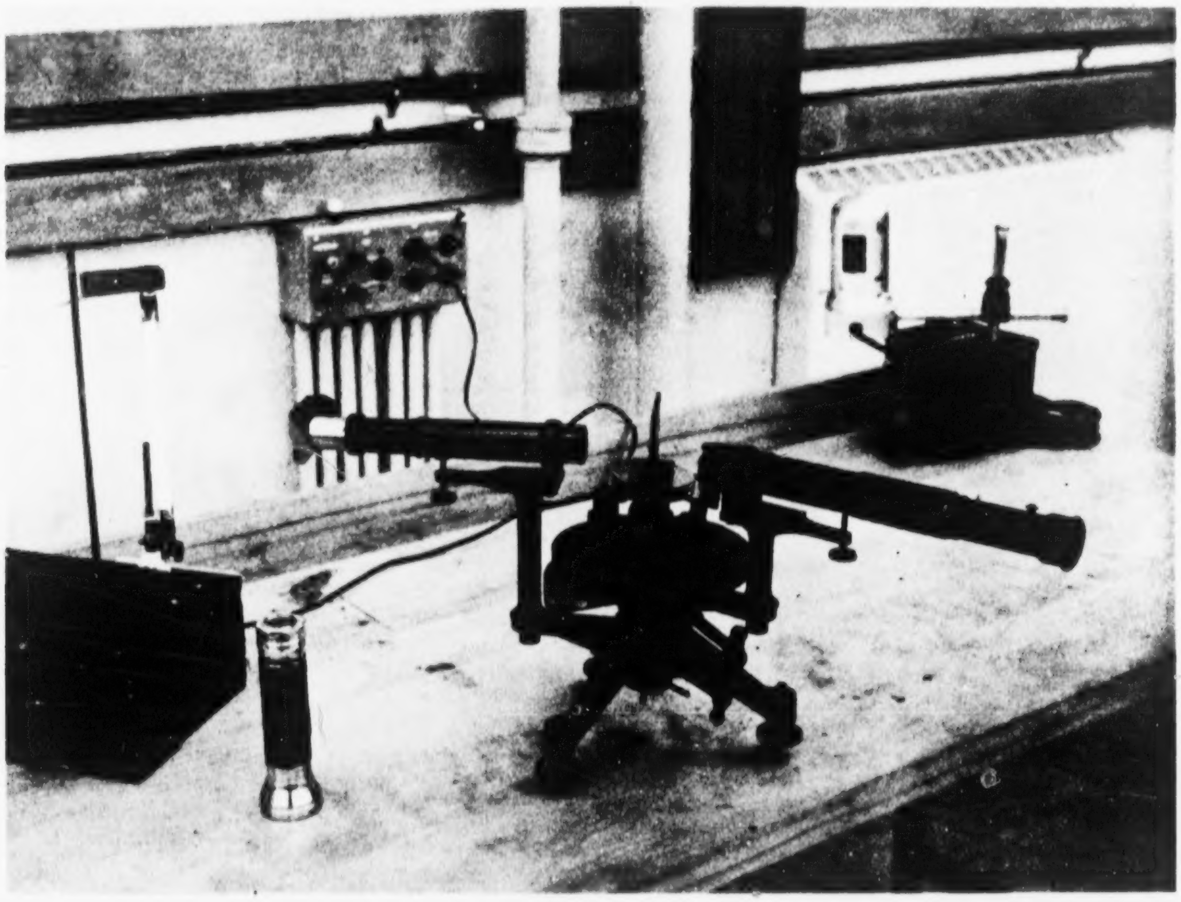
Post	Distance
1	6.48 cm
2	7.75
3	9.02
4	10.30
5	11.54

Table I



e/m of the Electron

Optical Spectrum of Hydrogen



THE ZEEMAN EFFECT (OPTICAL DETERMINATION OF e/m)

Stanford University

References

Harnwell and Livingood, Experimental Atomic Physics (McGraw-Hill Book Co., Inc., New York, 1933), first edition, pp 246 et seq. (Poor).

Lummer and Gehrcke, Ann. Physik 10, 457 (1903) (in German).

Simeon, J. Sci. Instr. 1, 298 (1924).

J. K. Robertson, Introduction to Physical Optics (D. Van Nostrand Co., Inc., 1930).

*W. E. Williams, Applications of Interferometry (London, Methuen and Co., 1950), Section on Lummer plate.

*Harrison, Lord, and Loofbourow, Practical Spectroscopy (Prentice Hall, New York, 1948), pp 547-558.

Handbook of Chemistry and Physics, section on spectroscopic data, wavelengths, etc.

*G. Herzberg, Atomic Spectra and Atomic Structure (Dover Publishing Co., New York, 1944), second edition, pp 74-114.

Pauling and Goudsmit, Structure of Line Spectra (McGraw-Hill Book Co., Inc., New York, 1930), first edition.

H. E. White, Introduction to Atomic Spectra (McGraw-Hill Book Co., Inc., New York, 1934), first edition.

*Recommended

Object

To measure by optical interferometry the small wavelength shifts induced when a source of monochromatic radiation is subjected to a magnetic field. For radiations which show the simplest effects, the shift is given by

$$\Delta\lambda = \frac{He\lambda^2}{4\pi c m_0}, \quad \text{(why?)} \qquad (1)$$

where H is the field strength in gauss, e is the electronic charge in emu, and m_0 is the electron mass. Even for the strongest fields we can produce, this is a small quantity, requiring optical apparatus of high resolving power. We use here a Lummer-Gehrke interferometer coupled to a simple prismatic monochromatizer to select the desired optical line.

If $\Delta\lambda$ and H are measured, it is apparent that this experiment can be regarded as a determination of the ratio e/m.

Procedure:

The interferometer is set up to observe light from the helium discharge tube in a direction perpendicular to the magnetic field. Observe the patterns formed by several of the helium lines (H = 0) noting the separation of the orders and their resolution. Adjust the interferometer for observation of the green line (5015.7 A) which shows the normal Zeeman effect. Turn on the field and observe the effect. Interpose a polarizing agent and observe and record the results.

- The discharge tube should be cooled by a blower at all times. Operate the tube at the lowest usable intensity. At the higher magnetic fields where it may be necessary to increase the tube current, do not operate continuously.

Report

1. Describe your method of magnetic field measurement and construct a plot of H vs magnet current.

2. For the calculation of $\Delta\lambda$ you will need information about the dispersion of the Lummer plate, and the following data (from spectrometer measurements on the plate) may be used:

λ	n
4060 A	1.5370
4358 A	1.5330
4916 A	1.5275
5461 A	1.5235
5780 A	1.5223
5893 A	1.5220

   Plot a dispersion curve from these data and determine $\partial\mu/\partial\lambda$ at the necessary point, either graphically or numerically. The thickness of the plate is 0.1250 inch.

3. Calculate values of $\Delta\lambda/\lambda^2$ for the shifts observed.

4. Calculate e/m for your data and compare with a modern determination.

5. State your polarization observations and account for them by a simple theoretical explanation.

THE RATIO OF e/m FOR THE ELECTRON

Massachusetts Institute of Technology

I. Introduction.

The best introduction to this experiment which can be offered is to quote passages from the original paper on the subject written by J. J. Thomson, Cavendish Professor of Experimental Physics, Cambridge, and published in Philosophical Magazine, volume 44, page 293, 1897. Before reading these passages it should be realized that the last decade of the 19th century marked the climax of classical physics; many physicists of that era felt that any problem could be solved by applying to it the basic principles of physics which had already been discovered. This feeling on their part, although erroneous, stemmed from the many successes of the atomic theory. Matter was supposed to be built up from some 90 different indestructible elements, called atoms, and the properties of matter were to be explained as the dynamic motion of these atoms moving under the influence of the forces which they exerted upon one another. Certainly these ideas had scored tremendous successes in explaining chemical reactions, and the properties of gasses as well as the thermal properties of all matter when combined with the statistical techniques of Maxwell and Boltzmann. Electricity and magnetism had been unified by the greatest theoretical breakthrough of the 19th century, the formulation by Maxwell of his famous equations. The Maxwell equations not only explained the electrical and magnetic properties of matter in terms of polarization of the individual atoms, but also described all of the results of optical experiments on the assumption that light consisted of coupled electric and magnetic fields propagating through space with a velocity of approximately 3×10^8 m/sec.

Thomson's experiments, then, are important because they provide the first unambiguous evidence that atoms were not indivisible. It was one of the first links in the chain of evidence which ultimately led to the concept of the modern nuclear atom, the concept according to which all matter is composed of protons, neutrons and electrons. Of course, it turns out that protons and neutrons also have internal structure, and it is the object of present day high energy physics to elucidate the relationship between neutrons, protons and electrons and the many new particles which have been discovered to result from high energy (greater than 500 Mev.) nuclear collisions.

The Experiment.

II. A. The Cathode-Ray Tube

Thomson obtained his electrons by means of an electrical discharge through the residual gas contained in a bulb at one end of his apparatus, see his Figure 2, and he detected the motion of his electron beam by means of the weak fluorescence produced in glass when bombarded by high energy electrons. Moreover, he had to produce his own apparatus and supply the pumps necessary to evacuate it. It is our good fortune to live in an era in which mass-produced devices are available which have provisions for producing electrons, contain an electron sensitive screen, have built in electrodes for accelerating the electrons and electrodes for deflecting the beam similar to the plates D, E in Thomson's Figure 2, and are very highly evacuated. These devices are called cathode - ray tubes, (CRT) and Figure 1 is a schematic diagram of the internal structure of the tubes used in this experiment. The operation of the "electron gun" component of the cathode ray tube, by means of which a narrow beam of high velocity electrons is produced, is described in the Cathode-Ray Oscilloscope supplement to these laboratory notes, and will not be repeated here.

Exercise 1.

Using simple kinetic theory arguments estimate the maximum tolerable pressure in a vessel such that an electron has approximately unit probability of being scattered by an air molecule in traveling 50 cm. Assume a radius of 10^{-8} cm for an air molecule.

The dynamics of the electrons as they traverse the C.R.T. will be described with reference to Figure 1. The initial velocities of the electrons at F are small; their velocities are spread out over an interval corresponding to energies from 0 to 6 electron volts. These velocities are negligable as compared to the velocity at the accelerating electrode D, a velocity corresponding to an energy of the order of 1000 electron volts, so that all the electrons leaving D have essentially the same velocity. From potential theory the kinetic energy of the electrons at D, E_f, is related to their energy at F, E_1, by

$$E_f - E_1 = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_1^2 = eV_f - eV_1$$

where m is the mass of an electron, and e is the charge on an electron taken without regard to sign, V_f, V_1 are the potentials at the electrodes D and F respectively, and v_f, v_1 are the electron speeds at D and F respectively. Since V_1 is negligable compared to V_f it follows that v_1 is also negligable compared to v_f so that the above equation reduces to

$$v_f = \left(2\,\frac{e}{m}\,V_f\right)^{\frac{1}{2}} \qquad (1)$$

After passing through the opening in electrode D the electrons travel down the tube in a region of constant potential, and hence with the constant speed v_f, until they strike the fluorescent screen H. A bright spot appears at the point where the electrons strike the screen.

Exercise 2.

It is fundamental to the success of this experiment that each electron find itself moving between electrode D and the screen in a region free from any forces other than those known fields applied by the experimenter in order to observe the response of an electron to a known force. However, in a beam of electrons there must be coulomb forces of repulsion acting between each electron and its neighbors. What is the largest beam current density which one can tolerate if the potential at an electron due to its nearest neighbors is to be less than 1 volt? Assume an electron velocity corresponding to a potential difference of 1000 volts. Is this space charge effect likely to cause any significant source of error in this experiment?

The object of this experiment will be to investigate the trajectories of electrons subjected to forces caused by their interaction with a uniform magnetic field directed along the axis of the cathode-ray tube. Since the magnetic force is given by:

$$\vec{F} = -e(\vec{v} \times \vec{B}),$$

where e is the magnitude of the charge on an electron, the electrons must have a component of velocity perpendicular to the magnetic field and, hence, to the tube axis. This velocity component is produced by means of an electric field between the plates E - E. The potential difference between these plates is obtained from the alternating current supply of Figure 2. The plates G - G could also be employed to provide the electrons with a transverse velocity, but these electrodes will not be used in this experiment.

B. The Magnetic Field of a Solenoid.

The cathode-ray tube is mounted in a solenoid so that the tube axis coincides with the solenoid axis as shown schematically in Figure 3. When a current I (obtained from the control circuit shown in Figure 4) flows through the solenoid winding, a magnetic field is set up in the solenoid parallel to its axis. The direction of the field reverses when the direction of I is reversed, and the field at any point in space is proportional to I. At the center of the solenoid the field, B_s, is;

$$B_s = \mu_o \frac{NI}{L} \left[\frac{1}{1 + (D/L)^2} \right]^{1/2} \qquad (2)$$

where μ_o = $4\pi \times 10^{-7}$ henries/meter.
N = the total number of turns on the solenoid
L = the length of the solenoid in meters
D = the diameter of the solenoid in meters

The field on the axis decreases at either end of the region of interest by approximately 7% of its value at the center of the solenoid. This variation introduces a small error ($\sim$ 1%) in the results when the value of B_s given by Equation 2 is used to calculate the ratio e/m. In principle, of course, one should use the average magnetic field which an electron experiences in its journey from the plates E - E to the screen of the C.R.T. The derivation of Equation 2 can be found in reference.(1) p. 205, (2) p. 131

C. Electron Ballistics in a Uniform Magnetic Field.

With an alternating voltage between the plates E-E the electrons emerge from the space between these electrodes with a

z - component of velocity, v_f, given by Equation 1, and a component of velocity in the y-direction, v_t, which depends upon the magnitude of the electric field between the deflection plates at the time the electron passes between them (see Figure 3). Since the electrons have velocities of the order of 10^9 cm/sec the time they spend between the deflection plates is very short compared to the period of the 60 c.p.s. field. Consequently, a whole spectrum of transverse velocities, ranging from $+v_o$ to $-v_o$, is produced as the voltage applied to the plates E - E goes through a complete cycle. It can also be shown that the effect of the finite length of the deflection plates is very nearly the same as if the electrons were given a single impulse, i.e. a discontinuous change in transverse velocity, midway along the length of the plates. It is for that reason that the origin of coordinates in Figure 3 has been drawn at the center of the electrodes E - E.

The only forces acting on the electrons in the region between the deflectron plates and the screen are due to the magnetic field because the plates have the same average potential as the screen. Setting each component of the magnetic force equal to the mass multiplied by the acceleration in that direction gives the following differential equations when the field is taken to be along the positive z direction:

$$\begin{aligned} \ddot{x} &= -(e/m)Bv_y \\ \ddot{y} &= (e/m)Bv_x \\ \ddot{z} &= 0 \end{aligned} \qquad (4)$$

The solution of these equations when the velocity is initially in in the yz plane is:

$$\begin{aligned} x(t) &= (v_t/\omega)(\text{Cos } \omega t - 1) \\ y(t) &= (v_t/\omega)\ \text{Sin } \omega t \\ z(t) &= v_f\ t \end{aligned} \qquad (5)$$

where $\omega = eB/m$, $v_f = (2\ e/m\ V_f)^{\frac{1}{2}}$, and v_t is the initial velocity given to the electron in the y-direction. Note that at any time t

$$\left(x + v_t/\omega\right)^2 + y^2 = \text{const.} = \left(v_t/\omega\right)^2.$$

This is the equation of a circle, radius $R = v_t/\omega$, and centered at the point $x = -R$, $y = 0$. In other words the electron describes a counter-clockwise helical orbit as seen by an observer looking into the C.R.T. screen.

Let x_s, y_s be the coordinates of the point on the screen struck by an electron whose path is given by the Equations 5. x_s and y_s can be obtained from 5 by setting the time of flight, t, equal to $t = S/v_f$, where S is the distance from the origin to to the center of the screen. If $\delta = \omega S/v_f$, then in terms of δ

and ω,

$$x_s = (v_t/\omega)(\mathrm{Cos}\ \delta - 1)$$
$$y_s = (v_t/\omega)\ \mathrm{Sin}\ \delta \qquad (6)$$

As a consequence of the a.c. voltage applied to the deflection plates there will be electrons having all possible values of v_t lying between $+v_o$ and $-v_o$. Therefore, the locus of all such points x_s, y_s will form a straight line on the screen whose end points lie at $((v_o/\omega)(\mathrm{Cos}\ \delta - 1), (v_o/\omega)\mathrm{Sin}\ \delta)$ and $((-v_o/\omega)(\mathrm{Cos}\ \delta - 1), (-v_o/\omega)\mathrm{Sin}\ \delta)$.

Exercise 3.

Prove that the locus of points on the C.R.T. screen given by Equations 6 for initial y-velocities lying between $+v_o$ and $-v_o$ is a straight line whose slope is:

$$dy_s/dx_s = \mathrm{Sin}\ \delta/(\mathrm{Cos}\ \delta - 1).$$

It is interesting to examine the behavior of this straight line image as the magnetic field is increased from zero. At $B = 0$ one has also $\delta = 0$ and the image on the screen is a straight line parallel to the y-axis, and of length $L_o = 2v_oS/v_f$. This result makes sense, since with $B = 0$ the electrons travel in straight line paths from the mid-point between the deflection plates to the screen. To obtain this result from Equations 6 it is necessary to expand $\mathrm{Sin}\ \delta$ and $\mathrm{Cos}\ \delta$ in a Taylor series, divide by ω, and then go to the limit $\omega = 0$.

$$\lim_{\omega \to 0} \frac{\mathrm{Cos}(\omega S/v_f) - 1)}{\omega} = \lim_{\omega \to 0} \frac{(1 - \frac{1}{2}\omega^2 S^2/v_f + \cdots - 1)}{\omega} = 0$$

$$\lim_{\omega \to 0} \frac{\mathrm{Sin}\ \omega S/v_f}{\omega} = \lim_{\omega \to 0} \frac{(\omega S/v_f - \omega^3 S^3/6v_f^3 + \cdots)}{\omega} = S/v_f.$$

At $\delta = \pi/2$ one obtains a line rotated 45° counter clockwise, looking into the screen, and of length $L = (2\sqrt{2}/\pi)L_o$. To obtain this last result set $\omega = v_f\delta/S$. Similarly the length of the line on the screen and the angle through which it has been rotated can be calculated as functions of B, or the parameter $\delta = (e/m)(S/v_f)B$ The results of such a calcuation are listed in Table 1.

Table 1.

δ	Length of the Image L	Angle of Rotation ϕ
0	$L_o = 2v_o S/v_f$	0
$\pi/2$	$0.90\ L_o$	$\pi/4$
π	$0.64\ L_o$	$\pi/2$
2π	0	π
$2\pi + \epsilon$	$\epsilon L_o/2\pi$	2π where ϵ is very small
3π	$0.21\ L_o$	$5\pi/2$
4π	0	2π

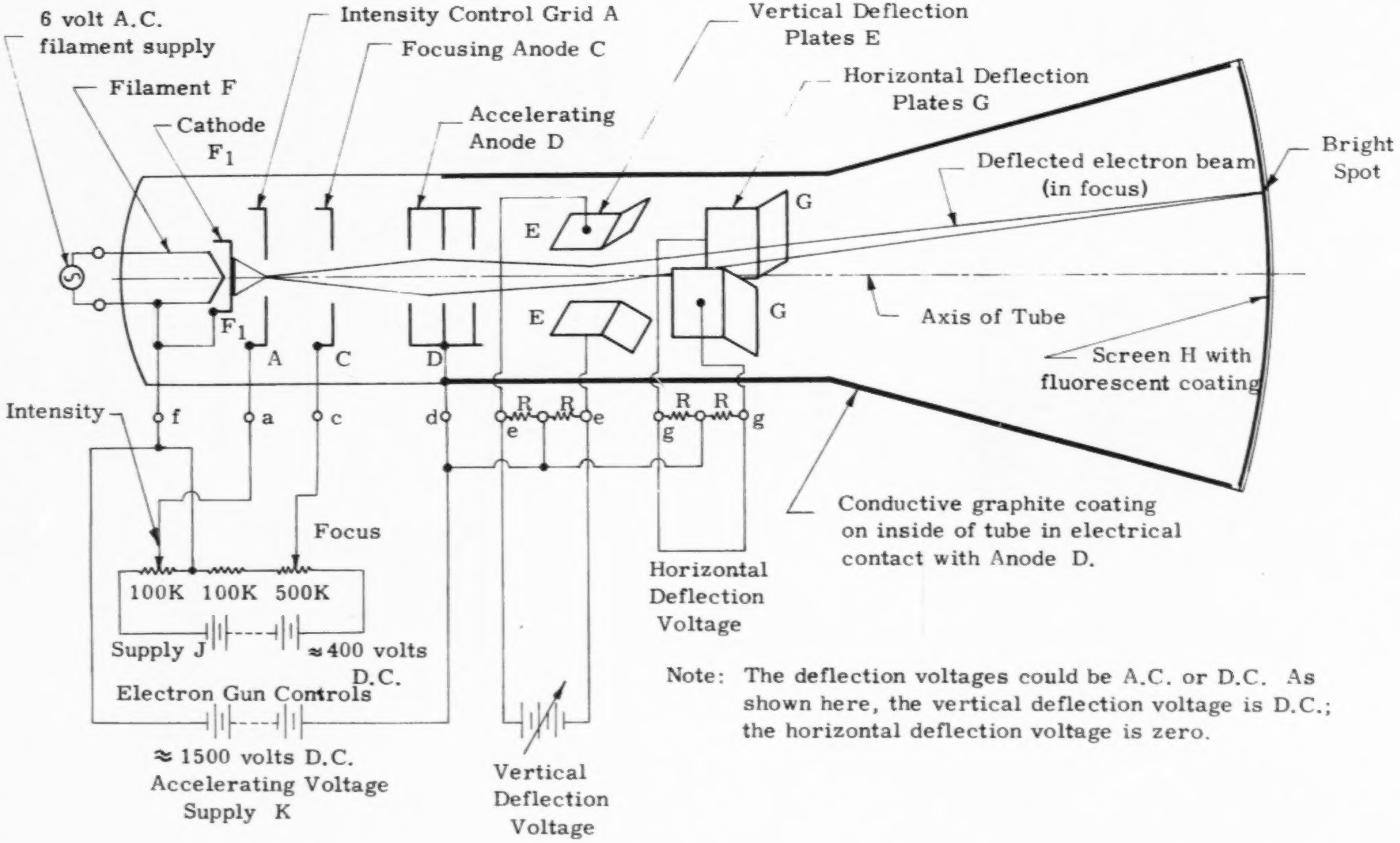
It is apparent that as B increases, the line on the screen rotates counter-clockwise, and the length of the line goes to zero whenever δ is a multiple of 2π. Note that a positively charged particle would give a line which rotated clock-wise as the field was increased along the +Z -direction. The observation of a counter-clock-wise rotation of the line for increasing fields consequently constitutes a proof that electrons carry a negative charge.

Similar results to the above are obtained if the field B is reversed in direction, except that the image on the screen rotates in the clock-wise direction as B is increased from 0.

D. The Effect of the Earth's Magnetic Field.

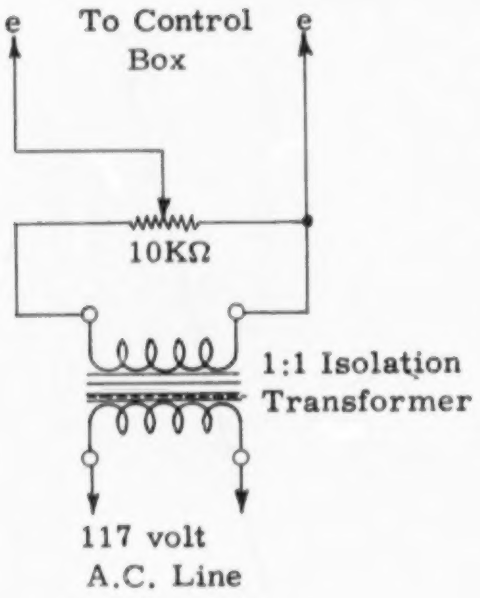
The results of the last section have been derived under the assumption that the field acting upon the electrons is parallel to the axis of the C.R.T. However, in the laboratory there is a weak magnetic field due to the magnetic moment of the earth. This field has a magnitude of approximately 0.5 Gauss, $5 \times 10^{-5}\ \omega/m^2$, and is inclined at an angle of approximately 70° to the horizontal in the north-south direction. The effect of the earth's field when added to the solenoid field is to give a resultant uniform field which is tilted at an angle to the axis of the cathode-ray tube. For the student with a mathematical bent the solution of the equation of motion of an electron moving in an arbritrary magnetic field is set forth in the Appendix. This Appendix also justifies the physical arguments which will be used to describe the behavior of the electron beam in a field which is not parallel to the C.R.T. axis. In the laboratory you will find that the apparatus has been oriented so that the earth's magnetic field lies in the x -Z plane, referring to the axes of Figure 3. This in turn means that the y-axis has been turned so that it lies in a horizontal plane. Consider two extreme cases, both with no voltage applied to the plates E - E;

Case (1) - the solenoid current is zero. Clearly the beam will be


6 volt A.C. filament supply
Filament F
Cathode F₁
Intensity Control Grid A
Focusing Anode C
Accelerating Anode D
Vertical Deflection Plates E
Horizontal Deflection Plates G
Deflected electron beam (in focus)
Bright Spot
Axis of Tube
Screen H with fluorescent coating
Conductive graphite coating on inside of tube in electrical contact with Anode D.
F₁
A
C
D
E
E
G
G
Intensity
f
a
c
d
R
R
R
R
e
e
g
g
Focus
100K 100K 500K
Supply J
≈400 volts D.C.
Electron Gun Controls
≈ 1500 volts D.C. Accelerating Voltage Supply K
Vertical Deflection Voltage
Horizontal Deflection Voltage

Note: The deflection voltages could be A.C. or D.C. As shown here, the vertical deflection voltage is D.C.; the horizontal deflection voltage is zero.

Fig. 1. Schematic Diagram of the Cathode Ray Tube and its Control Circuits.


e
To Control
Box
e
10KΩ
1:1 Isolation
Transformer
117 volt
A.C. Line

(b) A.C. Supply

Fig. 2. Deflection Voltage Supply

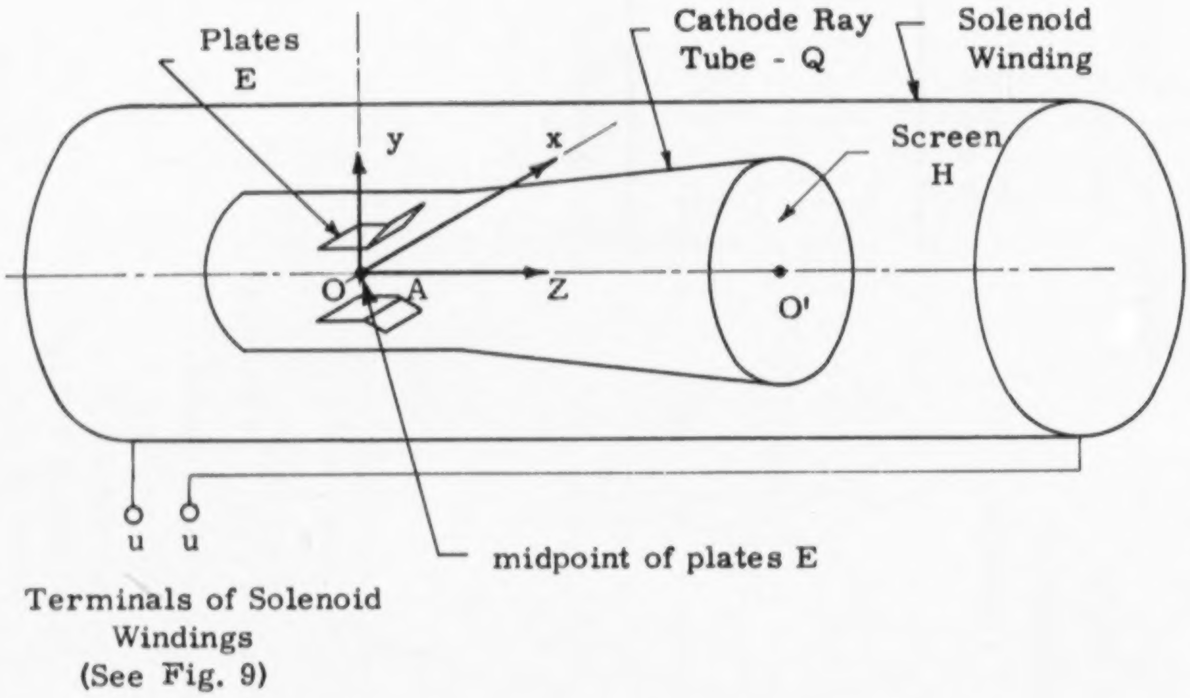

Plates
E
Cathode Ray
Tube - Q
Solenoid
Winding
y
x
Screen
H
O
A
Z
O'
u
u
midpoint of plates E
Terminals of Solenoid
Windings
(See Fig. 9)

Fig. 3 . Arrangement of C.R.T. in the Solenoid.

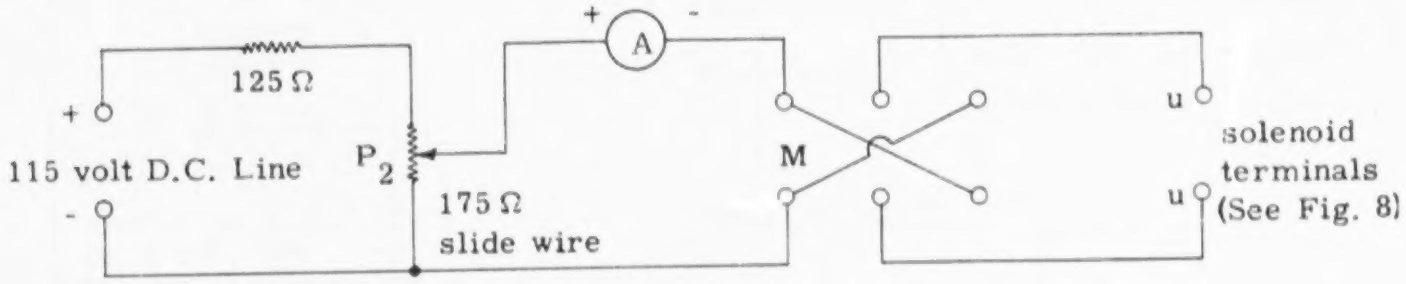
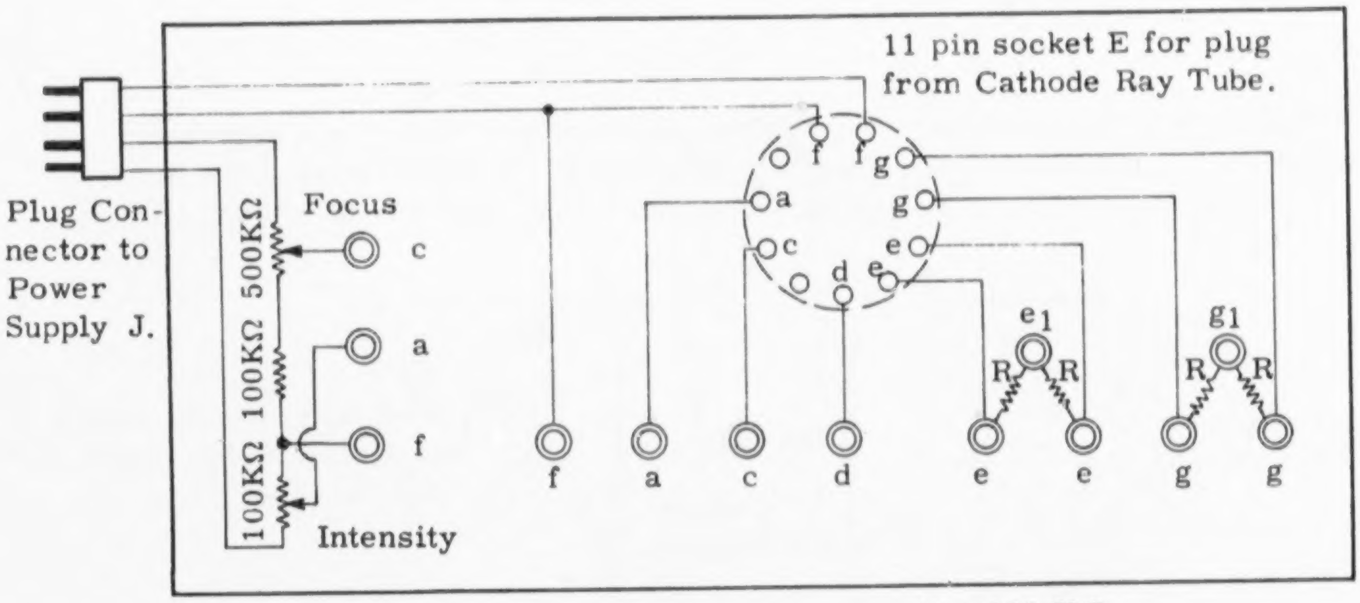

+
A
-
125 Ω
+
115 volt D.C. Line
-
P_2
175 Ω
slide wire
M
u
u
solenoid
terminals
(See Fig. 8)

Fig. 4. Control Circuit for Solenoid Current.


11 pin socket E for plug
from Cathode Ray Tube.
Plug Connector to
Power
Supply J.
Focus
500KΩ
100KΩ
100KΩ
Intensity
c
a
f
f
f
g
g
a
g
c
e
d
e
e_1
g_1
R
R
R
R
f
a
c
d
e
e
g
g

R = 220 KΩ

Fig. 5. Control Box.

All the wiring shown above is internal. The double concentric circles represent terminals to which you will attach external leads. An eleven (11) prong plug from the C.R.T. with all the tube connections fits into the socket. The 4 prong plug on the left goes to Power Supply J which supplies both filament voltage and electron gun control voltages.

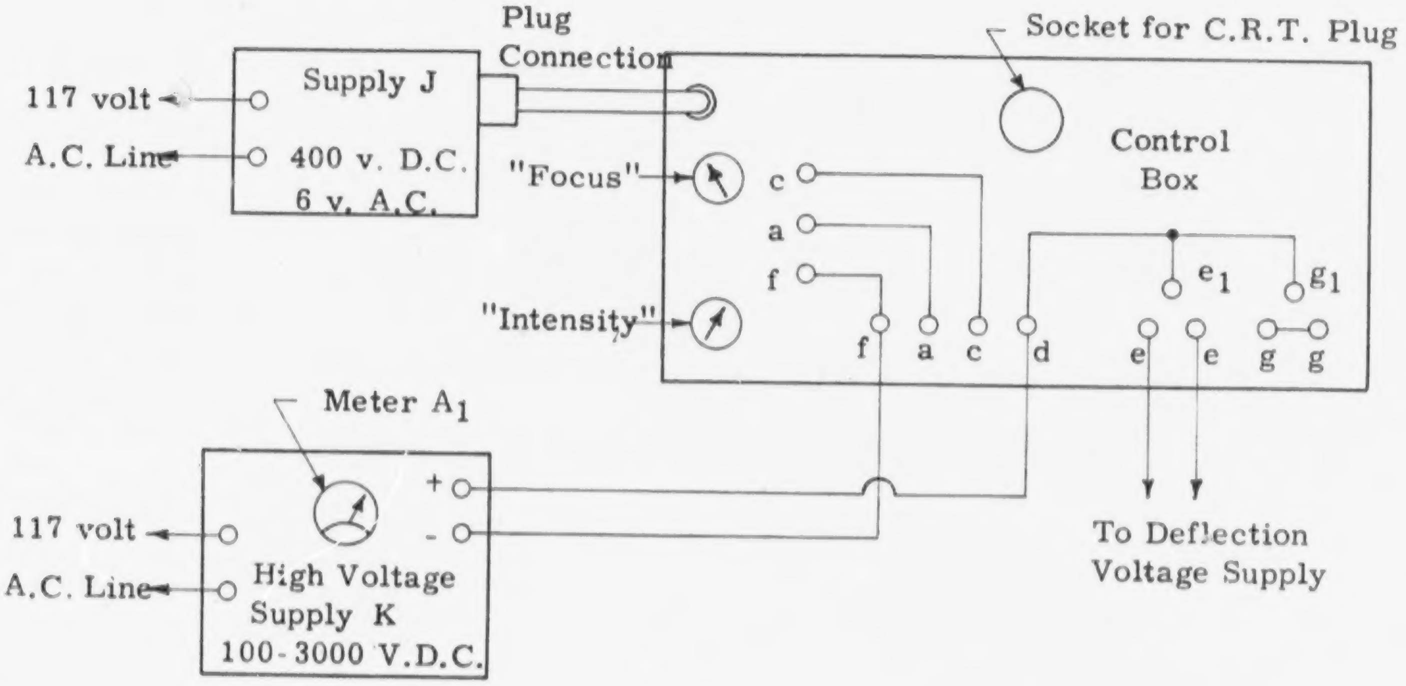
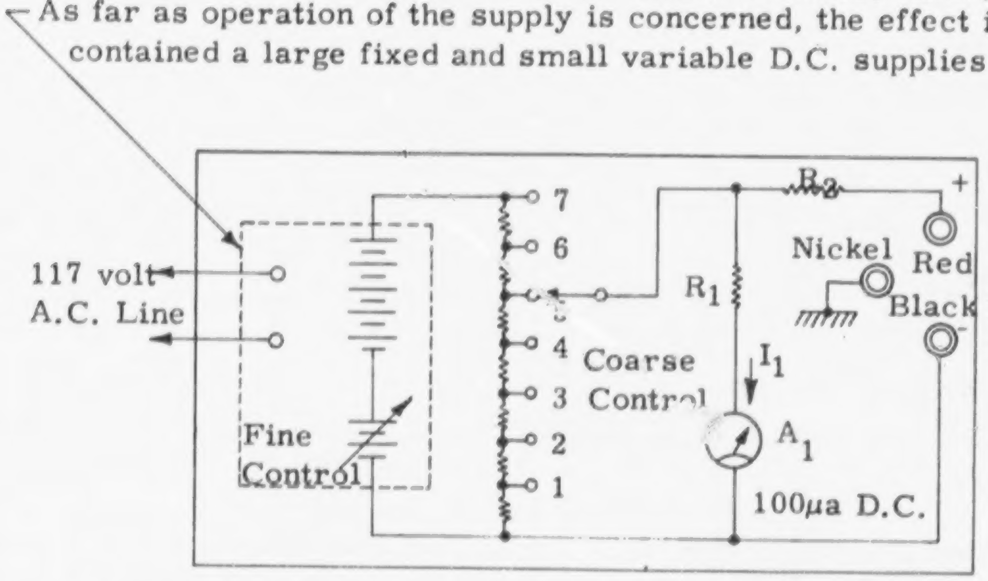

Plug Connection
Socket for C.R.T. Plug
117 volt
A.C. Line
Supply J
400 v. D.C.
6 v. A.C.
"Focus"
"Intensity"
Control Box
c
a
f
f
a
c
d
e
e
e_1
g_1
g
g
Meter A_1
+
-
117 volt
A.C. Line
High Voltage Supply K
100-3000 V.D.C.
To Deflection Voltage Supply

Fig. 6. Control Circuit

This box actually contains a transformer, rectifier, filter, and regulator. As far as operation of the supply is concerned, the effect is as if the box contained a large fixed and small variable D.C. supplies as shown.


117 volt
A.C. Line
7
6
5
4
3
2
1
Coarse Control
Fine Control
R_2
+
Nickel
Red
Black
-
R_1
I_1
A_1
100μa D.C.

Note: Nickel plated terminal is connected to case.

Note: All wiring and metering shown is internal.

$R_1 = 10 \text{ meg } \Omega \pm 1\% + 20 \text{ meg } \Omega \pm 1\%$
$R_2 = 2.7 \text{ meg } \Omega$

Fig. 7. High Voltage Power Supply K.

deflected in the y-direction as a consequence of the field component Hx and the Z-component of velocity, v_f. Of course, as the beam is deflected it developes a y-component of velocity which interacts with Hx, H_Z , but clearly this is a very small effect if the deflection in the y-direction is small. It is easy to estimate the magnitude of this y-deflection.

$y \approx \frac{1}{2}at^2$, $a = e/m\ v_f Hx \approx 10^{14}$ m/sec^2

$= 4.5$ cm $t = S/v_f = 3 \times 10^{-8}$ sec

where we have used $S = .3$ m; $v_f = 10^7$ m/sec corresponding to an acceleration potential of roughly 1000 v, and Hx = .5 Gauss = $\frac{1}{2} \times 10^{-4}$ Weber/m^2. The earth's field, then, produces a marked effect, and this is the principal reason that the spot, which you will obtain on the scope screen, will be found off-center.

Case (2)- the solenoid current is large, and produces a field in the Z-direction which is very much stronger than the earth's magnetic field. In this case the resultant magnetic field is inclined at some small angle $\phi \approx Hx/H$ to the Z-axis. Let this direction be a new axis, the Z'-axis. In this new co-ordinate system one has just the description of the electron motion given by Equations (5), in which $v_t = v_f \sin \phi$ and the velocity in the Z' direction is $v_Z' = v_f \cos \phi \approx v_f$, since ϕ is small. Consequently the coordinates of the spot on the screen will be given by Equations 6, except that the origin of coordinates must be taken at that point where the Z' axis pierces the screen. As the solenoid current is increased the spot will gyrate according to (6), but will approach more and more closely the intersection of the electron gun axis (Z -axis of Figure 3) and the screen.

One may now inquire into the effect of a small alternating potential difference between the plates E - E. It seems reasonable, and this is justified in the Appendix, that the resulting line on the screen behaves just as described in the previous section, that is, rotates and shrinks as the solenoid current is increased, but with this difference - the center of the line moves along a path on the scope screen corresponding to the traversal by the beam of all deflections intermediate between Case (1) and Case (2) above. In particular, the ratio of e/m can still be obtained from the value of the field required to bring the line to its first focus,

$$e/m = 2\pi v_f / B_Z S \ . \qquad (7)$$

Note that B_Z is the sum of the solenoid field and the Z-component of the earth's magnetic field.

III. Experimental Procedure.

The various resistors shown in the control circuits of the C.R.T. of Figure 1 are arranged in a box called the "Control Box", Figure 5. Make the various connections to this control box as shown in Figure 6. A schematic diagram of the power supply K is shown in Figure 7. You will be given, in the laboratory, a calibration curve for the meter on supply K. Connect the A.C. supply to the deflection plates E - E (terminals e - e on the control box), but leave the 10K ohms potentiometer set at 0. Connect the solenoid to its control circuit, Figure 4.

When your circuit has been checked switch on supply J and allow a few minutes for the tube filament to warm up. Turn on the high voltage supply, K, and increase its output until the accelerating voltage is of the order of 1000 volts. Adjust the intensity and focus controls to obtain a spot on the screen. You will find that as the intensity of the beam is increased, the accelerating voltage drops. This occurs because of the large internal impedance of supply K. Each C.R.T, has some optimum combination of accelerating voltage, focus, and intensity and the object is to find this combination after which the controls are not changed for the duration of the experiment.

Apply a 60 cps signal to the deflection plates by turning the knob on the 10K ohms potentiometer. You should see a straight line on the screen spread out equally on both sides of the original position of the spot. Readjust the focus, intensity and accelerating voltage controls in an effort to obtain a narrow, well-defined, and uniformly illuminated line. You will probably discover that the best line obtainable is approximately 1/8 inch thick. The settings of focus, etc. which give you the best line should not be changed during the course of the experiment. If you experience any difficulty in obtaining a well-defined straight line, apply to your instructor for help.

The experiment now proceeds in two parts.

Part I - Verify the predictions of Equation (6) as listed in Table 1 by plotting the length of the image on the scope as a function of the angle of rotation. The angle of rotation is produced by increasing the current through the solenoid. Make a graph of the angle of rotation of the line as a function of the solenoid current. The slope of the resulting line is related to e/m. How? Estimate e/m from this slope.

Part II - Increase the solenoid current until the line image on the screen is reduced to a spot. As this focusing condition is approached turn up the voltage applied to the deflection plates so as to be able to determine the focusing current more precisely.

As discussed in Section (D), Part II, one has at focus

$$\frac{e}{m} = \frac{2\pi v_f}{B_z S}, \qquad \text{and} \qquad v_f = (2e/m\ V_f)^{\frac{1}{2}} \qquad (8)$$

where V_f is the accelerating potential obtained from the meter on supply K, S is the distance from the midpoint of the deflecting plates E - E and the screen and is obtained from an X-ray photograph of the C.R.T. supplied with the experiment. Scale lengths on this photograph are obtained from the length of a 30 cm ruler which was held in the plane of the tube and photographed along with it.

$B_z = Bs + B_h$, where Bs is the field of the solenoid calculated from Equation 2 and B_h is the component of the earth's field parallel to the axis of the C.R.T. This component is unknown, but can be obtained by measuring the current necessary to obtain a focus with the solenoid field first in one direction and then reversed. The field corresponding to the resulting current difference must be $2B_h$. Calculate a value for e/m, with an estimate of the precision of your measurement obtained from estimates of the uncertainties in the individual elements which enter into Equation 7.

Finally, obtain the magnitude of the component of the earth's magnetic field transverse to the tube axis. This component may be obtained from Equations (A2) of the Appendix, which give the coordinates of the spot on the screen at focus,

$$x_s = (B_x/B_z)S \qquad \text{(A2)}$$
$$y_s = 0$$

It is clear then, that if focus is obtained with the solenoid field ($\approx B_z$) first in one direction, and then in the other, the spot moves a distance $x_s = (2B_x/B_z)S$, from which B_x can be easily obtained.

(IV) The Results

Your results should include:

(1) A graph showing the length of the line on the scope as a function of angle of rotation. Your graph should also include the theoretical curve.

(2) A graph of the angle of rotation as a function of solenoid current; both theoretical and experimental results should be included. This graph should also have on it an estimate of e/m obtained from the slope of angle of rotation vs current.

(3) A value of e/m, with its uncertainty, obtained from the focussing condition, Equation 7.

(4) An estimate of the magnitude and direction of the earth's magnetic field.

References

(1) Currents, Fields, and Particles, F. Bitter. Chapters 3 and 5, 1956 Edition.

(2) Introduction to Electricity and Optics, N.H. Frank. Chapters 6 and 7, Second Edition.

(3) Physics for Students of Science and Engineering, Part II, D. Halliday, R. Resnick. Chapter 33.

Appendix - The Motion of an Electron in an Arbritrary Magnetic Field.

Refer to the co-ordinate system of Figure 3. It will now be assumed that an electron leaves the origin of co-ordinates with an initial velocity in the Z -direction of v_f, an initial velocity in the y-direction of v_t, and an initial velocity of zero in the x-direction. Furthermore, the whole apparatus is immersed in a uniform magnetic field whose components are B_x, B_y, B_z. The equation of motion of the electron is $m\vec{a} = -e(\vec{v} \times \vec{B})$. That is

$$\ddot{x} = -e/m(v_yH_z - v_z\ H_y)$$

$$\ddot{y} = -e/m(v_zH_x - v_xH_z)$$

$$\ddot{Z} = -e/m(v_xH_y - v_yH_x)$$

For the sake of convenience we introduce the following notation:

$$\omega = eB/m$$

$$\omega_x = eB_x/m$$

$$\omega_y = eB_y/m$$

$$\omega_z = eB_z/m$$

The solutions of the above differential equations which satisfy the required initial conditions that

$x = 0$, $y = 0$, $Z = 0$, $v_x = 0$, $v_y = v_t$, $v_z = v_f$, at $t = 0$ are:

$$x(t) = \frac{(\omega_y v_f - \omega_z v_t)}{\omega^2}(1 - \mathrm{Cos}\ \omega t) + \frac{(\omega_x\omega_z v_f + \omega_x\omega_y v_t)}{\omega^3}(\omega t - \mathrm{Sin}\ \omega t)$$

$$y(t) = \frac{v_t}{\omega}\mathrm{Sin}\ \omega t - \frac{\omega_x v_f}{\omega^2}(1 - \mathrm{Cos}\ \omega t) + \frac{(\omega_y\omega_z v_f + \omega_y^2 v_t)}{\omega^3}(\omega t - \mathrm{Sin}\ \omega t)$$

$$Z(t) = \frac{v_f}{\omega}\mathrm{Sin}\ \omega t + \frac{\omega_x v_t}{\omega^2}(1 - \mathrm{Cos}\ \omega t) + \frac{(\omega_z^2 v_f + \omega_z\omega_y v_t)}{\omega^3}(\omega t - \mathrm{Sin}\ \omega t).$$

These expressions, which give the position of the electron at any time t after leaving the origin, are complicated, and therefore uninteresting. The first element of simplification can be introduced by rotating the whole cathode ray tube so that the magnetic field lies entirely in the x - Z plane, i.e. $\omega_y = 0$. Then one obtains

$$x(t) = \frac{\omega_z v_t}{\omega^2}(\mathrm{Cos}\ \omega t - 1) + \frac{\omega_x\omega_z}{\omega^3} v_f\ (\omega t - \mathrm{Sin}\ \omega t)$$

$$y(t) = \frac{v_t}{\omega}\mathrm{Sin}\ \omega t - \frac{\omega_x v_f}{\omega^2}(1 - \mathrm{Cos}\ \omega t) \qquad \text{(A 1)}$$

$$Z(t) = \frac{v_f}{\omega}\mathrm{Sin}\ \omega t + \frac{\omega_x v_t}{\omega^2}(1 - \mathrm{Cos}\ \omega t) + \frac{\omega_z^2 v_f}{\omega^3}(\omega t - \mathrm{Sin}\ \omega t).$$

No further simplification can be obtained except by considering the order of magnitude of each term and simply ignoring small quantities. Before, however, proceeding in that direction let us note one thing: if $\omega t = 2n\pi$, where n is an integer, and one has

$$Z(t) = 2n\pi \frac{\omega_z^2 v_f}{\omega^3} = S$$

then the co-ordinates of the point on the screen struck by the electron are:

$$x_s = 2n\pi \frac{\omega_x}{\omega_z} \frac{\omega_z^2 v_f}{\omega^3} = \frac{\omega_x S}{\omega_t} = \frac{B_x}{B} S$$

$$y_s = 0 \qquad \text{(A2)}$$

These co-ordinates are independent of the initial transverse velocity v_t, of the electron, consequently it is rigorously true even in this case that with an applied voltage between the deflection plates E - E the electron beam is brought to a focus whenever the field is such that

$$2n\pi \frac{\omega_z^2}{\omega^3} v_f = S = 2n\pi \left(\frac{B_z}{B}\right)^2 \frac{v_f}{\omega} .$$

For the apparatus used in this experiment B must be of the order of 10 Gauss at the first focus, given by n = 1 above, whereas B_x is of the order of the earths' field, 0.5 g. It follows that $B_z/B = 1$ to within (2/10)%. Hence to sufficient accuracy one obtains a focus at the screen when

$$S = 2n\pi \frac{v_f}{\omega_z} . \qquad \text{(A3)}$$

Now let us attack the equation A1. The maximum complication arises when $\omega_x \approx \omega_z \approx \omega$, that is when the solenoid field is zero. However, ωt is then small because at the first focus $\omega t = 2\pi$, and the earths' field is much smaller than the field required to focus the electron beam. Expanding the Sin ωt and Cos ωt terms in $Z(t)$, Equation A1, there results

$$Z(t) = v_f t + \frac{\omega_x v_t}{2} t^2 + \text{terms of the order } (\omega t)^3$$

$$\text{or,} \quad Z(t) = v_f t \left[1 + \tfrac{1}{2}(\omega_x t)(v_t/v_f)\right].$$

The maximum voltage applied to the deflection plates is of the order of 10 volts, where as the accelerating potential for the beam is some 1000 volts. This implies that v_t/v_f is of the order of 1/10 The result is, that to approximately 1%, we can write $Z(t) = v_f t$, and this approximation is valid over the whole range of solenoid fields, from zero to arbitrarily large values. The physical reason for this result is just that if the beam deflection is small (in practice this means that the beam image must remain on the scope screen) the distance the electrons travel from the deflection plates to the screen remains very closely equal to $\mathcal{S}$. Moreover, the velocity of the electron along its trajectory must remain approximately equal to v_f (exactly v_f if $v_t = 0$) since the force exerted by a magnetic field on a charged particle cannot alter the particles' kinetic energy.

One can now substitute $t = \mathcal{S}/v_f$ in equations A1 to obtain the co-ordinates of the point where the electron strikes the screen and the resulting expressions will be valid to within a few percent for any solenoid current. If the solenoid current is large enough so that in addition one can set $\omega_z/\omega = B_z/B = 1$, then the co-ordinates of the spot on the screen are:

$$x_s = \frac{v_t}{\omega}(\text{Cos } \omega t - 1) + \frac{\omega_x}{\omega} v_f t - \frac{\omega_x}{\omega^2} v_f \text{ Sin } \omega t \qquad \text{(A4)}$$

$$y_s = \frac{v_t}{\omega}\text{ Sin } \omega t - \frac{\omega_x v_f}{\omega^2}(1 - \text{Cos } \omega t).$$

The first terms of x_s and y_s in equations (A4) are identical to equations (6) in the text. If an alternating voltage of amplitude v_o is applied to the deflection plates there will be electrons in the resulting beam having initial y-velocities distributed between $-v_o$ and $+ v_o$. These terms in x_s and y_s consequently result in a line on the scope screen which, as the solenoid field is increased, rotates and shrinks in length exactly as described in the text for the case when the scope axis is rigorously parallel to the direction of the magnetic field. The other terms in (A4) represent a displacement of the center of this line as the field is increased.

PLANCK'S CONSTANT

Newark College of Engineering

THEORY.

When light quanta of frequency f are incident upon a surface, electrons may be emitted. The maximum possible kinetic energy of any ejected electron is the energy hf of the photon reduced by an amount W, the work function of the surface. That is,

$$KE_{max} = hf - W \quad .$$

For a given emitting surface, this maximum kinetic energy depends on the frequency of the light, not on its intensity. The intensity influences only the number of electrons ejected. If the incident light has several frequencies, the highest frequency applies.

As the frequency of the light decreases, the photons have less and less energy until a "threshold" frequency f_o is reached where $hf_o = W$. Below this frequency no electrons can possibly be emitted from the surface.

When the ejected electrons reach a collecting plate, the resulting current can be detected. This current will be reduced if a retarding, external potential difference is applied between the emitter and the collector. A sufficiently high potential difference prevents even the fastest electrons from reaching the collector, and the current reduces to zero. If the fastest electron just fails to reach the collector it must have had an initial kinetic energy

$$KE_{max} = eV_{crit} \quad ,$$

where V_{crit} is called the critical retarding potential.

Then $$eV_{crit} = hf - W$$

or $$V_{crit} = \frac{h}{e}f - \frac{W}{e} \quad .$$

A plot of V_{crit} versus f is a straight line:

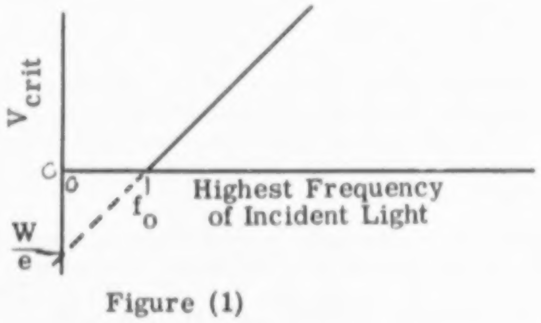


Figure (1)

The slope is h/e, the f axis intercept is f_o, and the V_{crit} axis intercept is $-W/e$.

An idealized form of an experiment to measure V_{crit} is illustrated in Figure (2) .

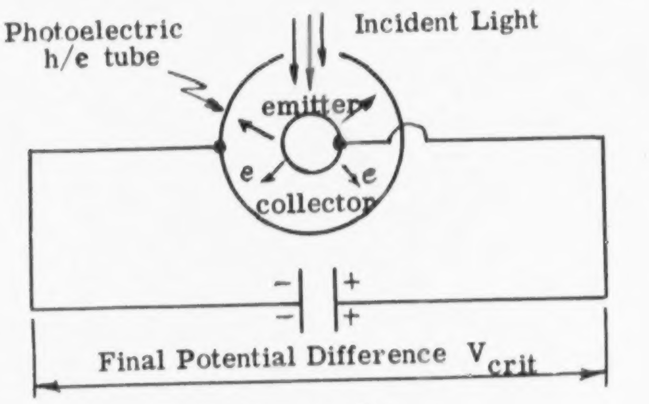


Figure (2)

The emitted electrons gradually charge the capacitor, building up a retarding potential. The final potential difference across the capacitor will be V_{crit} .

However, in actual practice the collector and emitter of the h/e tube are of different materials. An unknown, but constant, "contact" potential difference between these two materials enters into the attempt to measure V_{crit} . In fact, the contact potential difference is usually of such size and polarity that for visible light none of the emitted electrons ever reaches the collector.

This difficulty can be eliminated by introducing into the circuit a battery whose emf E just compensates for the contact potential, Figure (3) .

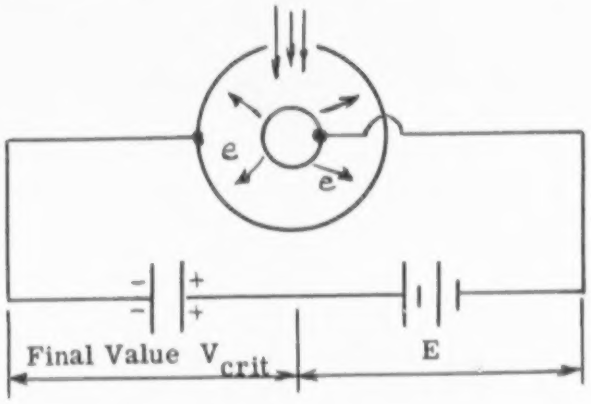


Figure (3)

Since the value of the contact potential difference is originally unknown, an arbitrary, constant value may be selected for E and the final potential difference across the capacitor will be an apparent critical retarding potential V .

A plot of V versus f will allow a determination of h/e from the slope. However, the respective intercepts will no longer be f_0 and - W/e . Note that the data obtained in this experiment will not permit an accurate measurement of f_0 and a subsequent determination of W and the contact potential difference. But the use of an intense, continuous spectrum light source in conjunction with a monochrometer (in place of the mercury vapor lamp and filters) could possibly permit this determination.

The potential difference across the small capacitor can be read with a voltmeter which draws extremely small currents. Such an instrument is an electrometer type direct current vacuum tube voltmeter (VTVM) , shown symbolically in Figure (4) .

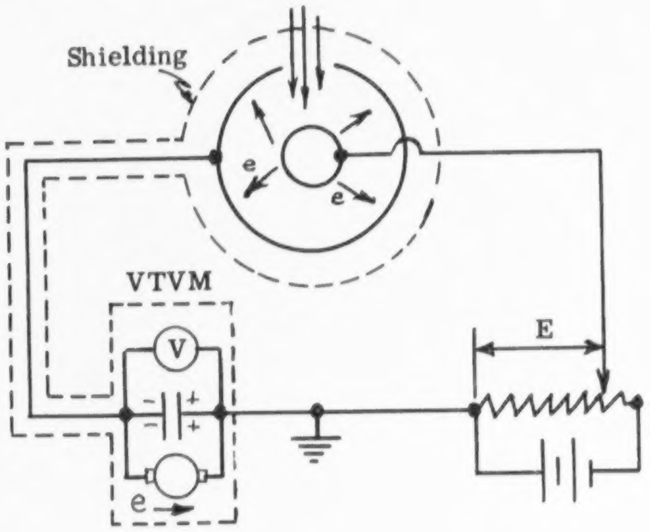


Figure (4)

The "small capacitor" is actually the input capacitance of the VTVM , approximately 20μμf . In the equivalent circuit of the VTVM , the small generator symbolizes the grid-drift current of about 10^{-14} amperes which exists in this type VTVM and which moves electrons as shown.

The photoelectrons gradually charge the capacitor of Figure (4) until a final, steady value V is obtained for which a photoelectron current of nearly zero (10^{-14} amperes) just balances the grid-drift current of the VTVM. Therefore "zero" current with this apparatus is 10^{-14} amperes.

The final value V can also be obtained by initially charging the VTVM to a potential difference above V. In this condition none of the emitted photoelectrons will reach the collector, and the grid-drift current will slowly discharge the capacitor until the potential difference is reduced to V .

The h/e tube used for this experiment consists of an emitting cylinder inside a collecting cylinder. It was made following Olpin's description (Physical Review, 2nd Series, Vol. 36, 1930, p. 251). The emitter was coated in a separate chamber to avoid any contamination of other tube surfaces which could result in secondary emission of electrons.

A housing shields the h/e tube* from extraneous light and also serves as an electrostatic shield. Because of the low current, care must be taken to assemble the circuit a sufficiently long time (several hours) before the performance of the experiment. This allows the dissipation of any electrostatic charges which may have been induced in handling. The h/e tube housing must be firmly fixed to the VTVM to prevent any motion which may induce electrostatic charges and may cause a variation in capacitance which could affect the VTVM readings.

---

*Information about this tube and its characteristics can be obtained from Madison Associates, Inc., 207 Greenwood Avenue, Madison, New Jersey.

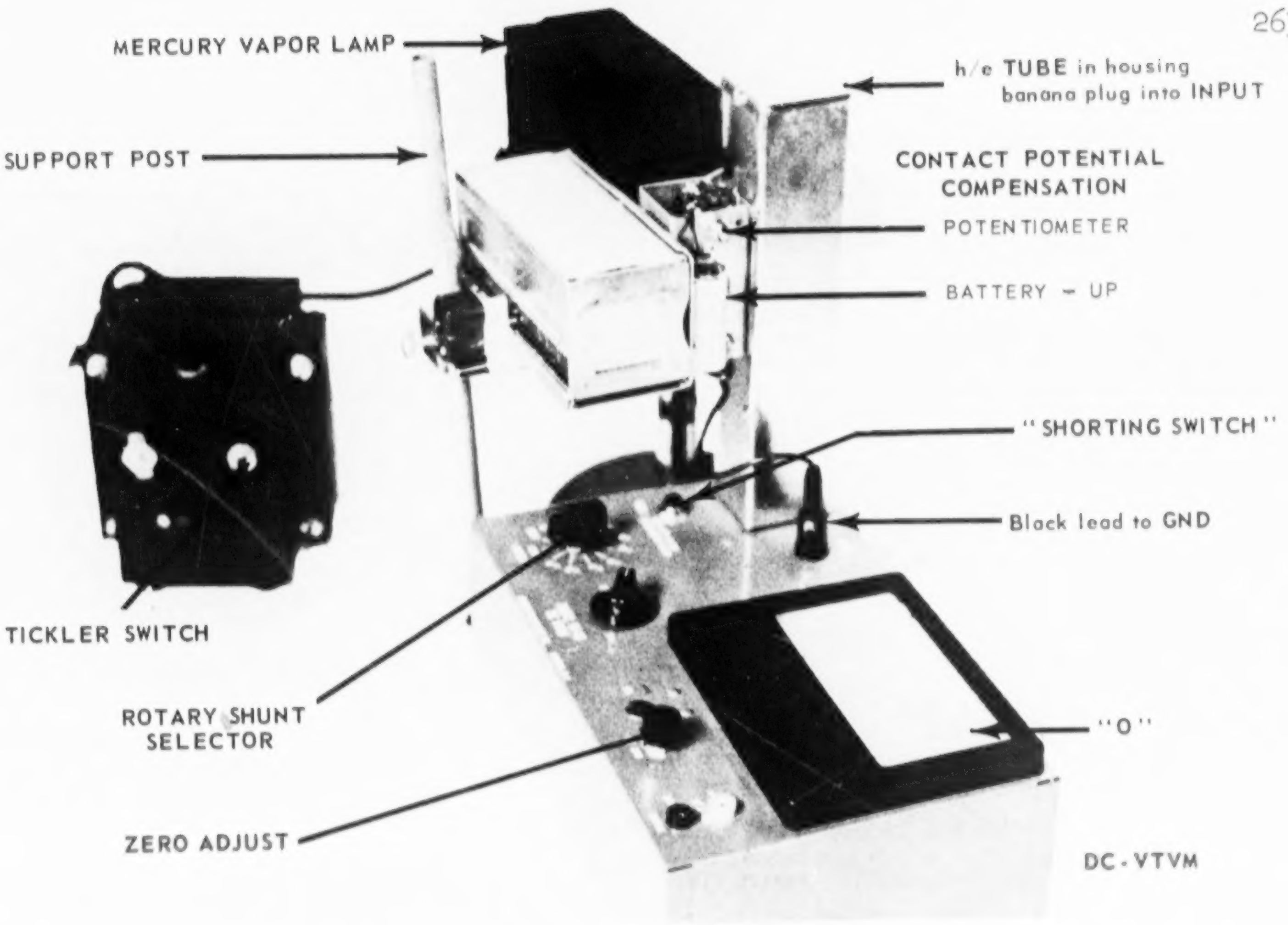


Figure (5)

PROCEDURE.

The following filters are available to be used with a quartz mercury vapor lamp.

Filter No.	Shortest λ Hg-line passed
KW-22+70	6889A°
KW-22+23A	5770
KW-74	5461
KW-3+4	4916
KW-45A+2B	4339
KW-2B+2C	4048
KW-2C+32	3906
KW-38A+32	3650
KW-46	3540
KW-35	3400
KW-31	3342
1AQ	3127
	3027
Pyrex	2967
	2925
	2893
	2847

The transmittance properties of the 1AQ glass (.75 mm thick, clear Belgium glass) and the pyrex h/e tube envelope are not controlled in production. Also, their transmittance cut-off's, which are in the ultraviolet, are gradual and affected by the thickness of the glass. Therefore, the effective cut-off wavelengths will vary.

Recommended form for data sheet:

Label column headings as described below; each filter will require 2 or 3 lines.

Col. No.	Column Heading	
(1)	Filter Number	
(2-6)	VTVM readings at 1 min. int'vls	volts
(7)	VTVM zero reading	volts
(8)	VTVM reading corrected for zero	volts
(9)	V App. crit. retarding pot.	volts
(10)	f Highest frequency passed	cps

1. The complete apparatus, h/e tube in housing, the VTVM, mercury vapor lamp, and contact potential compensation battery should have been assembled several hours before the experiment is to be performed. See Figure (5).

The VTVM should have been turned on at least an hour before and the mercury vapor lamp at least 15 minutes before. This procedure will avoid spurious readings due to static charges and thermal emf's, reduce the zero-drift of the VTVM (which would make it difficult to decide that a final steady reading had been reached), and allow the mercury vapor lamp to reach its maximum intensity.

Do not look directly at the mercury vapor lamp. The ultraviolet it emits is harmful to the eyes.

Check the wiring to see that it agrees with the circuit diagram of Figure (4). The collector of the h/e tube is connected directly to the input terminal of the VTVM inside the (light and electrostatic) shield housing the h/e tube. Make certain the h/e tube housing is fixed relative to the VTVM.

2. Select the "negative" polarity of the VTVM as indicated in Figure (4). For the HE-03 VTVM, this is accomplished by holding down the "shorting switch" while the needle is adjusted to zero at the right hand side of the scale. If the VTVM has current shunts, make certain the shunt selector is at OPEN.

3. With slide No. 22 - AIR inserted between the mercury vapor lamp and the h/e tube, adjust the contact potential compensation potentiometer on the h/e tube housing to obtain a steady reading (wait several minutes) between 2.8 and 2.9 volts on the VTVM.

4. To make certain that all static charges have been dissipated, insert slide No. 23 - OPAQUE in place. Press down the VTVM shorting switch and release gently. Note that the slight strain put on the teflon insulation of the shorting switch will usually cause a static charge resulting in approximately a ± .05 volt reading when the switch is released. This static charge disappears in about a minute.

Movement of the needle either to the left or rapidly to the right means that the apparatus is NOT READY for performance of the experiment.

If all the static charges have disappeared, the needle, due to the grid-drift current of the VTVM, will move very slowly (1 division in 20 secs, or slower) to the right until it goes off scale. It is necessary to wait until this condition is obtained before proceeding. Waiting for all the static charges to disappear can be accelerated by holding down the shorting switch for 1/2 to 1 minute intervals, repeating several times.

5. Slide the first filter, slide No. 24 - KW-22+70, in place. However, as only a few of the h/e tubes respond for this filter, it may be desirable to start with filter No. KW-22+23A.

6. Hold down the shorting switch of the VTVM, adjust the zero reading, and release. Read and record the VTVM readings about every minute until the VTVM reading is constant, i.e. does not change for 3 consecutive readings. Read to the nearest 1/10 of the smallest scale division, .005 volts.

7. Again press the shorting switch of the VTVM. Read and record the zero reading.

8. In turn, repeat Procedures 5, 6, and 7 with each succeeding filter and finally with slide No. 36 - AIR (pyrex glass envelope alone).

(a) For any filter it can be shown that the VTVM reading does not increase indefinitely but does approach a limit. To show this, slide AIR in slightly until the VTVM reads several divisions beyond the limit reached in Procedure 6. Now reposition the filter and notice that the VTVM readings decrease very slowly, approaching the same limit as in Procedure 6.

(b) The filters should be used in the sequence listed above. Reversing any part of the sequence will not alter the final results, but reaching the final, steady limit of Procedure 6 will be delayed due to the excess of space charge electrons between the collector and the emitter.

9. Leave the VTVM and the mercury lamp ON. The laboratory assistant will cut the power if another laboratory section does not follow.

ANALYSIS OF DATA.

1. For each of the final, constant voltage readings, correct the VTVM reading for its zero reading and for the meter calibration (use calibration chart supplied with the VTVM). This will give the apparent critical retarding potential V.

2. From the value of the shortest wavelength of the mercury spectrum transmitted by each filter, compute the corresponding highest frequency f of the transmitted light.

3. Plot a graph of V versus f. Knowing that the slope of the line will be useful, select scale factors (consistent with good practice) to obtain a slope of roughly 45°. Remember that the f values for the 1AQ filter and the pyrex envelope are not known accurately and, at best, should be plotted at several values of frequency.

Fit the best straight line to the plotted points, keeping in mind several factors.

(a) With the KW-22+70, or even the KW-22+23A filter, photoelectron emission may be very low. This could result in values of V which are too small.

(b) In manufacturing the h/e tube every effort is made to avoid depositing the emitting material anywhere but on the emitter. However, if even a very slight amount did reach the collector, this could result at the highest frequencies in values of V which are too small.

4. From the slope of the graph and the fact that the elementary electron charge is $e = 1.6021 \times 10^{-19}$ coulombs, determine h.

5. From the steepest and the flattest but still acceptable straight lines through the plotted points, determine an estimated deviation for your value of h.

CONCLUSION.

State your value of h with its estimated deviation.

DISCUSSION OF RESULTS.

State whether or not the range for your value of h includes the accepted value. Cite the reference for the accepted value, with its estimated deviation if available.

State whether or not any of the factors mentioned in Analysis of Data 3 were apparent and were allowed for.

If the leakage resistance (even a resistance of 10^{12} ohms across the h/e tube is a very low value that would seriously affect the experimental results) is too low due to excessive humidity or other cause, the straight line through the plotted points would be flatter. Do the results seem to indicate that the leakage resistance was too low?

MASS SPECTROMETER

University of California, Berkeley

INTRODUCTION

General Principles

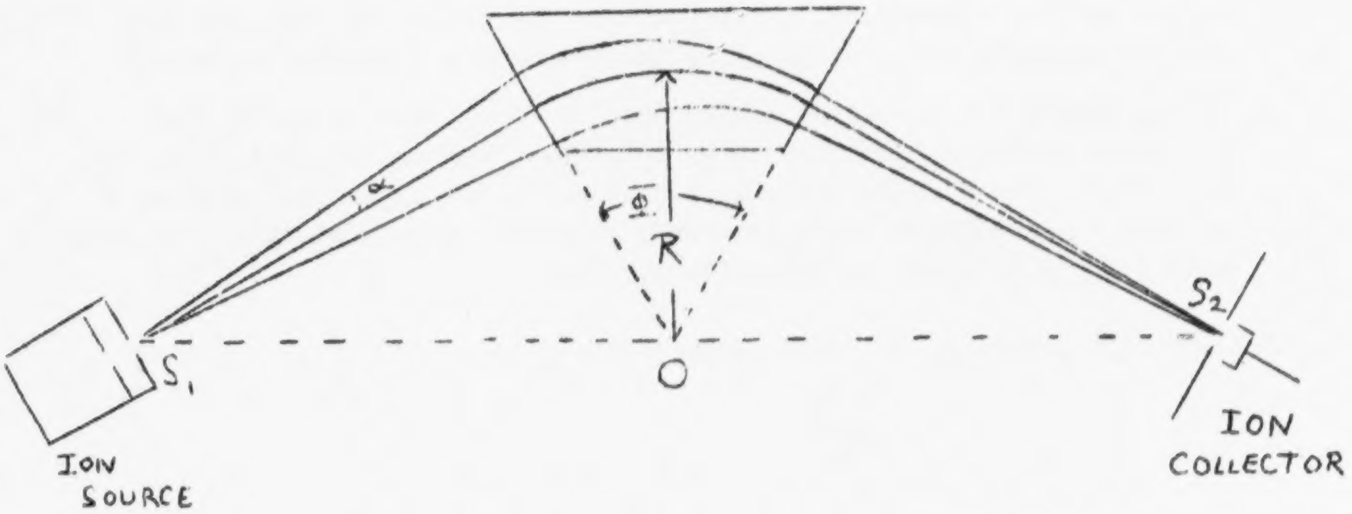


Fig. 1

The mass spectrometer used in this experiment consists basically of:

(a) an ion source which emits positive ions in a beam collimated by the pair of slits S_1. The ions are mostly singly charged (i.e., lacking one electron) although doubly charged ions also occur in small abundance. Triply and higher charged ions are very rare. The ions have been accelerated through a potential drop V in the ion source.

(b) a wedge-shaped magnetic field perpendicular to the paper and with apex at O. (See fig 1). Suppose that this field is of such a strength that an ion of mass M, and energy eV, and entering the field at right angles to the pole boundary travels on a circle of radius R with center at O and emerges from the field also perpendicular to the field boundary (central ray in Fig. 1). Now consider a particle which leaves the source on a trajectory somewhat higher than the ray we just considered - - but again having energy eV. Upon entering the field this particle moves in a circle again of radius R, but not centered at O. Because this particle has a longer path in the field, it is deflected through a larger angle and upon leaving the field intersects the primary ray again at S_2. A particle leaving the source on a lower trajectory spends less time in the field, is thus bent through a smaller angle and also intersects the primary ray. Detailed theory shows that there is a first-order focus at S_2 - - that is the width at S_2 of a pattern resulting from monoenergetic ions diverging from a line source at S_1 has no zero or first order terms when expanded in a power series of α, the half angle of divergence from the source. Theory also shows that the points S_1 , 0, and S_2 lie on a straight line. The particular instrument used in this experiment has a wedge angle, Φ , of 60° and has the source and collector arranged symmetrically with respect to the field, but the focussing principle holds for any angle and for unsymmetrical as well as symmetrical arrangements. The technical name for this type of analyzer is a "single direction-focussing magnetic analyzer."

(c) a cup-shaped ion collector behind the slit S_2. There is a 45 volt potential difference maintained by a battery between the slit plate and collector. The purpose of this is similar to the suppressor grid in a pentode - - namely to repel secondary electrons (produced by ion impacts within the cup) back to the collector cup.

The equation governing the ions in focus at S_2 is:

$$\frac{M}{Z} = 4.8 \times 10^{-5} \frac{B^2R^2}{V}$$

where M = mass in atomic mass units
Z = charge of ion in electron charges
B = magnetic field in gauss
R = radius is cm.
V = accelerating voltage in volts
$4.8 \times 10^{-5} = 300\, e\, N_o/2c^2$ where N_o = Avogadro's number.

The Vacuum and Sample System

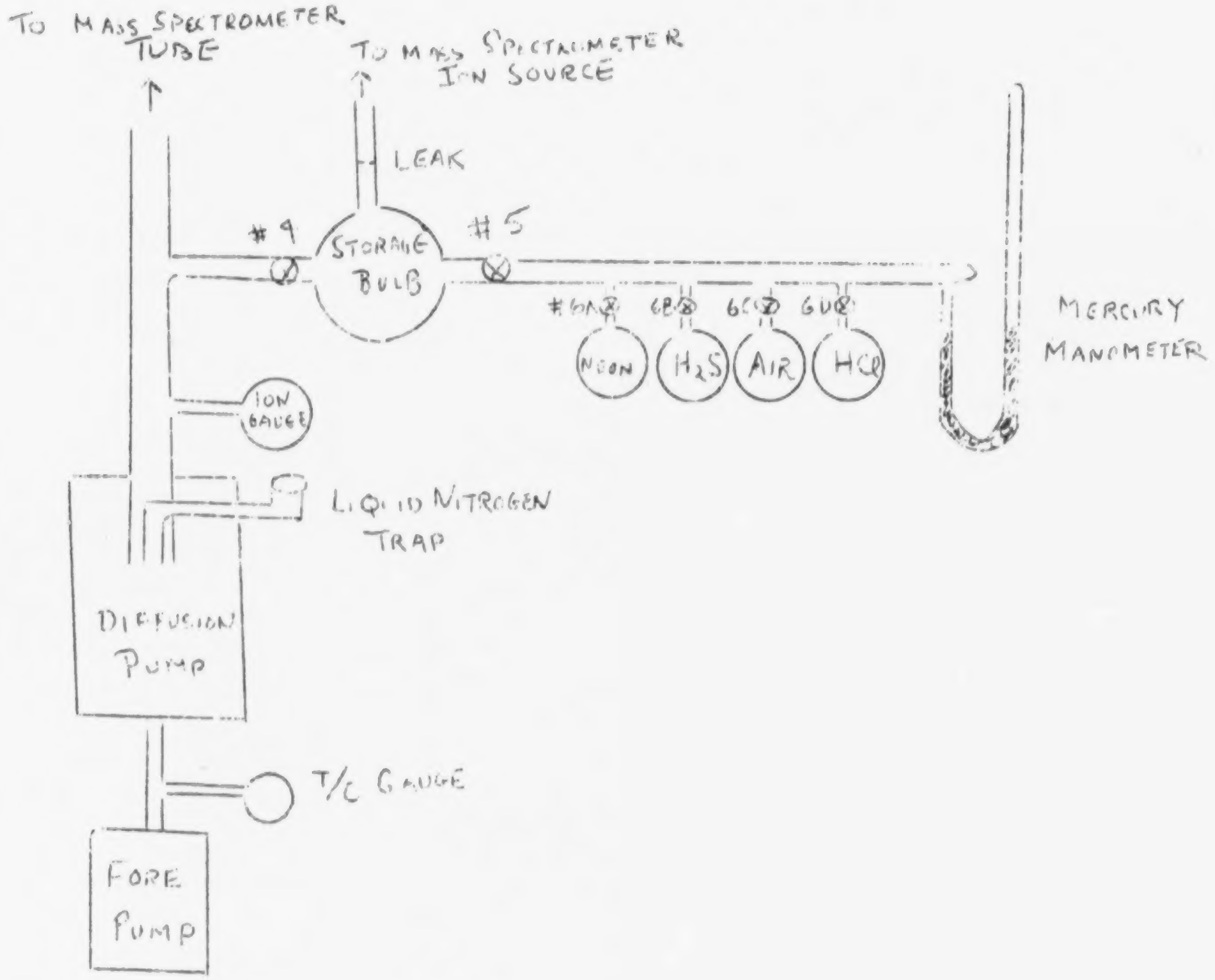

TO MASS SPECTROMETER TUBE
TO MASS SPECTROMETER ION SOURCE
LEAK
# 4
STORAGE BULB
# 5
#6A
6B
6C
6D
NEON
H_2S
AIR
HCl
MERCURY MANOMETER
ION GAUGE
LIQUID NITROGEN TRAP
DIFFUSION PUMP
T/C GAUGE
FORE PUMP

Fig. 2

The vacuum and sample system is shown in Fig. 2. (Stopcocks which the student will not need to operate are not shown). The vacuum system consists of a mechanical fore pump backing an oil diffusive pump which is electrically heated and water cooled. Thermocouple and ion gauges are provided to read fore-vacuum and high-vacuum pressures respectively. A liquid nitrogen trap condenses out vapors. Gas samples are transferred from the bulbs to the storage bulb by manipulating stopcocks no. 6 and no. 5. A tiny hole in a piece of thin glass serves as a leak between the storage bulb and ion source. The storage bulb and manifold can be pumped out when measurements are completed by opening stopcocks no. 4 and no. 5.

The Ion Source

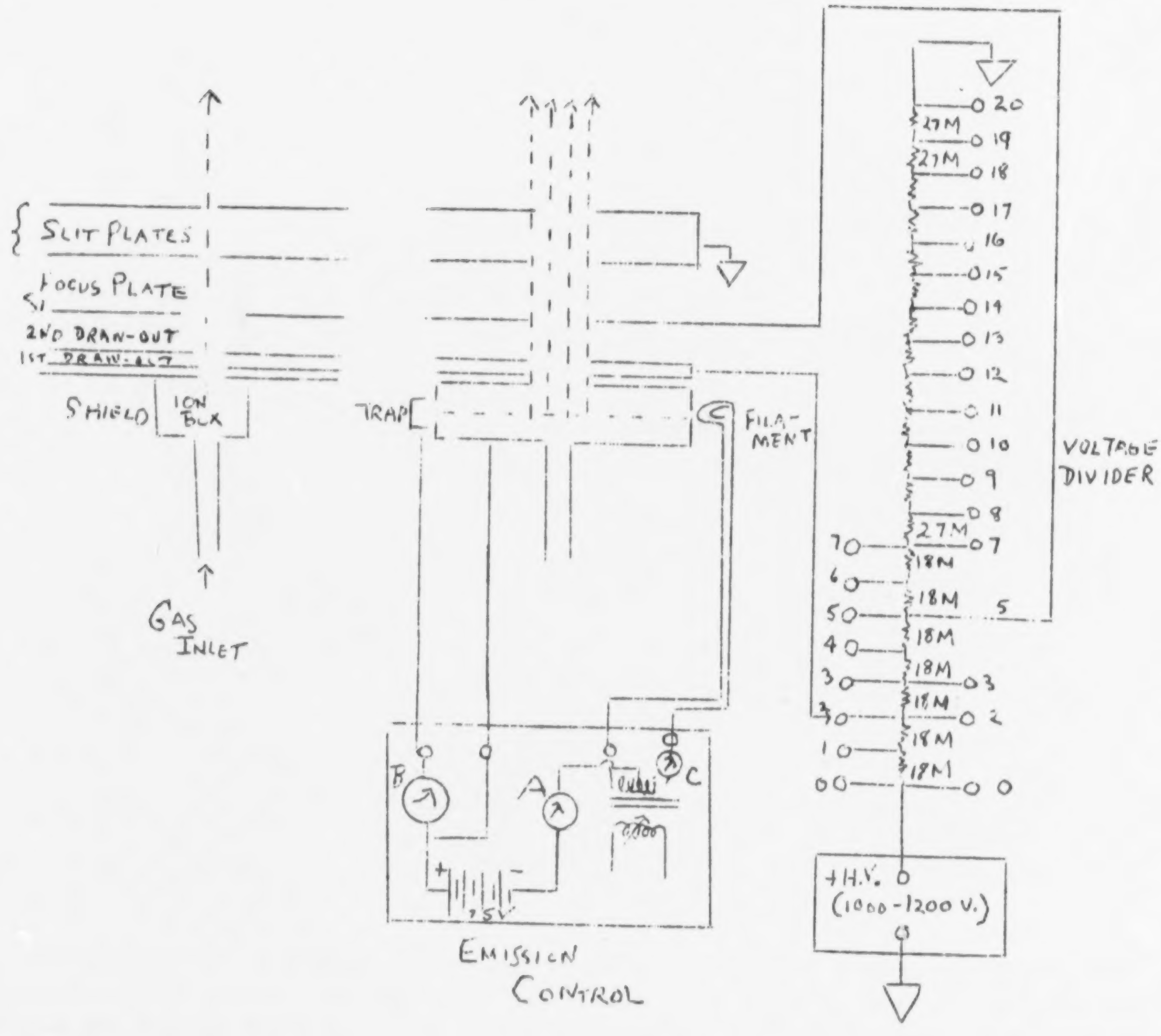

SLIT PLATES
FOCUS PLATE
2ND DRAW-OUT
1ST DRAW-OUT
SHIELD
ION BOX
GAS INLET
TRAP
FILA-MENT
VOLTAGE DIVIDER
27M
18M
20
19
18
17
16
15
14
13
12
11
10
9
8
7
5
3
2
0
70
60
50
40
30
20
10
00
B
A
C
75V
+H.V.
(1000-1200 V.)
EMISSION CONTROL

Fig. 3

The emission control heats the filament and provides a 75 volt acceleration potential for electrons emitted from the filament. Meter A reads the total electron emission. A small fraction of the electrons reach the trap. This current is measured on meter B. Meter C measures the A.C. filament heating current. (A new emission control is under construction. When installed this will electronically stabilize the electron emission and the meters will be arranged somewhat differently).

Positive ions produced in the ion box are withdrawn by a field, due to the drawing out plates, which penetrates slightly through the slit into the box. The focus plates, when adjusted to the proper potential, produce a focus of the ion beam at the position of the slit plates. The potentials of the drawing out and focus plates can be adjusted empirically for maximum beam current by **tap switches** on the 477 megohm voltage divider. The high voltage supply is adjustable from 1000 to 1200 volts.

The Ion Detector

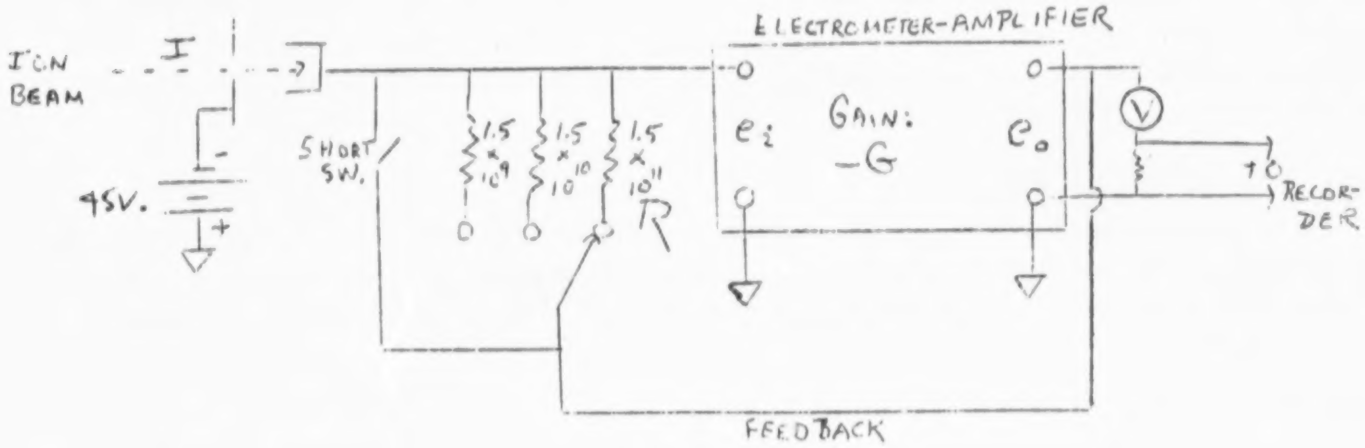


Fig. 4

The ion detector circuit is shown in Fig. 4. Ions are detected with a 100% feedback electrometer. If the ion current is I, the voltage input to the amplifier is:

$$e_i = IR + e_o = \frac{e_o}{-G}$$

Solving:

$$e_o = \frac{-\ IR}{1 + 1/G} \approx -\ IR \qquad \text{since G is a large number such as 1000.}$$

Thus the electrometer has an effective gain of unity. Its main function is that of an impedance-changer, developing the voltage IR across a low impedance (that of the voltmeter) instead of the extremely high impedance of the input resistor. A selector switch enables any of three input resistors to be used. (A shorting switch is also provided for testing the electrometer). The "x1" range of the voltmeter is 90 millivolts full scale on the 30 μ amp meter. Less sensitive ranges up to x 100 are provided. With most ion detectors it is usually best to use the lowest R feasible and operate on the more sensitive voltage scales. In this electrometer, however, the more sensitive voltage scales are noisy. Thus students will probably get best results by selecting an input resistor which puts the largest peak on the x 100 scale. When desired, the output of the meter can be recorded on a Varian self-balancing potentiometer recorder which is matched to the meter scale.

The Magnet

The magnet is an electromagnet operated from a 220 volt D.C. generator in the basement. A current regulator is in series with the generator and magnet coils so that the magnet current passes through a group of 6 AS7 vacuum tubes in parallel. Within the current regulator, a comparator circuit compares a voltage proportional to the magnet current with a fraction (the fraction determined by the setting of a ten-turn helipot) of a stabilized reference voltage. The difference between these voltages is amplified and fed to the grids of the 6AS7's. The action of the circuit is to stabilize the magnet current at the value for which the difference voltage is zero. Consequently the current can be varied smoothly by varying the helipot setting. The available range is 0 to 2 amperes. Sweep motors of 1/4, 1/2, and 1 RPM are available to sweep the helipot dial through a reduction gear and thus to sweep the mass spectrum.

The High Voltage Supply

The high voltage supply is also electronically stabilized. The range is 1000 to 1200 volts. There is a single sweep motor which sweeps this voltage range in about four minutes, thus providing an alternative method of sweeping the mass spectrum.

Operating Instructions

It is important to start-up and shut-down the apparatus in the proper sequence Start-up instructions follow. Shut-down instructions will be found at the end of the write-up.

I. Vacuum preparations

(See the teaching assistant. Some of this may have been started for you ahead of time).

Start the fore-pump after making sure that all stopcocks which vent the system to the atmosphere are closed. Open stopcocks no. 4 and no. 5. Turn on electrometer power, after making certain that range-switch on chassis is in "OFF" position. (This is because electrometer needs to be well warmed-up before it is stable). Check the fore pressure with the thermocouple gauge. If below 100 microns, turn on the diffusion pump cooling water and then turn on the diffusion pump.

Progress of the pumping can be followed by occasional use of the Tessla coil on the glass manifold. As pressure decreases from atmospheric pressure (no glow), the Tessla discharge will change from strong purple (about 1 cm Hg) to light purple (about 1 mm Hg) to soft blue (less than 100 μ or 10^{-1} mm) to no glow except for fluorescence of the glass (below 10^{-4} mm). After diffusion pump has been on long enough to produce these changes, attempt to start the ion gauge. If system is tight, ion gauge will soon indicate a pressure below 10^{-4} mm. Pour liquid nitrogen into trap. Pressure should soon fall below 10^{-5} mm.

II. Ion source preparations

When ion gauge is operating and pressure is below 10^{-5} mm, ion source may be operated. (There is an interlock which prevents the ion source filament from being turned on unless the ion gauge is operating). The emission control has a knob and a switch on the panel. (These remarks refer to the old emission control. When the new control is installed, the student will have to consult supplementary instructions for this control). Turn knob all the way counter-clockwise. Now turn on switch. The A.C. current meter should read, but emission (upper right meter) will be zero. Turn knob clockwise until emission reaches 0.2 ma. Later the student may wish to lower this current to adjust spectrometer sensitivity. There are two knobs on the voltage divider panel. Settings of draw-out-2 (right switch) and focus-5 (left switch) should be appropriate (see Fig. 3) and are recommended as initial settings. The high voltage supply can now be turned on.

WARNING: With high voltage on, never touch bare terminals on ion source or voltage divider panel. Never touch circuit elements inside emission control circuit.

To turn on high voltage, put "D.C. High Voltage" switch in "off" position and turn on "A.C. Power." After one minute of warm-up, "D.C. High Voltage" switch may be turned on. A warning light should light up near ion source. Magnitude of high voltage is 1000 volts plus reading of panel meter on high-voltage supply (200 volts full scale). Leave "Fine voltage control" dial at zero since it is not needed.

IMPORTANT RULE: Never operate any stopcock on this apparatus without first switching off emission switch on emission panel and D.C. high voltage switch on high voltage supply. This is the "stop-cock rule". Failure to observe this rule may lead to filament burn-out!

III. Magnet Operation

The magnet must be turned on properly to prevent possible damage to the windings or to the control circuit.

Turn helipot knob (knob with arrow) all the way to left. Turn "Ext 220 v. D.C." switch off. Turn "Inst. power" switch "ON." After 1 minute warm up, turn "Ext. 220 v. D.C." switch "ON." (It may be necessary to turn on Safety Switch no. 5 on bench also). Now magnet current can be controlled with helipot knob.

IV. Sample system operation

To admit a gas sample to the ion source, proceed as follows:

Don't forget "stop-cock rule!"

Close stopcork no. 5.

Cautiously "crack-open" one of the no. 6 stopcocks - - all the while watching manometer - - until manometer begins to rise slowly. Close stopcock in question when manometer indicates about 5 cm pressure.

Close stopcock no. 4. Open stopcock no. 5. Close stopcock no. 5. Manometer pressure should now be about 0.5 cm due to expansion of gas into storage bulb. This is an appropriate sample pressure. The ion gauge pressure will probably rise to about 10^{-4} mm with sample in.

To remove a gas sample from the ion source:

Don't forget "stopcock rule!"

Open stopcock no. 4. Open stopcock no. 5. Allow 10 minutes for pump out.

Note: When using H_2S or $HC\ell$ samples, much of the sample condenses on the liquid nitrogen trap. It is important to keep this trap cold with these gases. Otherwise enormous amount of vapor may be released from the trap very abruptly when it warms up.

V. Ion detector operation

With shorting switch (on mass spectrometer housing) in unshorted position and with range switch (also on mass spectrometer housing) in $1.5 \times 10''$ ohm position, begin to increase electrometer sensitivity with sensitivity switch on chassis. Zero as may be necessary with coarse and fine controls. Ion currents should now be observed intermittently on meter when magnet current is increased slowly.

The Varian recorder will record the meter current when it is connected. "Power" switch on recorder can be left on throughout the laboratory period. "Servo" switch should be on when recorder is to be operated or zeroed. "Paper" switch should be on "high" when spectra are being recorded. Recorder should read the same as does meter. If it does not, see teaching assistant. Recorder may need a new mercury cell.

Experimental Procedure

Required work:

Admit air sample.

Find highest peak. This should be mass 28. Verify as follows: With magnet current constant, we have $M_1V_1 = M_2V_2$. See if supposed 28 peak and supposed 29 peak obey this relation.

With spectrometer set on mass 28, now verified, measure V and deflection of ballistic galvanometer when flip coil is suddenly pulled out of magnetic field. Since deflection, D, is proportional to B, we have:

$$M = \frac{K D^2}{V} \quad .$$

Calculate K . Now it is possible to determine the mass of any observed peak.

Remove air sample.

Admit neon sample. Determine mass of two major isotopes (should be 20 and 22).

Make six or eight **manual** (i.e. without recorder) determinations of the 22/20 ratio.

Using recorder, record a sweep of the neon spectrum for each lab partner.

Optional Work: (any of the following "projects")

1. Using recorder make a careful recording of the mass spectrometer background from mass 12 to mass 50. Use the maximum reasonable sensitivity for this work. Pay special attention to mass 21. H_2O peaks (16, 17, 18) should be noticeable in background.

2. Sweep the 21, 22 portion of the neon spectrum at high sensitivity. Making any background corrections needed, calculate abundance of Ne^{21}.

3. Admit an $HC\ell$ sample. $C\ell^+$ and $HC\ell^+$ ions will be observed. Use either or both to determine $C\ell$ isotope abundances.

4. Admit an H_2S sample. S^+, HS^+, and H_2S^+ ions will be observed. See if you can unravel the resulting complex spectrum to determine S isotope abundances.

Questions

1. The hole in the leak is about .001" in diameter. Is the gas flow through this leak effusive flow or viscous flow?
2. The flow of gas out of the ion box is effusive flow. How does the relative density of isotopes in the ion box compare with the relative density in the storage bulb? How does the isotopic composition of the gas in the storage bulb vary with time? Make a quantitative calculation of this effect, if any, for neon after 1 hour.
3. Estimate the over-all mass spectrometer efficiency for the 28 peak. Assume that 50% of the ion source electrons enter the ion box and travel 1 cm there. Take the ionizing cross section to be 10^{-16} cm^2. Suppose that the area through which gas leaves the ion box is 0.2 cm^2.
4. Why are there so amy ion peaks in the background of this instrument? How might the background be reduced?

References

M.G. Inghram and R.I. Hayden, "A Handbook on Mass Spectroscopy," N.A.S./N.R.C. publication 311, Nuclear Science Series Report No. 14. (1954)

M.G. Inghram, "Modern Mass Spectroscopy", in "Advances in Electronics Vol I," Academic Press, New York City (1948).

"Mass Spectroscopy in Physics Research", National Bureau of Standards Circular 522 (1953).

H. Ewald and H. Hintenberger, "Methoden und Anwendungen der Massenspektroscopie", Verlag Chemie, Weinheim (1953).

G.P. Barnard, "Modern mass spectrometry," Institute of Physics, London (1953).

H. Ewald, "Massenspektroskopische Apparate," in Handbuch der Physik, Springer-Verlag, Berlin (1956).

Shut-Down Procedure

Turn off magnet as follows: (Sequence is important.)
Turn heliput all the way down.
Turn off D.C.
Turn off A.C.

Turn off high voltage, emission control, electrometer (first putting sensitivity on "Off" position), recorder (all three switches), ion gauge, diffusion pump.

Open stopcocks no. 4 and no. 5.

Leave cooling water on until diffusion pump has been off at least 20 minutes.

THE SINGLE CRYSTAL X-RAY SPECTROMETER

University of Pennsylvania

The spectrum of X-rays consists of a continuous spectrum on which are superimposed sharp lines. The continuous spectrum extends all the way from very long wavelengths up to a fairly well defined short wavelength limit.

From Planck's equation

$$E = h\nu = \frac{hc}{\lambda} \qquad (1)$$

$$\lambda_{min} = \frac{hc}{E_{max}} \qquad (2)$$

since h and c are constants, the quantum of X-radiation is a maximum when it equals the kinetic energy of the electrons bombarding the target.

$$E_{max} = K.E. = eV \qquad (3)$$

and

$$\lambda_{min} = \frac{hc}{eV} = \frac{1.24 \times 10^{-6}}{V}\, m = \frac{12400}{V} \times 10^{-10}\, m \qquad (4)$$

Thus the short wavelength limit depends upon the voltage accelerating the electrons which produce the X-rays. The character of the continuous spectrum is dependent only on the voltage across the X-ray tube. The material of the target influences the intensity of the continuous spectrum at a given voltage, but not its intensity distribution on a wavelength scale.

The line X-ray spectrum of an element used as the target in an X-ray tube is typical of that element, just as the optical spectrum of a substance identifies it. Because of similarities which exist among X-ray spectra of different elements, there is identified in each X-ray spectrum a group of lines called the K series, another called the L series, and M series, etc. The K series lines are the hardest (shortest wavelength, greatest frequency, greatest energy) and the series are named in order of increasing wavelength. In this experiment we shall study the continuous spectrum and the lines of the K series.

The X-ray tubes used are of the Coolidge type having a heated tungsten filament as the source of electrons, and a water cooled target. There are several different metals used as target materials such as copper, molybdenum, and tungsten. Find out from the instructor what kind of target is being used. The tubes are shielded in such a way that X-rays can escape in only two directions through holes in the shielding. Be sure to cover one of these holes with a lead diaphragm if only one side of the tube is being used. The high voltage is supplied from a step-up transformer and is rectified by a valve tube.

The spectrometer contains two collimating slits which define the X-rays into a sharp beam. This beam falls upon a single large crystal and some of the X-ray photons are diffracted in accordance with the Bragg equation

$$n\lambda = 2d \sin \theta \qquad (5)$$

(See section on Theory of X-Ray Diffraction)

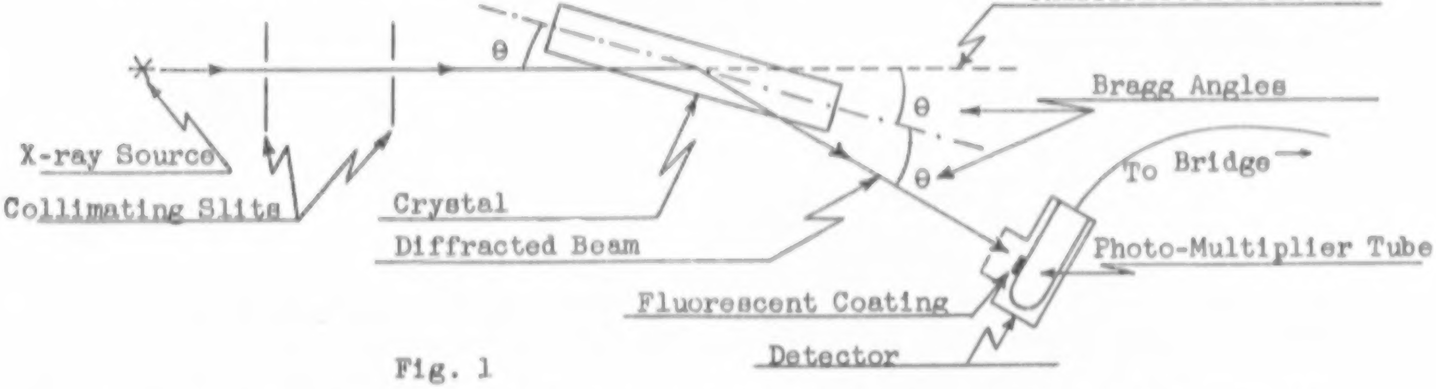

Undiffracted Beam
θ
Bragg Angles
θ
θ
X-ray Source
To Bridge
Collimating Slits
Crystal
Diffracted Beam
Photo-Multiplier Tube
Fluorescent Coating
Detector

Fig. 1

From Fig. 1 it is apparent that in order to detect any diffracted X-rays the detector must move about the axis of the rotating crystal through an angle 2 θ when the crystal moves through an angle θ. The spectrometer is fitted with gears to accomplish this after the position of the crystal has once been properly adjusted relative to the detector. The instructor will have made this adjustment (a very tedious one) and it should not be changed. The detection of the X-rays is achieved in the following way. The diffracted X-rays enter the slit of the detector and strike a fluorescent coating on a photo-multiplier tube. A cross section of such a tube is shown in Fig. 2. The visible light emitted by the fluorescent coating enters the photo-multiplier tube and ejects electrons from the photocathode. The electrons are then attracted from dynode to dynode with an increase of the number of electrons at each surface.

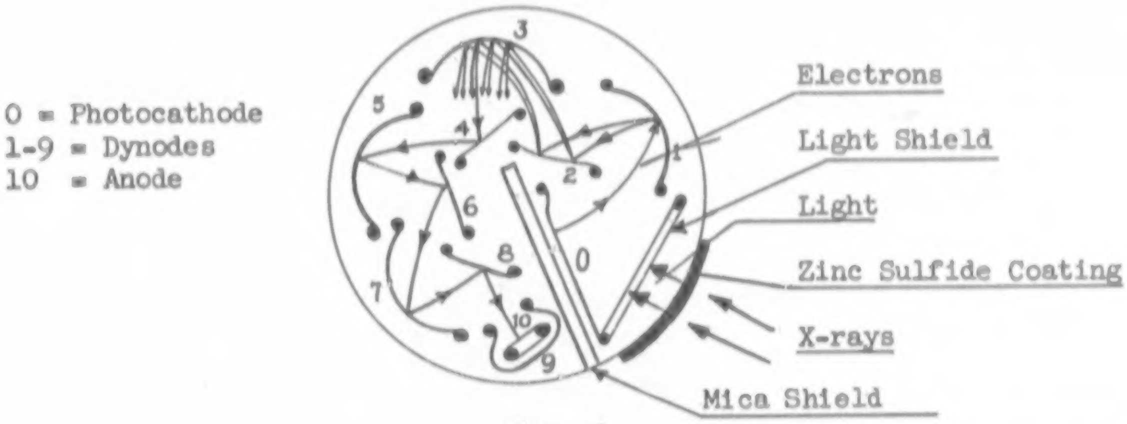

0 = Photocathode
1-9 = Dynodes
10 = Anode
3
5
4
1
2
6
0
8
7
10
9
Electrons
Light Shield
Light
Zinc Sulfide Coating
X-rays
Mica Shield

Fig. 2

The small output of the multiplier is then amplified.

WARNING: KEEP BEHIND SHIELDING WHEN X-RAY BEAM IS ON.

Procedure:

(a) Make certain that the water supply is turned on. Throw the main switch to "High." Set the kilovoltage on the KV meter to 40 KV. Warning: Turn main switch off while changing KV selector to prevent damage to the selector. Set the timer switch to 2 hrs. or more. X-rays will be produced when the key switch is turned on.

(b) Turn on amplifier and high voltage supply for photo-multiplier. Zero the meter after it has warmed up.

(c) Investigate the manner in which the crystal and detector are rotated and notice the ratio of the gears. In order to determine the zero position, the crystal is removed and the meter deflection is recorded every 6' over a short range on both sides of the central peak. It will be necessary to use very low sensitivity positions on the amplifier and reduce the filament current of the X-ray tube from the usual operating current of 13 ma. to get on scale deflection of the meter from the very intense direct beam.

(d) Replace crystal in holder and align it by eye, roughly parallel to the beam direction. Rotate the band wheel about 3 revolutions clockwise. Switch on beam, set detector to its most sensitive position. Tap brass crystal holder lightly with the back end of a screwdriver swinging from above by holding the blade between thumb and forefinger. Watch detector carefully while tapping. Tap to maximum signal. If no signal comes after much tapping, go back the other way.

(e) Starting on one side of the central position (where) the undiffracted beam is detected), rotate the crystal and detector, taking readings of amplifier for each ½ degree of chamber angle. In the region of the sharp peaks, take readings every 6' of chamber angle. Investigate the region over about 45° of chamber angle from the central position.

Report:

(1) Make a plot of the points taken without the crystal. Determine the zero position by finding the center of the peak. Please note that the center of the peak is not necessarily the point of strongest beam.

(2) Make a plot of the intensity as a function of chamber angle over the complete 45° range.

(3) Make an enlarged plot of the two peaks for a careful determination of the maxima. With the angular position of the spectral lines thus determined, compute their wavelength using the Bragg relation. (Eqn. 5) The grating space d for calcite is 3.03×10^{-10} m and for rock salt is 2.814×10^{-10}. Find out the type of target used in your X-ray equipment.

Compare your results with the accepted value. Note that the equipment does not resolve the $K\alpha_1$ and the $K\alpha_2$, etc. line but gives an average of $K\alpha$ and $K\beta$.

CHARACTERISTIC X-RAY SPECTRA

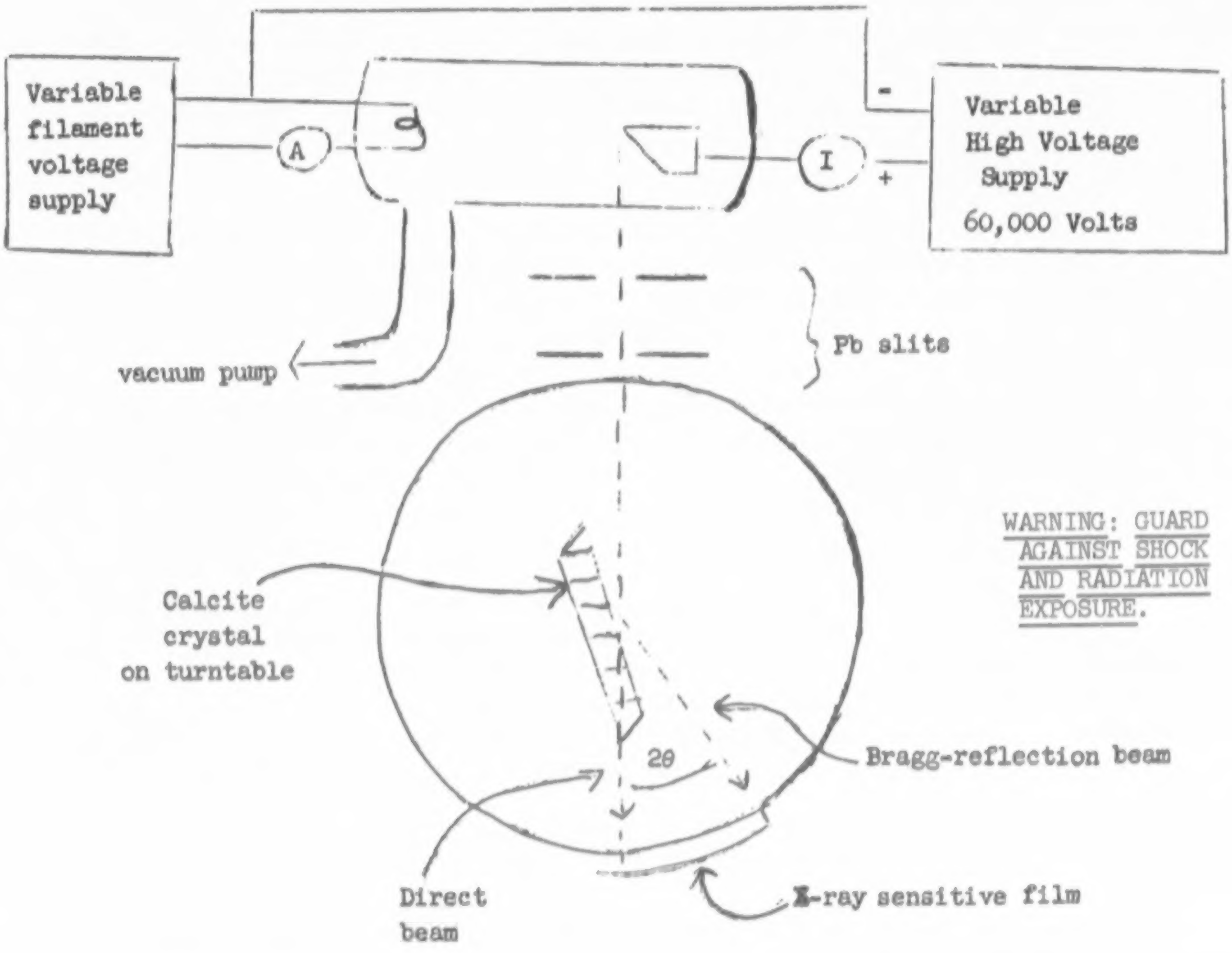
University of California, Berkeley

Reference: Harnwell and Livingdon, *Experimental Atomic Physics*, pp. 324-368
Richtmeyer and Kennard

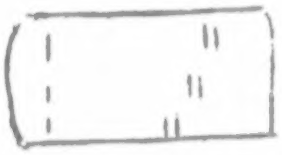
Apparatus: An X-ray tube with a rotatable anode, providing six different targets:
Fe, Co, Ni, Cu, Mo, and Ag;
A Bragg spectrometer using a rocking calcite crystal;
Kodak Super Speed X-ray film (occlusal code DF-45);
Power supplier and vacuum pumps.

Purpose: To measure the characteristic X-ray wavelengths for several elements and verify Moseley's Law.

Method: Electrons from a hot filament are accelerated by a high voltage towards the anode target, thus producing the characteristic X-rays of the element of which the target is composed, as well as a continuous X-ray spectrum. The X-rays are


Variable filament voltage supply
A
I
-
+
Variable High Voltage Supply
60,000 Volts
Pb slits
vacuum pump
Calcite crystal on turntable
2θ
Bragg-reflection beam
Direct beam
X-ray sensitive film

WARNING: GUARD AGAINST SHOCK AND RADIATION EXPOSURE.



developed film showing blackened lines due to the direct beam and to the K and K_o lines collimated (see Figure) by lead slits and strike a calcite crystal, mounted on a motor-driven turntable so as to periodically vary the angle of incidence. X-ray wavelengths which satisfy the Bragg relation $2d \sin\theta = n \lambda$ will be reflected at an angle 2θ from the direction of the incident (direct) beam and will strike a photographic film, thus producing a record of the X-ray spectrum. In the present apparatus only the first order ($n = 1$) spectrum is recorded, and furthermore only the K_α and the K_β X-ray spectral lines are recorded. The K_{α_1} and K_{α_2} lines are not resolved, nor are the K_{β_1} and K_{β_2} lines The developed film is measured with a comparator and the wavelengths are calculated.

Procedure:

A high vacuum should be established in the X-ray tube before turning on either the filament voltage or the high (anode) voltage. The vacuum will be provided by (1) a mechanical fore pump (which is left running continuously day and night) and (2) a water-cooled oil diffusion pump, which will normally be turned on by the laboratory assistants a few hours before the laboratory session. The fore-pump vacuum should be better than 50 microns before the diffusion pumps are turned on. Allow the diffusion pumps to pump down to a vacuum better than 10^{-5} mm Hg, as measured by the ion gauge connected to the X-ray tube. All tube voltages must be off and the housing raised when making this measurement. Rotate the anode to select the desired target element.

When a satisfactory vacuum has been obtained, disconnect the ion gauge meter and any other auxiliary connections to the X-ray tube and close the housing. Turn on the X-ray filament slowly and increase the current A to about 1.5 amps. Turn on the high (anode) voltage slowly until the primary voltmeter reads about 70 volts. Readjust the filament voltage until the anode current I is about 15 ma. Use a piece of florescent screen to verify that X-rays are coming out and cover the slit with the lead shutter. Calculate the crystal angle required to record the first order K_α and K_β lines of the target element and set the turntable to be varied about this angle by the large cam.

Mount a piece of X-ray film so that both the direct beam and the Bragg reflected beams will be incident on it. It is most convenient to use Kodak super speed occlusal (Code DF-45) X-ray film, which is already wrapped in paper, and, furthermore, contains two films so that each partner may have a record. The sensitive side of the film package is the embossed side; note the embossed dot for orientation purposes. Note that by displacing the film vertically it should be possible to record the spectra of several elements on one film package.

Remove the lead shutter and expose the film while the motor rocks the crystal. An exposure between 5 min. to 20 min. is usually sufficient. Close the shutter and carefully lift out the entire turntable; open the shutter and expose for the direct beam for about 1/2 minute. Turn off the tube voltages, open the housing and ground the anode with a clip wire for safety. Rotate the anode to a new target element and repeat the above procedure, thus recording the spectra of, say, 3 elements on a film package. Develop and fix this film before proceeding to the other 3 elements. Measure the films, calculate the wavelengths and make a Moseley plot($\sqrt{v}$ vs. Z) for both K_α and K_β lines. In the report describe the operation of an ion gauge and a thermocouple gauge.

CAUTION. Use suitable precautions against the hazards of electrical shock and X-radiation. Keep the lead shutter closed except during actual exposure. Do not leave the apparatus unattended during exposures, but avoid bodily exposure to the scattered X-radiation.

ABSORPTION OF X-RAYS

University of Pennsylvania

When a parallel beam of X-rays passes through matter the intensity of the emergent beam is less than that of the incident beam. The decrease in intensity in traveling through a thickness d of the material, see Fig. 1, is proportional to the thickness and the intensity I of the beam.

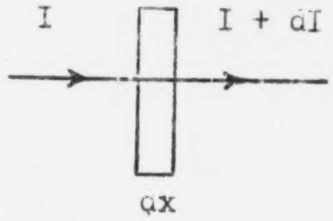


Fig. 1.

This decrease may be written as

$$- dI = \mu I \, dx \qquad (1)$$

where - dI represents the decrease in intensity and μ is the constant of proportionality known as the linear absorption coefficient. Eqn. (1) may be integrated at once to give

$$I = I_0 e^{-\mu x} \qquad (2)$$

where I_0 is the initial intensity of the beam.

There are several ways in which the atoms of a substance can remove energy from the incident beam: Scattering, fluorescent scattering, Compton scattering and pair production. The latter two are more important for energies in the neighborhood of a million volts so that their effects on beams of low energies may be neglected to a first approximation.

Scattering occurs because the transverse electric field associated with the X-ray photons accelerates the "free" electrons in the material. These accelerated electrons in turn radiate photons of the same energy (and frequency) as the exciting radiation. This radiation is scattered in all directions, although not uniformly. Absorption of this type is relatively more important for elements of low atomic number, and measurements of the absorption coefficient can be used to determine the number of electrons per atom (the atomic number).

If the energy of the incident photon is sufficiently large, there is the possibility that it can eject an electron from the atom according to the photo-electric equation

$$\tfrac{1}{2} mv^2 = h\nu - h\nu_0 \qquad (3)$$

where $\frac{1}{2}mv^2$ is the kinetic energy of the ejected electron, $h\nu$ the energy of the incident photon and $h\nu_0$ the energy required to ionize the atom (i.e. remove the electron). This ionized atom readjusts itself with the return of one of its other electrons belonging to an outer shell to the hole left by the ejected electron. The returning electron has a binding energy $h\nu_B < h\nu_0 < h\nu$ and there is a photon born with an energy of $h\nu_0 - h\nu_B$ whose frequency $\nu_0 - \nu_B$ is always less than ν and independent of ν. These scattered X-ray photons are called fluorescent X-rays and their frequency and hence wavelength depends only on the atomic number of the absorbing material. They are identical with the X-rays obtained when the absorber is made the target of an X-ray tube in which the cathode rays have an energy $Ve = h\nu$.

If $h\nu$ is less than $h\nu_0$, there can be no absorption by the photoelectric process for that particular shell. If $h\nu$ is much greater than $h\nu_0$, the actual probability of the ejection of a photoelectron is small even though there is sufficient energy for the process. As in a great many physical phenomena the probability is greatest when $h\nu$ equals $h\nu_0$, and resonance can be said to occur. Like the other type of scattering, the fluorescent radiation takes place in all directions.

The linear absorption coefficient may then be written as the sum of two terms

$$\mu = \sigma + \tau \tag{4}$$

where σ is the scattering coefficient and τ is the fluorescent absorption coefficient. If we consider an X-ray beam 1 m^2 in cross-section, then μ represents the total energy removed from the beam by 1 m^3 of the absorber, σ represents the energy scattered and τ represents the energy transformed into fluorescent radiation by this unit volume. It is sometimes more convenient to use the absorption coefficient per kilogram known as the mass absorption coefficient

$$\mu_m = \frac{\mu}{\rho} = \frac{\sigma}{\rho} + \frac{\tau}{\rho} \tag{5}$$

where ρ is the density of the absorber in kg/m^3; or the absorption coefficient per atom in the path of the beam,

$$\mu_a = m\mu_m \tag{6}$$

where m is the mass of an atom of absorber. Since $m = M/N$, where M is the atomic weight and N is Avogadro's number,

$$\mu_a = \frac{\mu M}{\rho N} \tag{7}$$

It is of fundamental importance for it represents the fraction of the X-rays absorbed by a single atom, when the X-ray beam has a cross-sectional area of 1 m^2.

Procedure:

The X-ray tube, controls, crystal, mounting and detector are the same as those used in Experiment 10, where a diagram of the apparatus and a brief description of the detector may be found. A large crystal is used to provide a monochromatic beam of X-rays.

(a) Make certain that the water supply is turned on. Throw the main switch to "High." Set the kilovoltage on the KV meter to 40 KV. WARNING: Turn main switch off while changing KV selector to prevent damage to the selector. Set the timer switch to 2 hours or more. X-rays will be produced when the key switch is turned on. Regulate the filament current to 13 ma.

(b) Turn on the amplifier and zero. The zero reading should be checked frequently and a series of readings repeated if it is found to have moved.

(c) Set the detector in such a position that the monochromatic beam of X-rays from the crystal is a maximum (cf. Experiment 10).

ALWAYS KEEP OUT OF THE DIRECT X-RAY BEAM.

(d) Record the X-ray intensity transmitted through various aluminum foils. Start with the thinnest and proceed to thicker foils until the intensity has dropped by a factor of about ten. Ten appropriately chosen thicknesses will suffice. Note that all foils of one element must be done on the same sensitivity range.

(e) Repeat for copper, nickel, tin, iron and silver.

Report:

(1) Plot the intensity of the X-ray beam transmitted through the foils divided by I_o, the intensity without an absorber on semilog graph paper against the surface density ρ_s of the foils. The surface density is the product of the density and the thickness of the foil. The points for small values of the surface density fall upon a straight line; put a straight line through these points.

(2) Since $I = I_o e^{-\mu x} = I_o e^{-\frac{\mu}{\rho_s} x \rho_s} = I_o e^{-\mu_m \rho_s}$, then $\ln \frac{I}{I_o} = -\mu_m \rho_s$.

Therefore the slope of the curve is $-\mu_m$. Determine the slope for each element.

(3) Knowing ρ_s, M and N compute the values of μ_a. Plot μ_a against Z, the atomic number of each of the elements. It is expected that $_a$ will be a rapidly increasing function of Z. Explain any discrepancy that you find.

References: Introduction to Modern Physics, Richmyer, Kennard & Lauritsen. Bragg Scattering: Page 357-359; Compton Effect: Page 386-392 (note that RK&L use notation $\phi \leftrightarrow \theta$)

X-Rays in Theory and Experiment, Compton & Allison. Width of Modified Line: Page 243-249.

Outline of the Theory: By considering X-rays as particles interacting with free electrons and by using the conservation of energy and momentum, the following equations can be derived:

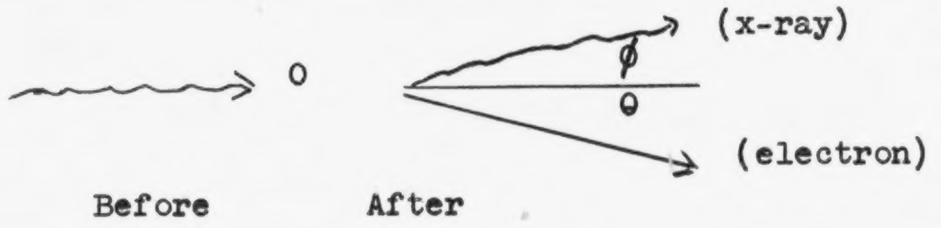


Figure 1 Interaction of X-rays with Free Electrons

$$\lambda_s - \lambda_o = \frac{h}{m_e c} (1 - \cos\phi) \qquad (1.1a)$$

$$\cot\frac{\phi}{2} = -(1 + \alpha)\tan\theta \qquad (1.1b)$$

$$E = \frac{h\nu\, 2\alpha \cos^2\theta}{(1+\alpha)^2 - \alpha^2 \cos^2\theta} \qquad (1.1c)$$

where $\alpha = \frac{h\nu}{m_e c^2}$, and m_e is the rest mass of the electron
E = kinetic energy of electrons.

Action of X-rays on Crystals

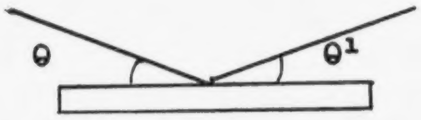


Fig.2 X-rays incident on a crystal

If a heterogeneous source of X-rays strikes a crystal at an angle θ, the interaction obeys the following equations:

$$\theta = \theta^1 \qquad (1.2a)$$

$$n\lambda = 2d\sin\theta \qquad (1.2b)$$

where d is the inter-atomic distance in the crystal. Equation (1.2b) is known as Bragg's Law. These equations say that, for a given incident angle θ, only that radiation of wavelength $\lambda = \frac{2d}{n} \sin \theta$ will be reflected, and that the angle of reflection $\theta^1=\theta$. It is evident that, if the crystal is rotated, the reflected radiation will be of a different wavelength. Consequently this device can be used as a spectrometer. This is commonly referred to as a Bragg spectrometer.

Experimental Setup: The experimental apparatus is shown schematically below. X-rays generated in the X-ray tube leave the tube and are columated by slit S_1. They then hit the target T interacting with its electrons. From this interaction radiation is emitted in all directions. A particular angle of observation ϕ with respect to the main beam is selected. At this angle of observation the spectrum of the scattered radiation is analyzed by means of a Bragg spectrometer (crystal and Geiger counter system). The entire spectrometer pivots about T, while the crystal and Geiger tube pivot about C. A gearing system provides that the crystal moves through θ, while the Geiger tube goes through 2θ.

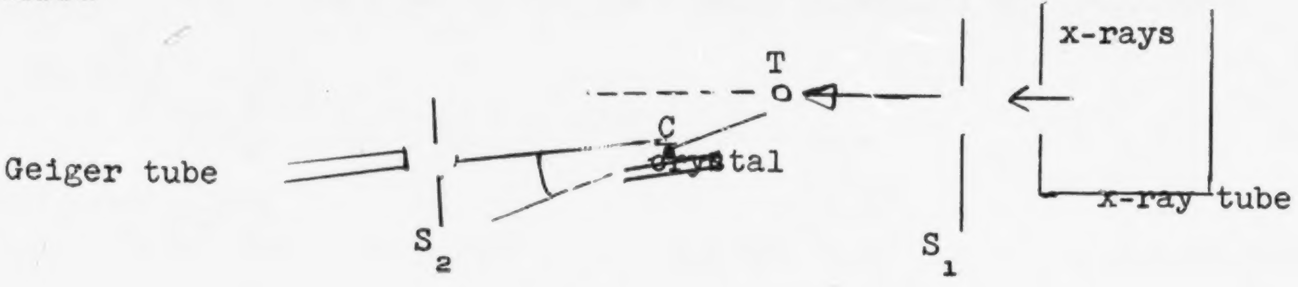
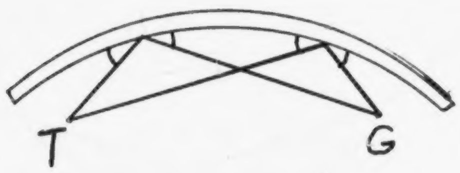


Fig. 3 Schematic of A. H. Compton Experiment

In the experiment the crystal is not flat but is bent along the circumference of a circle. It is bent to such a radius that

Fig. 4 Bent Crystal



all X-rays leaving T (target) with the same wavelength and striking the crystal anywhere come to focus at G (Geiger tube). This arrangement is equivalent to a flat crystal tangent to the circle at C and can be thought of as such for all calculations. The advantage of using a bent crystal is that is multiplies the intensity by a factor of approximately one hundred.

The radiation from the X-ray tube is from a copper anode. All wavelengths are emitted (above the short wavelength limit) but the CuK$_\alpha$ emission line is so intense that, in this experiment, it can be considered as a monochromatic source of that wavelength. The target material is carbon, and the crystal is LiF.

The crystal is aligned using an iron target in place of carbon. This gives a very intense line (Fe K$_\alpha$) which enables one to adjust the crystal quickly. This has been done in your set-ups.

Procedure:

1. Never put your hands or fingers in the X-ray beam, or stand or sit where your body can be exposed to the X-rays. If the procedure is followed correctly and the safety precautions observed, the experiment is completely harmless. When making any changes, put the lead shield in front of the X-ray tube window.

2. Before the day of the experiment come to the laboratory and look over the experiment. Also, before coming to do the experiment, calculate the first Bragg angle for CuK$_\alpha$ from a LiF crystal. Calculate the separation of the Compton peaks in minutes. (You will need these numbers while doing the experiment.) (Compton peaks = "modified" and "unmodified" lines).

3. Turn on the X-ray tube, Geiger tube, and power supply for Geiger tube. (Instructions provided near the experiment).

4. An iron rod is provided in the laboratory, and the equipment should be zeroed each day, and for each scattering angle ϕ. The zero of the spectrometer is set in a similar manner. Using the F_e K$_\alpha$ line, set the zero approximately by ear (i.e., by listening to the clicks in the mechanical scaler). Then determine zero more precisely by counting.

5. Wait until the X-ray tube, Geiger tube, and power supplies have warmed up for about an hour before taking relative intensity readings with the carbon target. This time can be utilized in locating the zero and getting an approximate spectrum.

6. When studying the CuK$_\alpha$ peaks, locate them first by scanning, and then count over them in intervals suitable to determine the shape. Take a sufficient number of counts at each position to minimize the uncertainty due to statistical fluctuations in the counting rate. As this goes as $k / \sqrt{N}$, a decision must be made in the lab as to the statistics which are needed. Always approach points from the same direction to avoid backlash in the gears. It is important that the voltage across the Geiger tube and the intensity of the X-ray beam be kept constant.

Report:

1. Make a plot of relative intensities vs angle of the crystal for each ϕ. Make the scale such that the peaks are shown in detail.

2. Calculate the λ of the peak at the smaller crystal angle and the $\Delta\lambda$ between the two peaks. Compare this with the theoretical values.

3. From quantum mechanics the following formula is derived:

$$\Delta\nu = \sqrt{\frac{E_k}{E_\ell}}\,\delta\nu \qquad (1.3)$$

where Δ v is the width of modified peak (in frequency), E_k = binding energy of electron, E_ℓ = first energy of recoil electron, and δ v is the separation of the peaks (in frequency).

Make an estimate of the width of your modified peaks - then compare with the value given by (1.3) to see whether you have resolved the peaks as well as possible. (The window of the Geiger tube subtends an angle of only 5 minutes).

Questions:

1. Derive equations 1.1a, b, c.

2. The Compton effect does not explain how the original radiation can be scattered through an angle ϕ without a change in wavelength. Explain.

3. Explain what the effect would be as the target element is changed in Z (going from Hydrogen to Uranium) assuming incident CuKα - radiation.

THE COMPTON EFFECT

University of Colorado

PURPOSE:

In terms of physics this experiment has the following aims:

(1) to verify the theoretical angular dependence of the energy of the scattered photons in a Compton collision.

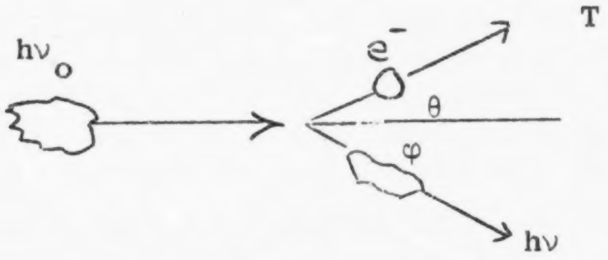
(2) to verify the theoretical angular distribution of the scattered photons.

(3) By observing the energy of the scattered electron, the conservation of energy may be verified.

The method of observation relies directly upon the simultaniety of the emission of the scattered photon and the emission of the struck electron, and therefore implicitly verifies that energy is conserved exactly in this atomic process.

The experiment performs the additional function of allowing the student to apply what has been previously learned about pulse electronics and pulse height analysis and to use some of the most modern techniques and equipment that are applied to atomic and nuclear experiments.

THEORY:



In the scattering of incident gamma radiation by free electrons, conservation of momentum and energy requires that the scattered photon have the energy hν given by

$$h\nu = \frac{h\nu_o}{1 + \left(\frac{h\nu_o}{m_{el}c^2}\right)\left(1 - \cos\varphi\right)}$$

The angular distribution (number of scattered photons scattered in a given direction) was first calculated by Klein and Nishina and, after appropriate averaging over all polarizations of the incident and outgoing photons, is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{r_o^2}{2}\right) \frac{(1+\cos^2\varphi)}{\left(1+2\alpha\sin^2\left(\frac{\varphi}{2}\right)\right)^2} \left[1+ \frac{4\alpha^2\sin^4\left(\frac{\varphi}{2}\right)}{\left(1+\cos^2\varphi\right)\left(1+2\alpha\sin^2\left(\frac{\varphi}{2}\right)\right)}\right]$$

where $\alpha = \frac{h\nu_o}{m_{el}c^2}$ and r_o is the classical electron radius.

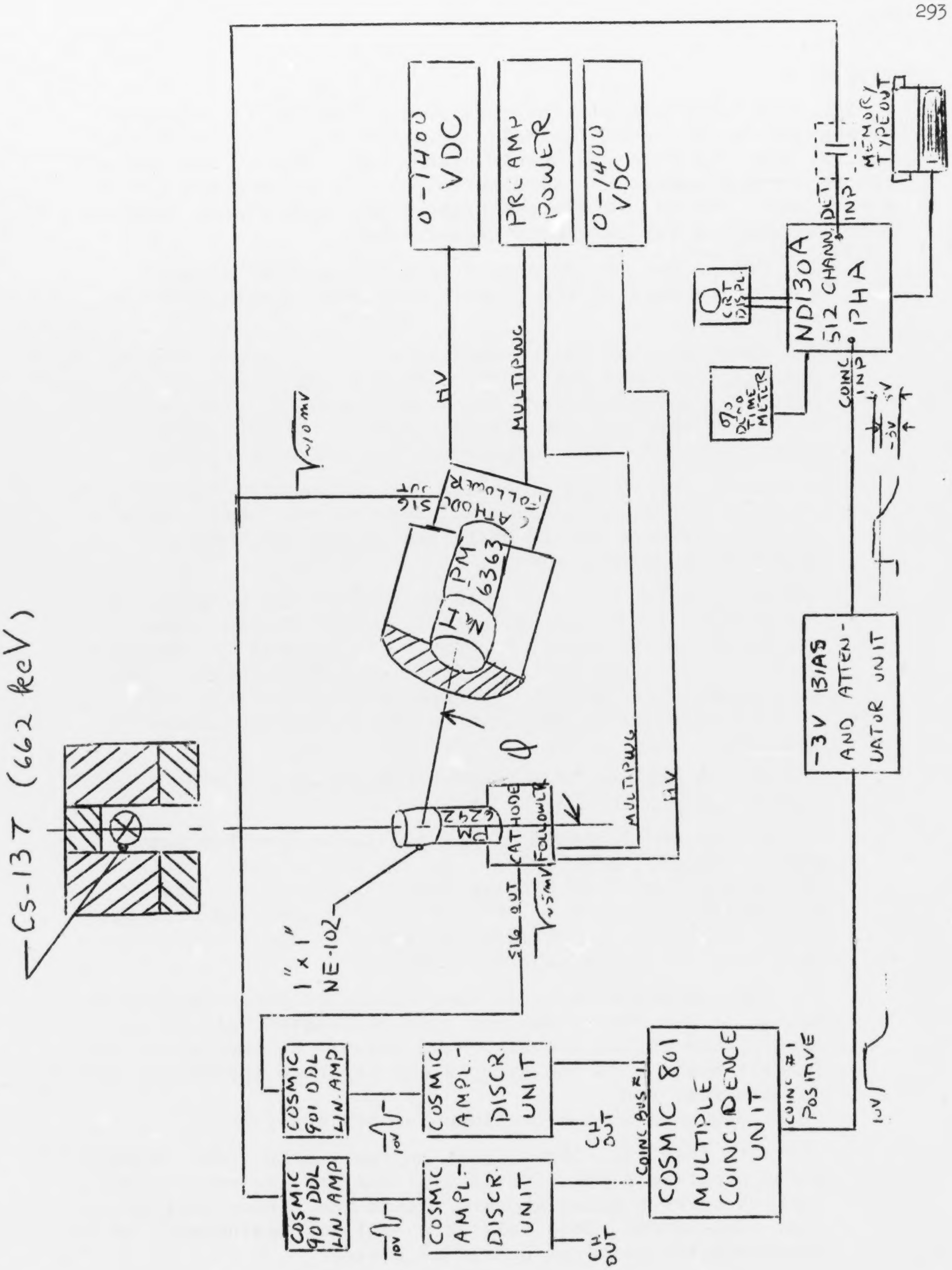
Note that as $\alpha \to o$ this cross-section approaches the classical (Thomson) angular distribution.

APPARATUS:

Incident gamma radiation of 0.662 MeV energy is provided by a C5-137 source of several millicuries. This source is collimated by lead slits and the resultant gamma ray beam impinges on the target, which is a block of plastic scentillator material (type NE-102). Scattered photons are detected by a NaI 2"x 2" crystal which may be moved through an angular ranged from 112½° to 0°. The light pulses are detected and amplified by a 6363 photomultiplier tube. The NE-102 has a low cross section for X-rays (consisting mainly of low Z material) but stops completely the scattered electrons. These produce light pulses which are detected and amplified by a 6292 photomultiplier tube.

Either the output of the 6363 or 6292 detector may be analyzed. The electron (6292) output pulse spectrum is a continuum when no coincidence requirement is made, since it includes all electron scattering angles. The γ spectrum (6363) output is also a continuum because of high counting rates of other processes taking place in the NE-102 and in the NaI crystal.

However, when a coincidence requirement is made between the scattered photon pulse and the electron pulse, the Compton events stand out clearly from other events in the pulse height spectrum. The method of analysis is shown below in block diagram form.


Cs-137 (662 keV)
1"x1" NE-102
PM 6242
CATHODE FOLLOWER
SIG OUT
~5mV
MULTIPLUG
HV
φ
NaI
PM 6363
CATHODE FOLLOWER
SIG OUT
~10mV
HV
MULTIPLUG
0-1400 VDC
PREAMP POWER
0-1400 VDC
COSMIC 901 DDL LIN. AMP
COSMIC 901 DDL LIN. AMP
10V
10V
COSMIC AMPL-DISCR UNIT
COSMIC AMPL-DISCR UNIT
CH OUT
CH OUT
COINC BUS #1
COSMIC 801 MULTIPLE COINCIDENCE UNIT
COINC #1 POSITIVE
10V
-3V BIAS AND ATTENUATOR UNIT
COINC INP
-3V
+V
ND130A 512 CHANN PHA
DET INP
CRT DISPL
ORTEC TIME METER
MEMORY TYPEOUT

PROCEDURE:

You will take both electron and photon spectra for a series of angular positions of the NaI detector. The coincidence rate is low, and therefore you should plan on at least 20 minutes per run at forward angles and 40 minutes per run at backward angles. A very short run at ~ 0 degrees without any coincidence requirement is required for an energy calibration.

(a) Obtain the Cs-137 source from the instructor and insert the base of the plastic container firmly into the collimator.

(b) Turn on power to the amplifiers, coincidence circuit, preamplifiers and phototubes. Do not adjust the gain settings, discriminators or HV settings initially, since they have been set up already. Later, in the course of the experiment, some adjustments may prove desirable.

(c) Check the instructions for use of the 512 channel analyzer before turning on AC power to this unit. Note that no switches should be thrown unless the function switch is in STOP 1 mode.

(d) Use a meter stick or rule to measure the geometry of the source, target and detector. NOTE: Unless these distances are measured, you may be in trouble the second week trying to fit the cross-section runs together. There is no guarantee that the source collimator or the NaI crystal will not be moved in or out during the course of the week.

(e) At ~ 0 degrees take a short non-coincidence run for energy calibration.

(f) Decide on the basis of the available time how many angles you can take data at, if the coincidence rate requires at least 20 minutes runs forward of 90° and 40 minutes runs back of 90°. Note that counting rate may be sacrificed to geometrical definition, and vice versa, by moving the source and NaI crystal.

(g) Data from two separate runs should be taken in the two halves of the memory and may then be printed out all at once. Make carbon copies of the output on the typewriter for all members of the group since separate individual write-ups are required.
NOTE: See special instructions for 512 analyzer.

(h) Note that the "CHout" jack on the Cosmic AMPL.-DISCR. units provides a pulse which will operate the coincidence jack whenever a pulse appears within the window setting of that AMPL-DISCR. This will be useful if readjustment of the discriminator settings proves necessary.

(i) Note also that if you are to check the conservation of energy, the NaI crystal must be set at the same angles for both photon and electron pulse analysis.

(j) In turning off power, turn off the high voltage to the PM tubes first (Note that you must not turn the lower supply to the negative HV position accidentally). Then turn off all AC power switches. (The 512 analyzer must be in STOP 1 mode when turned off.)

RESULTS:

(1) Compare your $h\nu$ (φ) with the theoretical relationship. (Use graphical representation).

(2) Compare your angular distribution with the theoretical Klein-Nishina relationship. (Use graphical representation). Discuss any discrepancies.

(3) Compare your valves for $h\nu + T$ with theory as a function of angle.

QUESTIONS:

(1) Derive the energy $h\nu$ of the scattered photon as a function of the photon angle φ. Also obtain the angle θ of the electron associated with photons at the angle φ.

(2) Discuss the polarization phonomena connected with Compton scattering.

(3) Derive from classical electrodynamics the angular distribution for scattering of an incident plane monochromatic electromagnetic wave by a free electron. This is equal to the quantum result in the limit as $\alpha \to 0$ for the latter.

ENERGY STATES OF SODIUM

Lehigh University

One of the basic concepts of quantum physics is that an atomic system, composed of negative electrons and a positive nucleus, normally exists in energy states which are discrete rather than continuously distributed. Furthermore, there is associated with the atoms of each element a characteristic set of states, referred to as an energy spectrum. The purpose of the present experiment is to build up the energy spectrum for neutral sodium from wave length measurements of its emitted radiation.

The connection between energy states and wave lengths of emitted radiation is given by Einstein's relation.

$$h\nu_{ij} = \frac{hc_o}{\lambda_{ij}} = E_i - E_j \qquad (1)$$

ν_{ij} is the frequency (cycles/sec) of the radiation emitted when the atom changes its energy (ergs) from state E_i to state E_j. h (erg-sec) is Planck's constant, c_o is the velocity (cm/sec) of electromagnetic radiation in vacuo, and λ_{ij} is the wave length (cm) corresponding to the frequency ν_{ij}. For convenience, this equation may be put in a simpler form.

$$\bar{\nu}_{ij} = T_i - T_j \qquad (2)$$

Here, the wave number, $\bar{\nu}_{ij}$ (= $1/\lambda_{ij}$), as well as the energy states, T_i (=E_i/hc_o) and T_j (E_j/hc_o), are expressed in units of reciprocal centimeters.

Having measured the characteristic wave lengths, λ_{ij}, of a particular atom, and calculated the corresponding wave numbers, $\bar{\nu}_{ij}$, the problem is to determine values for the energy states T_i. This is called a spectrum analysis. For most atoms, carrying out such an analysis is a far from easy task, despite the simplicity of equation (2). In a few cases, however, it is possible to pick out by inspection groups of characteristic wavelengths which form regular patterns referred to as series. When this can be done, the series can be used as a basis for analysis, and the problem is greatly simplified.

The potassium atom will be considered for purposes of illustration.

Energy Levels of the Neutral Potassium Atom

Fig. 1 is a schematic drawing of a spectrogram showing the characteristic radiation emitted by the potassium atom. For simplicity the wave length scale has been taken as linear, although this is far from true for a spectrogram taken with a prism instrument such as will be used in the present experiment.

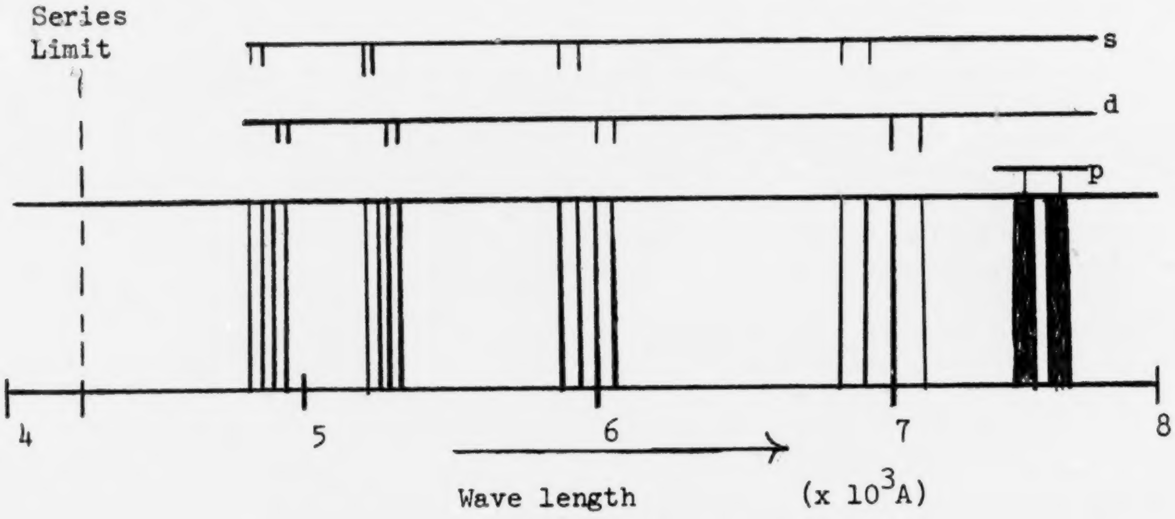


Fig. 1 Spectrogram of Potassium

It will be noted that there are two groups of doublets (marked s and d) which have similar characteristics. In both groups the interval in Angstroms between each doublet and the next decreases progressively in going to shorter wave lengths so there appears to be convergence to a common limit. Furthermore, the intensity of the radiation also decreases in the same order. These are typical characteristics of a series. Table I lists the relative intensities and wave numbers of the "lines" belonging to the s series, together with the relative intensity and wave number of the "lines" marked p.

TABLE I. Intensities and Wave Numbers of Potassium Lines

Identification	Relative Intensity	Wave Number (cm^{-1})	Differences (cm^{-1})
p	10	12985.1	57.7
p	10	13042.8	
s	8	14407.7	57.7
s	7	14465.4	
s	6	17230.2	57.8
s	5	17288.0	
s	4	18722.6	57.8
s	4	18780.4	
s	3	19605.5	57.8
s	2	19663.3	

If the lines designated s belong to a true series they should all originate from the same energy level (or, in this case, from the same pair of energy levels). Further evidence that these lines originate from the same levels comes from the fact that wave number difference is, within the accuracy of measurement, the same for all the doublets. On the assumption that this is true and using equation (2), relative values can be assigned to a tentative set of energy levels. This is done in Fig. 2, where the lowest level has abitrarily been assigned a value of zero for convenience. The slanting lines indicate the observed transitions.

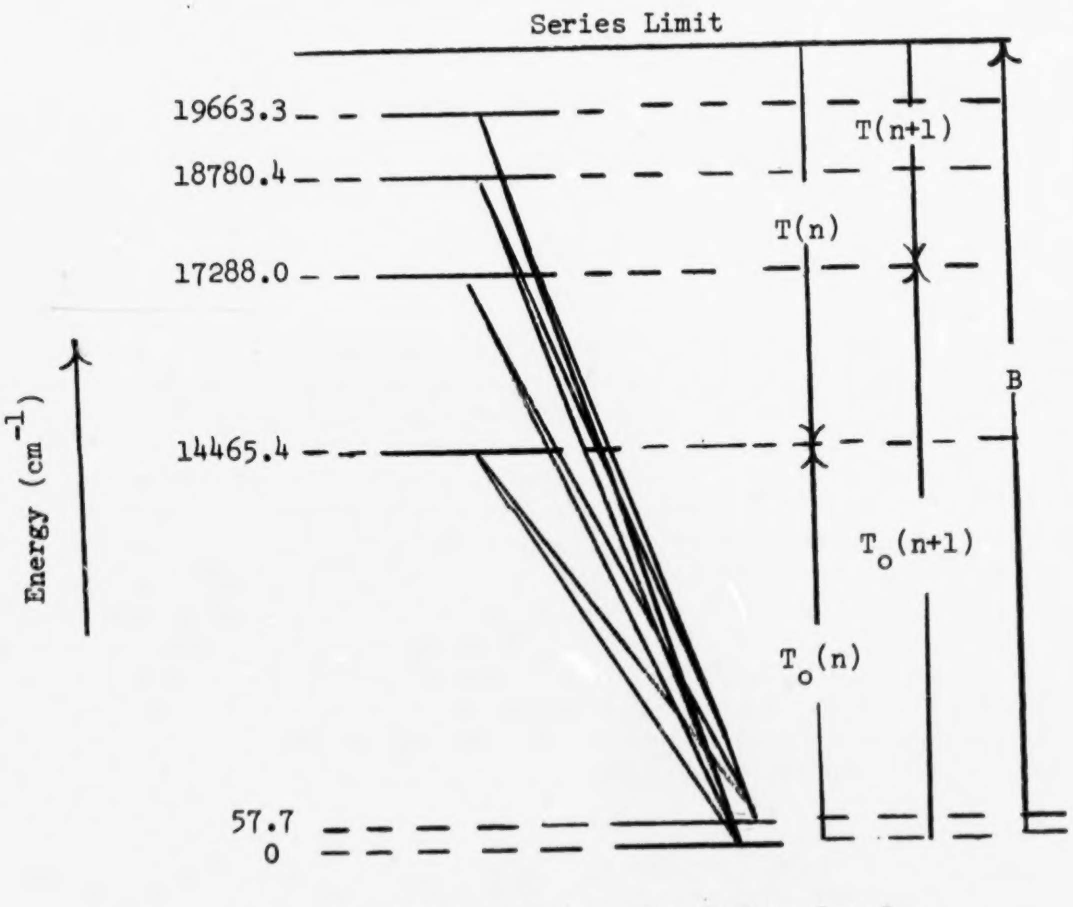


Fig. 2 Assigned Values for Energy Levels

If these assignments are correct it should be possible to represent the values for the energy levels to a good approximation by an equation of the following type.

$$T(n) = \frac{R}{(n-\delta)^2} = \frac{R}{n^{*2}} \qquad (3)$$

T(n) is the value for the energy level measured from the series limit, R is Rydberg's constant (=109,737.4cm^{-1}), n is an integer, and δ, the "quantum defect", is approximately constant for a given series.† n^*, the effective quantum number, need not be an integer but it, as well as n, change by one in going from one member of a series to the next.

---

† Note the similarity between this equation and the equation for the energy levels of hydrogen.

At this stage of the analysis the values of the energy levels are given only with respect to a level to which we have arbitrarily assigned a value of zero and not with respect to the series limit whose position is unknown. Let B and T(n) represent, respectively, the values for the series limit and the energy levels with respect to the arbitrarily chosen reference level Then equation (3) becomes,

$$B - T_o(n) = \frac{R}{n^{*2}} \qquad (4)$$

Considering the two lowest members of the proposed series, we have

$$B - 14465.4 = \frac{109737.4}{n^{*2}}$$

$$B - 17288.0 = \frac{109737.4}{(n^{*}+1)^2} \qquad (5)$$

where n* has been used to designate the effective quantum number of the lowest member of the series. These two equations could be solved simultaneously to obtain values for the unknowns B and n*, but this would be tedious to carry out by direct means since n* and B must be determined to a large number of significant figures. Fortunately, a table is available which simplifies the task. (1) This table (Rydberg Interpolation Table) gives values for R/n^{*2} as function of n*. Between the columns of this table are given differences corresponding to $R/n^{*2} - R/(n^{*}+1)^2$. Subtracting the second of the above equations from the first, we eliminate B and obtain 2822.6cm^{-1} as the difference $R/n^{*2} - R/(n^{*}+1)^2$. This number can be found in the table as the difference between $R/(3.807)^2$ and $R/(4.807)^2$. In the column to the left of this difference is the number 7571.6, which is the value of R/n^{*2}, and in the column to the right is the number 4749.0 which is the value of $R/(n^{*}+1)^2$. Substituting into equation (5),

$$B - 14465.4 = 7571.6$$

and $$B - 17288.0 = 4749.0$$

Thus $$B = 22037.0\text{cm}^{-1}.$$

We are now in a position to make predictions in order to test our assignments. Reading across the row in the table containing the difference 2822.6, we find

$$\frac{R}{(n^{*}+2)^2} = \frac{R}{(5.807)^2} = 3254.3$$

Thus $T_o(n+2) = B + \frac{R}{(n^{*}+2)^2} = 22037.0 + 3254.3 = 18782.7\text{cm}^{-1}$. This agrees reasonably well with the observed value 18780.4cm^{-1}. The assignment 19663.3cm^{-1} for $T_o(n+3)$ can be tested in a similar way. Furthermore, we can predict the positions of possible energy levels in this series below $T_o(n)$. The predicted values for $T_o(n-1)$ and $T(n-2)$ are

$$T_o(n-1) = 8109\text{cm}^{-1}$$

$$T_o(n-2) = -11571\text{cm}^{-1}$$

A transition from 8109cm^{-1} to the zero reference level would be in the infra-red and would ordinarily not be observed, but it should be possible to observe transitions from the levels at 0cm^{-1} and 57.7cm^{-1} to the lower level T_o (n-2). It is known, however, that in going from high to low levels in a series the predictions based on this method of representation become progressively worse. With this in mind, it is reasonable to associate T_o (n-2) with the p transitions 12985.1cm^{-1} and 13042.8cm^{-1}. This places an energy level 12985.1cm^{-1} below the zero reference level and this turns out to be the lowest (or ground) state of the energy spectrum.

Since the ionization potential is the difference between the ground state and the series limit, its value is

$$\text{IP} = 22037 + 12985 = 35022\text{cm}^{-1}$$

$$= 4.341 \text{ electron-volts}$$

Other series can be used to locate additional energy levels and to make independent evaluations of the series limit and therefore the ionization potential.

The steps to be followed in the present experiment are to obtain a spectrogram of the radiation emitted by the sodium atom, to measure the observed wave lengths and to use these measurements to build up an energy level diagram.

Spectrogram of Sodium

Using the constant deviation spectrograph, obtain a series of spectrograms with sodium and mercury arcs as radiation sources. The spectrograph is in adjustment. Do not turn any controls. (2)

First load the plate holder. This must be done in complete darkness, since the Super Pancro Plates are sensitive to all visible wave lengths. The emulsion side of the plate can be determined by holding the corner of the plate lightly between the thumb and forefinger. The thumb or finger touching the emulsion will slip more easily than the one touching the glass. Practice this with an exposed plate.

Turn on both arcs and allow about fifteen minutes for warm up. Then take a series of four spectrograms, shifting the plate holder between each exposure. Both sodium and mercury should appear on each exposure, since the mercury is to provide standard wave lengths for measurement. When using the mercury arc cover a portion of the slit with a card so the mercury lines can be distinguished from sodium. The exposure time for mercury should in each case be about 20 seconds, while those for sodium should be 1 sec, 10 sec, 1 min and 5 min.

Develop the plate for 5 minutes and transfer to a fixing bath for 10 minutes. Rinse in running water for 10 minutes, shake to remove excess water and allow a short time for drying. Throw away the developer but pour the fixer back into the half gallon container. Use a damp paper towel to clean up any developer or fixer which has been spilled.

Each of the four exposures should appear approximately as shown in Fig. 3, where the short lines correspond to mercury and the long lines to sodium. With magnification, it will be seen that all the sodium lines are double.

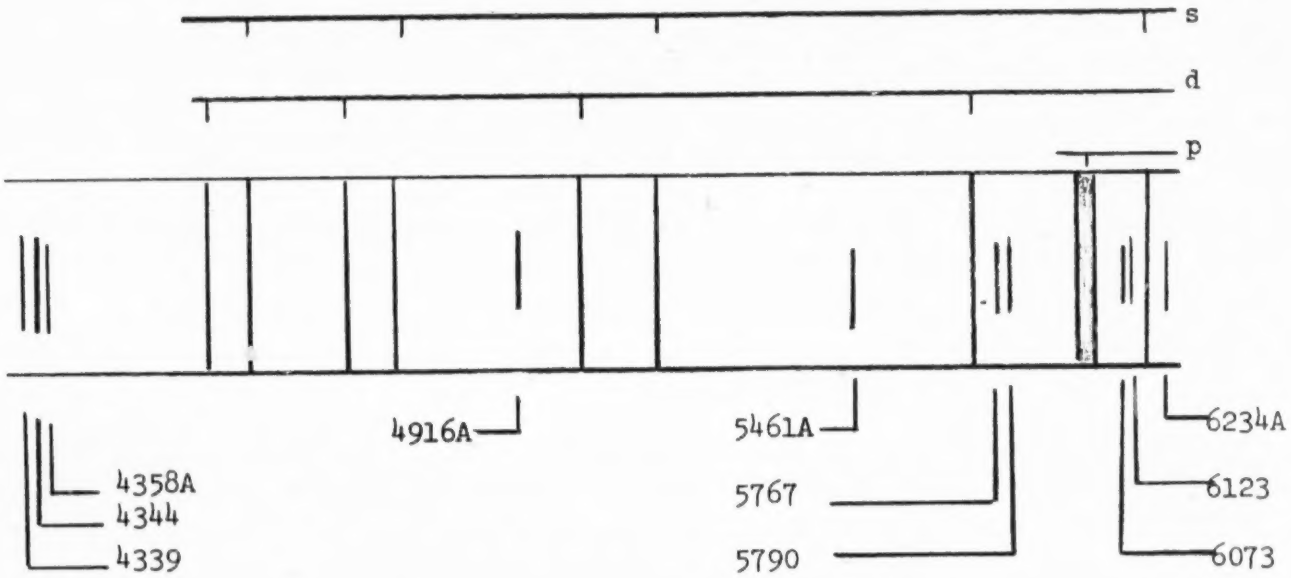


Fig. 3 Spectrogram of Sodium with Mercury Standards

Measurement and Reduction of Spectrogram

Wave length measurements are usually made by comparison with standards. In this case, the standards will be taken as the wave lengths of the mercury lines which are listed in FIG. 3.

Using a comparator, measure the positions of all mercury and sodium lines shown in FIG. 3. (There should be at least three members of both the s and the d series). This should be completed by the end of the first period.

With a prism instrument, the variation of wave length with distance along the plate is far from linear. (2) To a very good approximation, the relationship between the position on the plate, R, and the wave-length, λ, is given by the following equation.

$$\lambda = \lambda_o + \frac{C}{R-R_o} \qquad (6)$$

This contains three parameter, λ_o, R_o and C which must be determined from known wave length values for three lines λ_1, λ_2 and λ_3.

$$\lambda_o = \frac{\lambda_3 - \theta\lambda_2}{1-\theta} \qquad \theta = \frac{(\lambda_3-\lambda_1)\,(R_2-R_1)}{(\lambda_2-\lambda_1)\,(R_3-R_1)}$$

$$R_o = \frac{R_2 - \theta R_3}{1-\theta}$$

$$C = (\lambda_3-\lambda_o)\,(R_3-R_o) = (\lambda_2-\lambda_o)\,(R_2-R_o) = (\lambda_1-\lambda_o)\,(R_1-R_o)$$

For λ_1 and λ_3 choose two mercury lines near the ends of measurements, whose wave lengths bracket all the measured sodium lines. λ_2 should be somewhere near the middle of the range.

Having determined λ_o, R_o and C calculate the wave lengths of all measured lines. Deviations from the known values for the wave lengths of the remaining mercury lines will indicate the accuracy to be expected from the measurements for the sodium lines.

Finally, calculate the wave numbers of all observed sodium lines.

The Energy Spectrum of Neutral Sodium

(1) From the lines of the s series identified in FIG. 3, determine the energy levels giving rise to these transitions, using the method described in the first section of this report. Predict and identify as many lines of this series as possible.

(2) Repeat the procedure of part (1) for the d series. How does the series limit, determined from the s series, compare with that determined from the d series?

(3) Can you predict the positions of energy levels which would give rise to lines having the following wave lengths?

$\lambda_a = 3302.3A$

$\lambda_b = 3302.9A$

$\lambda_c = 2852.8A$

$\lambda_d = 2853.0A$

(4) Determine the ionization potential for neutral sodium.

(5) Draw to scale an energy level diagram and indicate all of the observed transitions as well as the transitions given in part (3).

---

(1) A "Rydberg Interpolation Table", listing values of $109737.4/(n+\mu)^2$ for all values of $(n+\mu)$ from 1.000 to 11.000 in steps of 0.001, was prepared by A. G. Shenstone, J. C. Boyce and H. N. Russell in 1934. To obtain copies of this table write to Prof. C. W. Curtis, Physics Dept., Lehigh University, Bethlehem, Pa.

(2) If the spectrum is photographed near the normal of a concave grating in a Paschen mount, it will be almost linear in wave length and much easier to reduce than a spectrogram produced by a prism. A linear interpolation between end standards, λ_o and λ_N, will provide approximately correct values for the wave lengths.

$$D = \frac{\lambda_N - \lambda_o}{R_N - R_o} \quad ; \quad \lambda_\ell = \lambda_o - DR_o + DR$$

This equation can be set upon a desk calculator so that values of λ_ℓ are obtained directly by entering corresponding values of R into the machine. Accurate values of λ are determined from a correction curve constructed from differences between known wave lengths of the comparison spectrum and the corresponding values obtained by linear interpolation.

FRANCK-HERTZ EXPERIMENT

University of California, Berkeley

1. Description

The Franck-Hertz tube is used for demonstrating the inelastic collisions of slow electrons of kinetic energy 4.9 electron volts with mercury atoms. It is a four-electrode tube with indirectly heated cathode. There is a mercury drop in the interior.

The tube has a seven pin socket which is inserted into a supplied tube-holder. From the tube-holder extends a cable containing 6 different color cords which are connected to six marked plugs.

The connections are as follows:

Electrode	Socket Pin	Colored Cord	Plug Marking
Collector Electrode	7	pink	A
Grid 2	2	yellow	g_2
Grid 1	10	white	g_1
Heating cathode	11)	brown	fk
Heating filament	12)		
Heating filament	1	green	f

The No. 6 pin leads over the grey cord to the plug marked S, corresponding to the shielding electrode inside the tube and located between collector electrode A and grid 2.

The required mercury pressure in the tube is 15 - 20 mm. Therefore the entire tube must be kept at a temperature of approximately 200°C in an electric tubular heater (555 81). The tube socket is separated from the glass envelope by a glass sleeve and remains outside the heater.

With insufficient mercury pressure in the tube, not all electrons of 4.9 electron volts give up their energy to mercury atoms. They can accumulate higher energies and, finally, form ions. The same effect takes place with a great stream of electrons. Therefore small currents and a sensitive indicator must be used, for instance, the mirror galvanometer (532 10) with D.C. amplifier No. (532 00), which enables the measurement of current down to 10^{-11} amp/mm/m. The necessary voltages forthe tube are as follows:

Heating voltage, 6.3 volts
Adjustable D.C. voltages between the grids and the cathode 0...4 volts and 0...30 volt,
The bias voltage between collector electrode and grid, 1 volt;
The power-supply No. 522 35a and a small dry battery or one cell of a nickel-cadmium battery have proved to be satisfactory voltage sources.

2. The experimental procedure

a) The Franck-Hertz tube is put into the oven.

b) The temperature of the electric heater is adjusted by a variable resistor, (sliding rheostat, 110 ohms, 2.5 A., cat. no. (537 24) in such a way that there are 50 volts on the terminals of the heater.

c) The amplifying unit is switched on and the sensitivity of the galvanometer is set to zero.

d) Set-up the connections according to the suggested circuit on diagram 1. The power supply used as voltage source has to be switched off during the setting up. The voltmeter (531 64a) with a special shunt, measuring range 30 volts, performs two functions: As a member of a voltage divider, it supplies voltages from 0 to 30 volts to grid g_2, when the power-supply is set to 0 - 270 volts. The dial of the voltmeter indicates 0 - 30 between g_2 and the cathode. If another indicating instrument is used, a resistance of at least 10,000 ohms must be included in the circuit for grid g_2 in order to limit the current in case of a gas discharge in the tube.

The collector electrode is connected to the terminal, "G[illegible] Eing.", on the amplifying unit. The two terminals on the right side of the amplifying unit lead to the galvanometer with an Ayrton shunt in parallel for sensitivity regulation.

e) Approximately one hour after the oven and the amplifier are switched on, the cathode is also switched on. At this moment, there should not be any voltage between the grids and the cathode.

f) About one minute thereafter, we determine the right voltage for g_1. The voltage on g_1 determines the space-charge around the cathode and, consequently, also the emission current. If there are 30 volts between the cathode and g_2, no gas discharge will take place is we have the right voltage on g_1. In order to find the voltage for g_1, we set the Ayrton shunt of approximately 1/10 of the full sensitivity of the galvonometer and bring the voltage on g_2 to 30 volts. Then under continuous observation of the galvonometer the voltage on g_1 is increased with caution. The current increases slowly and steadily. If ever the current increases rapidly, a gas discharge has taken place, and the voltage on g_2 must be brought down to zero immediately. The voltage on g_1 is lowered and the procedure is repeated until the gas discharge does not occur with 30 volts on grid 2, while the greatest possible emission current flows. This voltage is in general approximately 1/2 volt.

g) Restore the sensitivity of the galvanometer on the Ayrton shunt insuch a way that the deflection of the galvanometer covers the entire scale. Now the arrangement is ready for taking measurements.

h) The voltage on grid 2 is gradually increased, beginning from zero. As soon as approximately 4 volts are reached, the electron current between grid 2 and the collector electrode starts and reaches a maximum at ca. 7 volts. Further maxima can be found in intervals of 4.9 volts each. (The difference between 7 and 4.9 volts may be explained by contact difference of potential.

3. Special points

a) The Franck-Hertz tube contains metallic mercury. After the tube has been transported, there is a danger that some mercury has been retained between the electrodes, and may cause a short-circuit. The start of operation without pre-heating would therefore endanger the tube.

b) Too low heating of the filament also endangers the cathode. If for this reason emission is insufficient, the emission of the cathode can be improved by overheating it very briefly at approximately 8 volts.

c) Apart from the space-charge, the emission current and the occurrences of the inelastic collisions depend also upon the mercury pressure in the tube, and consequently, upon the temperature of the heater. If the regulation of the voltage on the grid g_1 is not sufficient, the temperature of the heater must be changed.

Should the tube be overheated, the emission current would be small, and maximas and minimas would hardly be recognized or seen at all.

For checking purposes, we take the tube out of the oven, let it cool off for approximately 30 seconds, and then, if the tube was overheated before, an increase in current takes place, and maximas can be found. If this is the case, the oven must be cooled off and operated with a lower voltage.

d) When the tube is not sufficiently heated, the emission current is large and the maximas, especially those of a higher order, can hardly be seen. Due to low mercury pressure in the insufficiently heated tube, discharge can take place. To prevent the gas discharge, the voltage on g_1 has to be lowered to such an extent that the emission current becomes too small and is again only a few 10^{-9} amp. When this has been ascertained, the tube must be heated to a higher temperature. However, the voltage at the oven must in no case exceed 60 volts.

In general, the adjustment of the heating temperature is not difficult. One should change the temperature of the heater only after one has ascertained that there is no mistake in the connections and that the facts described are really in accordance with one of the two cases. (par. c and d). To increase the temperature of the oven without reason may endanger the life of the tube.

e) The p.d. between grid 2 and cathode is not to exceed 30 V to avoid ionization by collision.

f) The life of the tube is limited by the strain the high operating temperature puts on the cathode material. If the tube is left in the hot tubular furnace for several consecutive hours, metal and glass parts may give off occluded gas thus deteriorating the vacuum. However, if these points are borne in mind, a life of several hundred working hours can be expected.

g) A detailed description of the experiment and its explanation is found in Leybold Physics Leaflet DC 535.352;a.

Notice:

The instructions given above are supplied by the manufacturer of the Leybold tube. The experimental setup in the lab is slightly different. Mainly the difference is that an electrometer is used as a current detector instead of an amplifier and a mirror galvanometer. Read the instructions carefully and analyze the circuit given below before attempting to take data.

The report should include the following:

1). Determination of the first excitation potential of Hg ($^1S_0$ - $^3P_1$) with error analysis and comparison with accepted value.

2). Discussion of contact potential, and of use of thermocouple to measure temperatures.

3). Is it possible to perform the experiment with a tube that does not have (G_1) grid? What is the purpose of (G_1)?

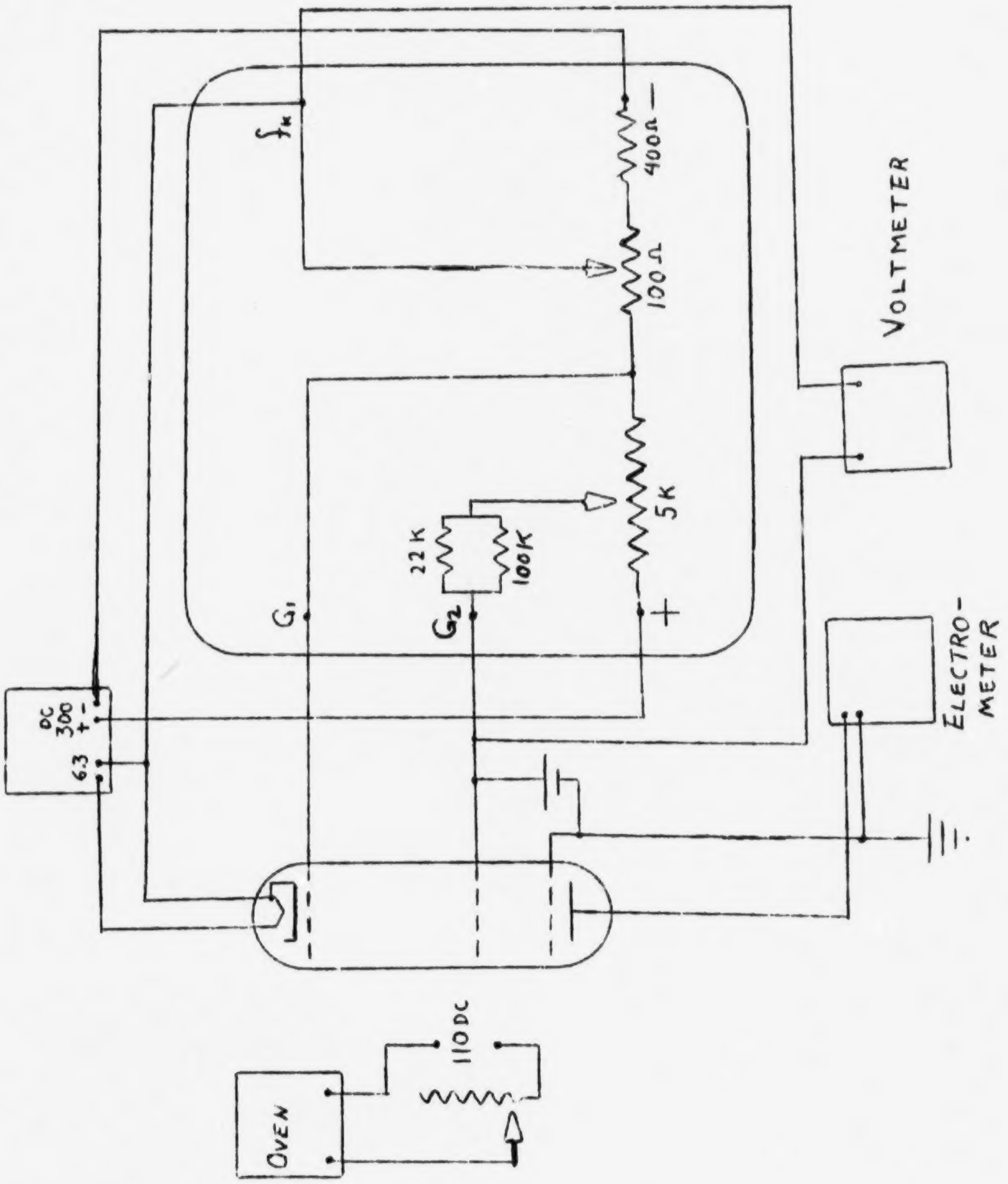
4). From your data you might notice that the metastable state $^3P_0$ is also excited (potential 4.66 volts); but this transition is prohibited by the spectroscopic selection rules. Explain.

5). Can the same tube be used to determine the ionization potential of mercury? If yes, give an appropriate circuit diagram. If not, explain why not.

Special Notes

1). Several articles in the American Journal of Physics will be of interest in connection with this experiment. The reader is referred especially to Bernheim, Gossard, and Pound, Am. J. Phys. 24, 630 (1956) and Dewdney, Am. J. Phys. 27, 645 (1959).

2). Several new tubes for performing experiments on the Franck-Hertz effect are on the market. For example, a tube containing mercury and neon is available from Klinger Scientific Apparatus Corp., 83-45 Parsons Blvd., Jamaica 32, N.Y., which makes it possible to observe the appearance of spectral lines as well as the resonance peaks.


VOLTMETER
ELECTRO-METER
OVEN
110 DC
400Ω
100Ω
5K
22K
100K
f_k
G_1
G_2
+
−
DC
300
+ −
6.3

OPTICAL PUMPING

University of California, Berkeley

References: DeZafra, Optical Pumping; American Journal of Physics, 28 646, (1960);
Bloom, Optical Pumping; Scientific American, Oct. 1960, p.72
Knapp and Aprahamian, Optical Pumping, Special Project Report - Physics 110D Laboratory, May 24, 1962

Apparatus:
- Rb electrodeless light source
- Circular polarizer
- D_2 filter
- Gas cell containing Rb^{85} and Rb^{87}
- Photocell and amplifier
- Oscilloscope
- Rf generator
- Helmholtz coils

Purpose: To study the optical pumping technique and to measure the nuclear spins of Rb^{85} and Rb^{87}, as well as the value of the earth's magnetic field.

Procedure: Before touching any of the experimental apparatus, you should read carefully both reference 1 and reference 2. It may also be helpful to read the report of Knapp and Aprahamian, who put this experiment into operating condition. This latter report is kept near the equipment and may never be removed from the laboratory.

During the first laboratory period inspect the light source to learn its construction and to insure correct orientation of the circular polarizer. Be very careful in handling the components since they are very costly to replace. The filter which passes the D_1 light costs about $300.

Turn on all of the equipment. Heat the oven to 50°C and then turn off the heater while taking data. (You should assess for yourself whether turning off the heater is necessary or not.) After the temperature drops to approximately 42°C, reheat the oven chamber for another series of observations. (CAUTION: Do not heat the oven over 55°C.) It is very doubtful if you can obtain resonances below 40°C because the rubidium vapor pressure is too low at that temperature.

By bucking out the earth's magnetic field with the Helmholtz coils, observe the zero field resonances. The zero field condition occurs when the resonance dips on the oscilloscope are equally spaced and of the same depth.

Observe the Rb^{85} and Rb^{87} resonances at many values of magnetic field produced by the Helmholtz coils. Incrementing the current through the coils by .2 amperes seems reasonable. Do not use too large a sweep on the modulation AC current, for then both the Rb^{85} and Rb^{87} resonance patterns appear superimposed on the oscilloscope screen. Record your results and the construction of the Helmholtz coils.

Report: Using the low field approximation of the Breit-Rabi formula, show that

$$\frac{\nu_1}{\nu_2} = \frac{2I_2+1}{2I_1+1}$$

where ν_1 and ν_2 are the resonance frequencies of Rb^{85} and Rb^{87}, respectively;
I_1 and I_2 are the nuclear spins of Rb^{85} and Rb^{87}, respectively.

From your data determine the best value of the ratio ν_1/ν_2 at zero magnetic field. From a knowledge of the construction of the Helmholtz coil obtain the approximate magnetic field at the resonance bulb and compute the nuclear spins of both Rb^{85} and Rb^{87}.(Do this at two values of coil current.)

Once I_1 and I_2 are known, use the atomic parameters and the resonance frequencies to obtain the magnetic field at the bulb as a function of current. From this graph you may determine the earth's magnetic field. Check the direction of current through the Helmholtz coils to be sure it is consistent with your understanding of the direction of the earth's field.

Plot a rough Breit-Rabi diagram (E vs H) for the Rb^{87} atom by using the energy and frequency tables in the laboratory.

ZEEMAN EFFECT

Purdue University

References: H. E. White, Introduction to Atomic Spectra, Ch. X, pages 149-170.
Richtmyer and Kennard, Introduction to Modern Physics, 75-83 and 365-386.

Pieter Zeeman found in 1896 that the emission of a light source can be influenced by a magnetic field. The spectral lines emitted by a source put into a magnetic field are split into several components and exhibit a definite polarization, depending on the direction of observation. In moderate fields, i. e., several thousand oersted (or gauss), the splitting, measured in wave numbers, is proportional to the field strength. This phenomenon is called the Zeeman effect.

I. TERMINOLOGY

The transverse effect is observed if the axis of the collimator is perpendicular to the magnetic lines of force.

The longitudinal effect is observed if the axis of the collimator points in the direction of the magnetic lines of force.

The normal Zeeman effect is the phenomenon predicted by the classical theory of H. A. Lorentz.

The anomalous Zeeman effect includes all cases in which the number of components and magnitude of splitting differs from the prediction of classical theory.

II. THEORY

A line exhibiting the normal transverse effect is split symmetrically into three components ν_0, $\nu_0 + \Delta\nu$ and $\nu_0 - \Delta\nu$ where ν_0 is the wave number of the line without magnetic field. $\Delta\nu$ is proportional to the field strength H. $\Delta\nu = a H$, where $a = 4.6699 \times 10^{-5}$ cm^{-1} gauss^{-1}. The components of this normal triplet are linearly polarized. The plane of vibration of the central component is parallel to H (π - component), while that of the displaced components is perpendicular to H (σ - components).

If the light is observed parallel to H, a line shows the normal longitudinal effect. In this case, the originally single line is split into two components, their wave numbers being $\nu_0 + \Delta\nu$ and $\nu_0 - \Delta\nu$. These components are circularly polarized, the first one being left handed, the latter being right handed circularly polarized, provided that the light leaving the source traveled in the direction of the magnetic lines of force, i.e., from north pole to south pole.

The normal Zeeman effect is found in all lines which are combinations between singlet-terms. All the other combinations exhibit much more complicated patterns and a great variety of Zeeman types, each being characteristic for the combination involved. All the types deviating from the "normal triplet" are called "anomalous types" and we speak of the anomalous Zeeman effect.

The splittings observed with the most common combinations are given in Table I. Table II contains the types of a selected number of neon lines. The splittings are given as fractional parts of the normal splitting.

III. EXPERIMENTAL DETAILS

Since the splitting of the lines in a moderate field is very small, it is necessary to employ an instrument of rather high resolving power, either a large concave grating or an interferometer in connection with a prism or grating spectroscope. For our purposes the latter is preferable, since we can obtain the necessary resolving power with relatively simple means. The setup is the same as used in Experiment 9, except that the light source is located between the poles of a small electromagnet, one pole shoe of which is provided with a bore parallel to the lines of force. In this way it is possible to observe either transversally or longitudinally. Furthermore, a Nicol prism can be inserted into the light path and in addition to it, a quarter wave plate. These means enable one to carry out the investigations of the states of polarization present in the transverse and longitudinal effects.

IV. PROCEDURE

Use a helium Geissler tube.

1) Line up the light source in the axis of the spectroscope.

2) Insert the objective which is to project the interference fringes on the slit of the spectroscope.

3) Adjust the objective, centered with the collimator axis, so that the slit is in the focus of the lens. (Method of reversed light path.)

4) Insert an achromatic lens, producing parallel light, in front of the Geissler tube, whose capillary must now be focussed on the slit sharply.

5) Place the Fabry-Perot interferometer between the two lenses and adjust it by tilting until the center of the ring system coincides with the center of the slit.

6) Check whether the fringes appear sharp together with the cross hair or the pointer. If not, refocus the objective.

V. OBSERVATIONS

Transverse Effect

1) Helium light. Observe and record by means of sketches the influence of the magnetic field on the appearance of the interference fringes of the different lines of helium (e.g., 6678, 5015, 5875 A). Translate these observations into a qualitative description of wave length changes.

2) Investigate the state of polarization by means of the Nicol prism. The red and green lines exhibit the normal effect; check it.

3) Repeat (1) and (2) with a krypton light source. The green and yellow lines should show a distinct splitting into a magnetic triplet without the Nicol prism, if everything is in good adjustment.

4) Repeat (1) and (2) with a neon light source. In this case we have the greatest variety of Zeeman patterns. The splittings of the different lines are not of the same magnitude or number; furthermore, certain lines exhibit a splitting of the parallel components, which reveals immediately the presence of an anomalous Zeeman effect. Observe and sketch carefully a few selected lines and compare your results with the table of splittings.

5) Employ a neon source and insert the micrometer eyepiece in place of the ordinary eyepiece. Measure the ring system of the yellow neon line, 5852 A, with and without magnetic field. Calculate the total splitting of the outer components and find the field strength, H, of the magnetic field employed.

Longitudinal Effect

Use a helium Geissler tube for this part of the experiment.

1) Arrange the magnet in such a way that the light travels through the pole shoe of the south pole of the magnet.

2) Determine the polarity of the poles by means of the ammeter and by observing the sense of winding of the magnetizing coils. The light should travel in the direction of the lines of force, from north to south pole.

3) Observe the effect of the magnetic field without the Nicol prism.

4) Insert the Nicol prism and, with the field on, observe the fringe patterns as the Nicol is rotated.

5) Careful investigation will show that no change of patterns is present. Neither a change of the shape nor of relative intensity can be found.

6) The conclusion drawn from (5) is the following:
Either, (a) the light is unpolarized or (b) the light is circularly polarized.

7) A decision whether (6a) or (6b) is correct can be made by inserting a quarter wave plate (mica) into the light path in front of the hole in the pole shoe.

8) With the "lambda quarter plate" ($\lambda/4$) in position it will be found that for certain definite azimuths of the Nicol prism, the one or the other component of the split patterns disappears. From this, one can conclude that the light is circularly polarized. If the light were unpolarized, the insertion of the $\lambda/4$ plate would have no effect. Check this by observing the patterns without magnetic field.

9) Decide which component is right handed and which component is left handed polarized. Check the theory according to which the component having greater wave number should be left handed circularly polarized.

Lambda Quarter Plate

Circularly polarized light can be transformed into linearly polarized light if a $\lambda/4$ plate is inserted in its path. A quarter wave plate is a plane parallel plate cut from a doubly refracting crystal. The thickness of the plate is such as to produce a path difference of a quarter of a wave length between the ordinary and extraordinary waves traversing the $\lambda/4$ plate perpendicularly to its faces. The direction of vibration of the slower wave (i.e., greater index of refraction) usually has been marked by the manufacturer with an arrow on the plate.

If linearly polarized light is incident, it will be decomposed by the $\lambda/4$ plate into two waves vibrating perpendicularly to each other. These two waves enter the plate with zero phase difference and leave the plate with a phase difference of 90 degrees. The emerging light exhibits, therefore, generally elliptical polarization. However, if the plane of vibration of the incident wave makes an angle of 45 degrees with the principal section of the $\lambda/4$ plate, the emerging light will be circularly polarized. The thickness of the $\lambda/4$ plate can be found from the relation,

$$t = (\lambda/4) / (n_1 - n_2),$$

which should be proved by the student in his report. Since the thickness depends on the refractive index, a certain plate can play the role of a $\lambda/4$ plate only for one wave length. Very often, commercial plates are correct for yellow sodium light.

A $\lambda/4$ plate can also be used for analysing elliptically and circularly polarized light. We discuss only the latter case since it is important for the Zeeman effect. Circularly polarized light can be described as consisting of two linearly polarized waves of equal amplitudes exhibiting a constant phase difference of plus or minus 90 degrees. If the y-vibration is lagging, the light travelling in the z-direction is left handed circularly polarized. If the y-vibration is leading (x-vibration lagging) the wave is right handed circularly polarized.

Circularly polarized light traversing a $\lambda/4$ plate will be transformed into linearly polarized light since the phase differences present in the circularly polarized wave can be composed with those produced by the $\lambda/4$ plate and can add up to zero or 180 degrees. The linearly polarized light produced in this manner can be investigated with a simple analyser (Nicol prism, e.g.). In certain azimuths of the analyser, the intensity observed will be a maximum (maximum position), in an azimuth 90 degrees different, the intensity will be zero.

It is possible to distinguish between right handed and left handed circularly polarized light if one observes the relative position of the plane vibration of the analyser and the arrow of the $\lambda/4$ plate. One can easily prove the following statement:

If the analyser is in maximum position, the sense of rotation of the circularly polarized light is determined by that sense of rotation which brings the arrow of the $\lambda/4$ plate into the position of the analyser's direction of vibration in the shortest way.

It is left to the student to prove this by sketches or analytical treatment. It should be proved analytically also, that a simple analyser (Nicol prism) has no effect upon the intensity of circularly polarized light traversing the prism.

Table I

ZEEMAN PATTERNS

Magnitude of Splitting Relative to Normal Splitting,

$$\Delta\nu = \pm \frac{e}{4\pi mc^2} H = 4.67 \times 10^{-5} \text{ H cm}^{-1}$$

		Pi-components	Sigma-components
Singlets "Normal pattern"		0	1/1
Doublets			
$^2S_{1/2}$	$^2P_{1/2}$	2/3	4/3
$^2S_{1/2}$	$^2P_{3/2}$	1/3	3/3 5/3
$^2P_{1/2}$	$^2D_{3/2}$	1/15	11/15 13/15
$^2P_{3/2}$	$^2D_{3/2}$	4/15 8/15	12/15 16/15 24/15
$^2P_{3/2}$	$^2D_{5/2}$	1/15 13/15	15, 17, 19, 21 /15*
Triplets			
$^3S_1$	$^3P_0$	0	4/2
$^3S_1$	$^3P_1$	1	3/2 4/2
$^3S_1$	$^3P_2$	0 1/2	2/2 3/2 4/2

* To conserve space the denominator is given only once.

Table II Zeeman Effect in Neon (Transverse Effect)

Wave Length	Pi-components	Sigma-components
5400	(0) / 15	22 / 15
5852	(0) / 30	31 / 30
5881	(0) (1) / 6	8 9 10 /6
5944	(2) (4) / 10	11 13 15 17 / 10
5975	(0) (1) / 2	2 3 4 / 2
6029	(2) / 15	20 22 / 15
6074	(0) / 15	22 / 15
6096	(0) (5) / 30	34 39 44 / 30
6128	(7) / 15	15 22 / 15
6143	(16) / 30	8 29 37 45 53 / 30
6163	(0) / 3	4 / 3
6217	(0) (5) / 6	4 9 14 / 6
6266	(0) / 1	1 / 1
6304	(0) (7) / 30	30 37 44 / 30
6334	(11) (22) / 30	23 34 45 56 / 30
6382	(12) / 15	10 22 / 15
6402	(0) (1) (2) / 6	6 7 8 9 10 / 6
6506	(0) (5) / 15	12 17 22 / 15
6532	(0) / 3	2 / 3
6598	(69) / 30	31 40 / 30
6678	(0) (8) / 30	31 39 47 / 30
6717	(1) / 30	30 31 / 30
6929	(0) (6) / 30	31 37 43 / 30
7032	(0) (1) / 2	2 3 4 / 2
7245	(8) / 15	22 30 / 15

Strongest effects are underscored.

The number after the / mark is the denominator for fractions whose numerators are the numbers preceding the / mark.

Solid State Physics

ELECTRICAL CONDUCTION BY SOLIDS

Brown University

The electrical resistivity ρ of a material is defined as the electrical resistance (in ohms) offered by a unit cube of the material to a current directed through the cube from one face to the opposite one. The usual units are ohm-cm or ohm-meter. The electrical conductivity σ is $1/\rho$. Here we shall use ρ, not σ, because the instruments are calibrated to read resistance.

The resistivity of a material, and particularly of a solid, is of interest in either pure or applied science -- not only the magnitude of ρ, but its dependence on such factors as temperature, structure and purity of the material, etc. By means of such distinctions, materials are classified as insulators, semiconductors, or conductors.

In solid insulators, the highest occupied band of energy states, the valence band, is exactly filled with electrons -- no empty states, no extra electrons -- and the next allowed (but unoccupied) band is higher in energy by several electron-volts. With no external electric field, electrons in the valence band move equally in all directions, and there is no current. When a field is applied, the Exclusion Principle forbids any electron's changing its momentum or energy, because all available quantum states are already occupied. At most, pairs of electrons could exchange states, which cannot produce any current. At ordinary temperatures, the Boltzmann distribution shows that practically no electrons could be excited past the energy gap of several electron-volts, to the next higher allowed band. At temperatures approaching the melting point, more electrons can be so excited, and the insulator may break down.

Intrinsic semiconductors -- semimetallic elements such as germanium or silicon, and various chemical compounds -- are like insulators in having a full valence band, but the next allowed band is only about 1 e-v higher. At low temperatures, ρ is very great, but around room temperature, or above, appreciable numbers of electrons reach the next higher band, where they are free to move (this band never becoming filled) and so can carry current. Simultaneously, unoccupied states or holes are left in the valence band. In effect, a hole acts like a charge +e, which can move and carry current (really, a neighboring electron fills the hole, but then the hole has merely been displaced to the former site of this electron). Thus such a semiconductor is marked by a resistivity that falls very rapidly as the temperature rises.

Doped semiconductors contain chosen impurity atoms (a few parts per million, ordinarily). A *donor* impurity atom has one more free or "freeable" electron than the atoms of the main solid; thus some electrons are provided in quantum states only a fraction of 1 e-v above the filled valence band, and hence at ordinary temperatures there are some free electrons. The result is called an *n-type* semiconductor (n for "negative" -- the current carriers are ordinary negatively charged electrons). An *acceptor* impurity atom has one less valence electron than its main neighbors; therefore it tends to take and keep one electron out of the filled valence band, leaving a *hole* to carry current, and producing a *p-type* semiconductor (p for "positive" -- the hole). Both n- and p-types have rather large ρ's, but smaller than those for intrinsic semiconductors, with less sensitive dependence on temperature.

Such solid-state electronic devices as diodes and transistors are combinations of n- and p-type doped semiconductors. Semiconductors are useful not because they conduct -- most metals far excel them in this respect -- but because of the peculiar ways in which they conduct.

Solid conductors are metals and alloys. The uppermost occupied band is only partly filled with electrons, and so at least some of these (roughly 1 percent, near room temperature) are free to move, changing their momentum and energy by going to unoccupied quantum states. In a perfect crystal lattice of this sort, free electrons would move with no hindrance at all -- i.e., $\rho = 0$. But no lattice is perfect: (a) thermal vibrations of the atomic particles make it irregular at any $T > 0$ °K; (b) at any temperature, including 0°K, impurity atoms and *defects* (displaced atoms) cause irregularities. Thus ρ is not zero. A conductor starts at 0°K with a finite value of ρ, caused by impurities and defects; as T rises, ρ first increases slowly, and then eventually in linear fashion with T. At elevated temperatures, ρ may rise faster than T. (This description ignores the special case of superconductivity.)

I. *Metal -- ρ versus T.*

The resistor used is a great length of very fine copper wire, to have an easily measurable resistance. *It is therefore quite fragile -- treat it carefully.* The resistance is measured with a Wheatstone bridge, at several widely separated temperatures. Representative cases are (a) liquid nitrogen (TREAT IT CAUTIOUSLY -- IT CAN CAUSE CONTACT INJURY SIMILAR TO A BURN), at 77°K; solid CO_2 at 195°K; the ice point (273°K); and the steam point (373°K).

For a given sample, ρ is directly proportional to the resistance R. Measure R at each temperature, plot a graph of R vs. T, and discuss its form.

II. Intrinsic Semiconductor: ρ versus T.

The semiconductor is in the form of a thermistor, chosen to have a rapidly changing resistance near room temperature. Thermistors can detect small changes of temperature, or small quantities of absorbed energy; they have other applications as well.

The thermistor and a thermometer are in a test tube of insulating oil, immersed in a beaker of water (the thermistor's resistance is large enough -- up to 10^5 ohms at 20°C -- so that conduction by water could affect the results). Start with cold water and oil. With a Wheatstone bridge, measure R about every 5°C from the original temperature to 30° higher. Heat the bath slowly; stir both water and oil; and remove the heater while reaching R -- all to insure temperature equilibrium.

Plot a graph of R vs. T and discuss it.

III. The Hall Effect.

(KEEP WATCHES AND ALL IRON OBJECTS WELL AWAY FROM THE MAGNET.)

When a conductor or semiconductor is carrying current, charges must be moving -- negative, positive, or both. Let us suppose that only electrons are present; they must be moving toward the end at positive voltage (Fig. 1). Now a magnetic field B straight across the line of motion (B indicated by crosses) must cause a side-thrust force driving the electrons to one edge of the conductor (the layer -e -e -e ··· shown in the Figure, if B is into the page. Thus a potential difference ΔV is built up at right angles to the current. When ΔV is large enough to repel any more accumulation of electrons, both the main current and ΔV remain fixed: the Hall effect.

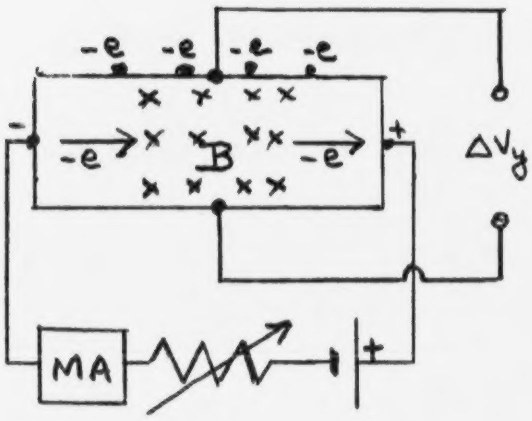


Fig. 1

If, instead, positive charges -- holes -- were carrying the current, their opposite charge and direction cause them to accumulate on the same side of the conductor, but now ΔV is reversed in sign. If equal numbers of holes and electrons were present, ΔV would nearly vanish (not quite, because holes and electrons move with different speeds). Hence the Hall effect can tell us what kinds of moving charges are present.

The equations are simple, for a single kind of charged particle. If n is the number of free charges per unit volume, each with charge e, the current density j_z (current in the z-direction, per unit area of cross section) is

$$j_z = n\ e\ v, \qquad (1)$$

where v is the average speed of the charges. (The derivation of Eq. (1) is exactly the same as for a mass flux.) Let the conductor dimensions be Δx, Δy, Δz as in Fig. 2. Then the total current I_z is $j_z\ \Delta x\ \Delta y = n\ e\ v\ \Delta x\ \Delta y$. (2) The resistance R is

$$R = \rho\ \Delta z/(\Delta x\ \Delta y). \qquad (3)$$

From Ohm's law ($V_z = I_z R$) and Eqs. (2), (3) we have

$$\mathcal{E}_z = n\ e\ v\ \rho, \qquad (4)$$

where $\mathcal{E}_z$ is the z-component of electric field intensity, equal to V_z/Δ_z.

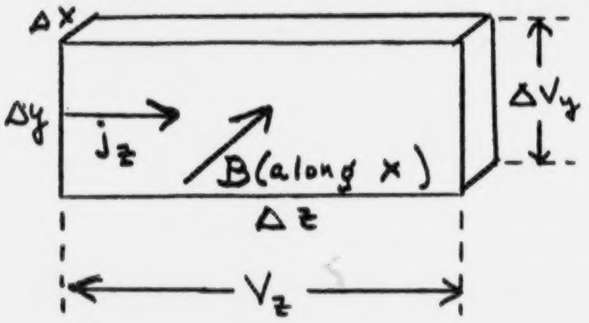


Fig. 2

The side thrust force on each charge, Bev, is balanced by the force e $\mathcal{E}_y$, where $\mathcal{E}_y = \Delta V_y/\Delta_y$:

$$Bev = e \cdot \mathcal{E}_y = e\ \Delta V_y/\Delta_y. \qquad (5)$$

The Hall coefficient R_H is defined thus:

$$R_H = \mathcal{E}_y/(B \cdot j_z) = 1/(n\ e) \qquad (6)$$

[multiply Eq. (5) through by n, and use (1)]. R_H is negative for free electrons (because "e" is negative). Evidently knowledge of ρ and R_H (which requires measurement of V_z, ΔV_y, $\Delta x, \Delta y$, Δz, I, and B -- fixes both n and v. Moreover, the direction of B and the polarities of V_z and ΔV_y show the sign of the major current-carrying charges.

Note from Eq. (5) multiplied through by n that

$$\Delta V_y = \text{const. } B \cdot I. \qquad (7)$$

Test this relation. B is constant (permanent magnet); I -- NOT TO EXCEED 0.3 ampere = 300 milliamperes -- is varied by means of a series rheostat, and ΔV_y is determined by a potentiometer (to avoid drawing any current). V_z can be read with a voltmeter of high resistance.

Plot ΔV_y vs. I., and discuss the graph. What is the sign of the current carrying charges here? Justify your answer. Compute R_H and ρ; then find n and v for a current of 100 milliamps. (The semiconductor is doped indium antimonide.)

In a gas at a temperature T, the molecules exhibit a random thermal motion. The velocities of the molecules are not uniform. In each gas we can find very fast as well as very slow particels. Characteristic to the temperature is, however, the velocity distribution of the particles. This tells us how many particles can be found in a velocity interval.

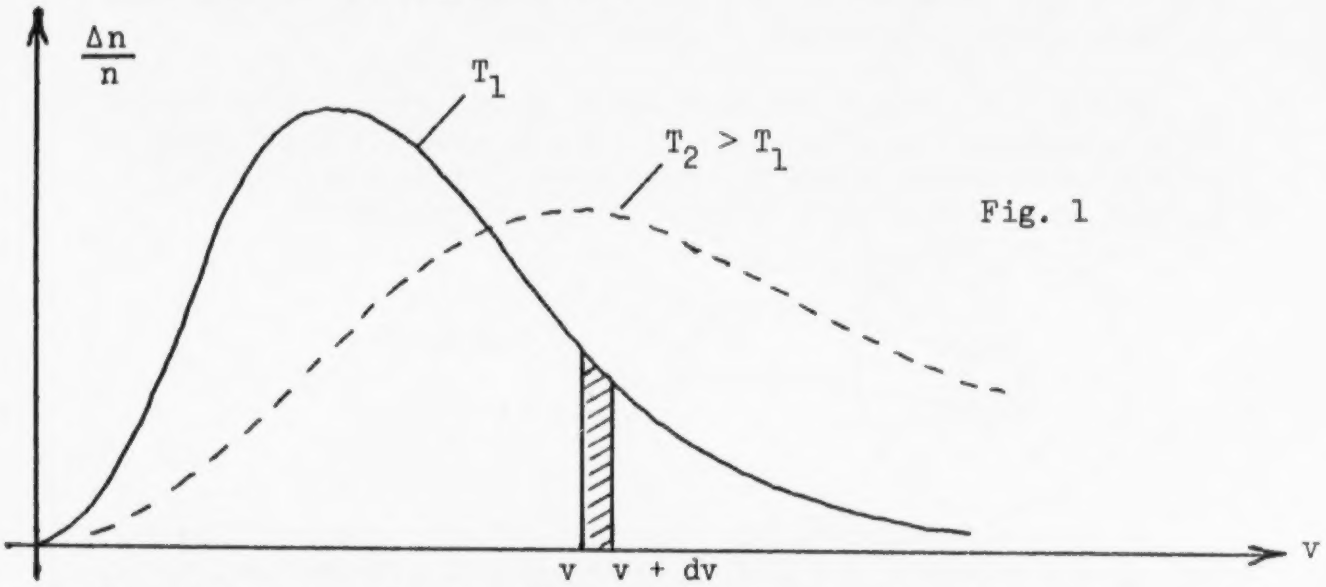


Fig. 1

Fig. 1 shows a velocity distribution curve for a gas at a certain temperature. The number of particles we find at a certain instant with the velocity greater than v, but smaller than v + v is proportional to the area marked in Figure 1. (Not a single particle has exactly a prescribed velocity v, in the mathematical sense, this is why we rather speak about particles occupying a velocity interval). The law for the distribution function of particles in a gas is due to Maxwell and Boltzmann, and named the Maxwell-Boltzmann distribution function.

For the same gas at different temperatures we get different distribution functions. The hotter the gas the larger the proportion of fast, energetic particles. From the velocity distribution, the average particle energy can be easily determined. It turns out that the average kinetic energy of the molecules of a gas is simply proportional to its temperature. In other words the temperature is the measure of the internal energy of the gas.

The form of the Maxwell-Boltzmann distribution function is the following:

$$\Delta n = C v^2 T^{-3/2} e^{-\frac{mv^2}{2kT}} \Delta v \qquad (1)$$

where Δn is the number of particles in the velocity interval Δv, and C is a factor characteristic to the gas. The temperature T is measured in degrees Kelvin, m is the mass of the particles, v velocity and k the Boltzmann constant.

A metal heated to sufficiently high temperatures emits electrons. This is the so-called thermionic emission and the cathodes of the vacuum tubes operate on this principle. The electrons leaving the filament act like a gas. The distribution function in such an electron gas is again described by Eq. (1). As the electron gas surrounding the cathode is in thermal contact with it, their temperatures are roughly equal.

Imagine a cathode filament emitting electrons, surrounded by a cylindrical anode (Figure 2). If the anode is kept negative with respect to the filament, electrons have to move "uphill" against an electric force in order to reach the anode.

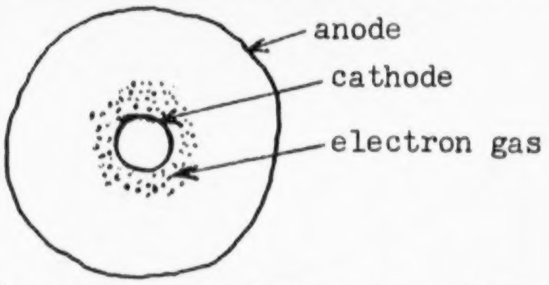


Fig. 2

It can be seen from Fig. 1 that some very energetic particles are always present in a gas. These particles in the "Maxwell tail" will reach the anode and produce a current in the anode circuit (Fig. 3).

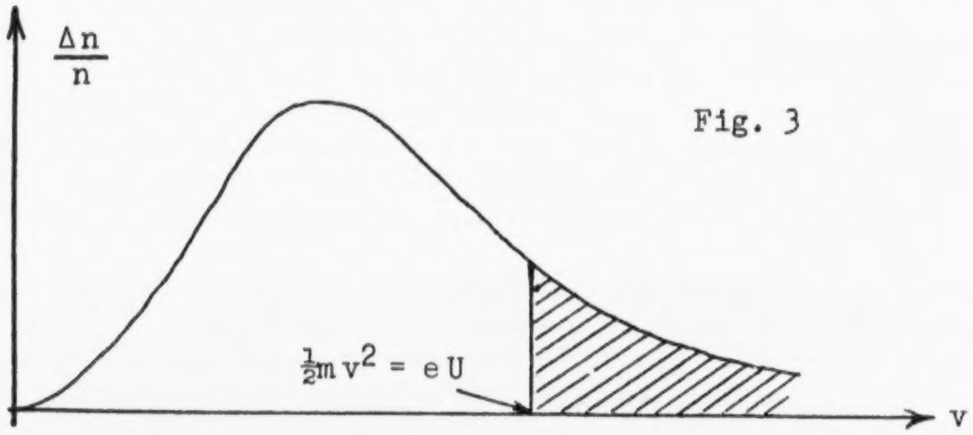


Fig. 3

The minimum kinetic energy an electron has to have in order to contribute to this current is determined by:

$$\frac{1}{2} m v^2 = eU \tag{2}$$

where e is the electron charge and U the retarding potential.

In calculating the electron current flowing to the cathode, the distribution function can be used, taking into consideration the distribution of the orientation of velocities as well. The resulting current is:

$$J = A T^2 e^{-\frac{eU}{kT}} \tag{3}$$

where A is a constant. The log of Eq. (3) yields:

$$\log J = \log A + 2 \log T - \frac{eU}{kT} \tag{4}$$

Plotting log J as a function of the retarding voltage at a constant temperature, we obtain a straight line. The slope of this line is

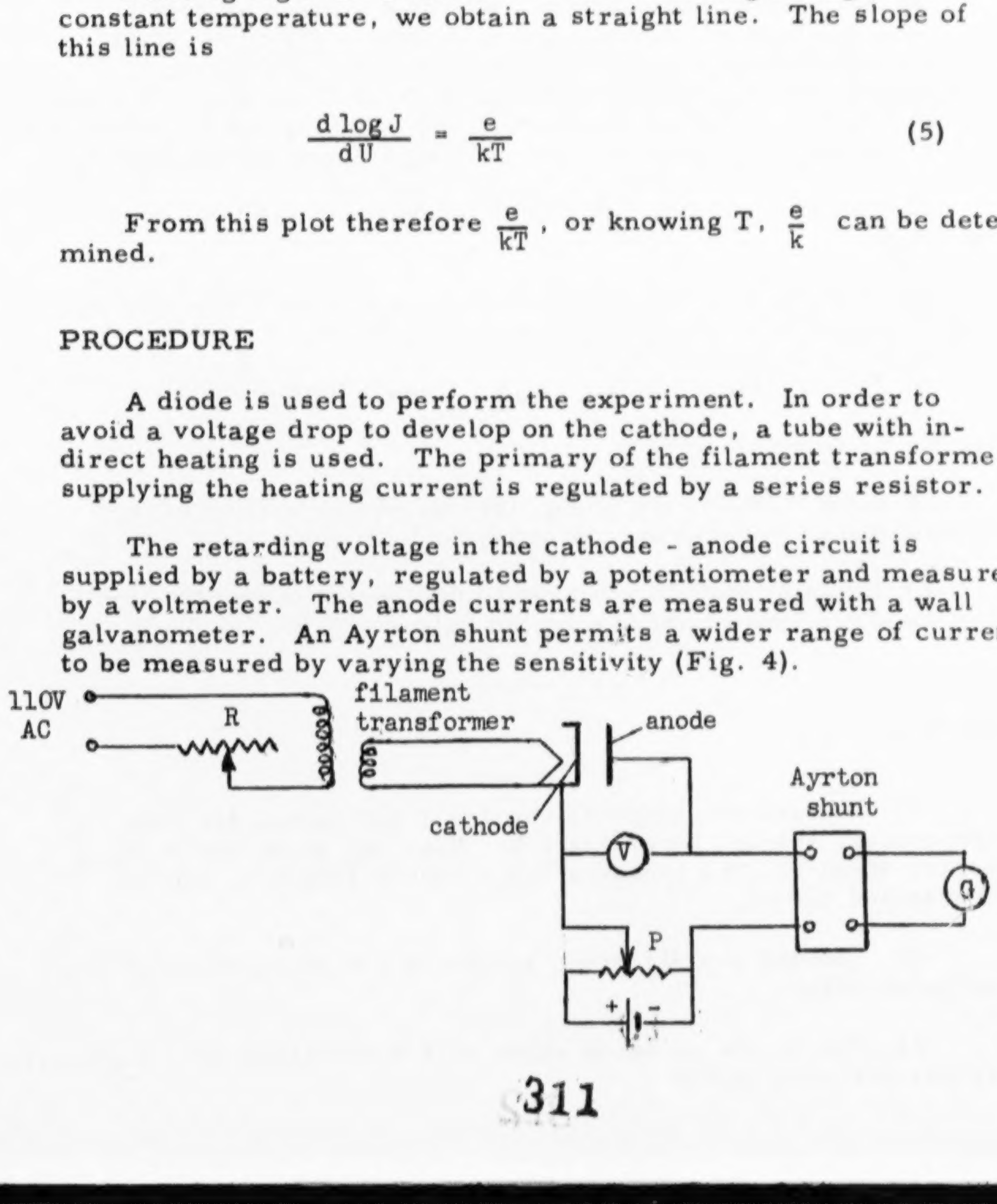
$$\frac{d \log J}{dU} = \frac{e}{kT} \tag{5}$$

From this plot therefore $\frac{e}{kT}$, or knowing T, $\frac{e}{k}$ can be determined.

PROCEDURE

A diode is used to perform the experiment. In order to avoid a voltage drop to develop on the cathode, a tube with indirect heating is used. The primary of the filament transformer supplying the heating current is regulated by a series resistor.

The retarding voltage in the cathode - anode circuit is supplied by a battery, regulated by a potentiometer and measured by a voltmeter. The anode currents are measured with a wall galvanometer. An Ayrton shunt permits a wider range of currents to be measured by varying the sensitivity (Fig. 4).



The temperature of the cathode is measured with an optical pyrometer. The principle of its operation is based on the fact that the radiation emitted by a body is characteristic of its temperature. In the optical pyrometer, we compare visually the light emitted by the cathode and an incandescent-lamp filament, built into the instrument. The light to be seen comes through a filter which transmits only a small wave length range from both sources. The temperature of the incandescent lamp can be varied by changing the heating current. If both filaments can be seen with the same brilliance, their temperatures are equal. The heating current of the built in filament is calibrated in a temperature scale and can be directly read on the instrument.

By taking measurements you are to follow this procedure:

1) Select a cathode temperature by setting resistor R. Caution: Don't overheat the cathode! Stay below the point marked on the resistor.

2) Measure filament with the optical pyrometer.

3) Starting with the largest retarding voltage available, measure the current with the galvanometer. Caution: Start always with the least sensitive shunt position, and increase sensitivity step by step until you get a current reading. You won't need to calibrate the galvanometer, the readings can be taken in an arbitrary unit. At each current reading disconnect the voltmeter.

Decrease retarding voltage by 0.1 V steps and take current readings.

4) Plot log J as a function of V on log paper. The resulting curve should be a straight line according to Eq. 4.

5) Repeat this procedure at 3 different temperatures.

Questions to be answered

1) Show on the basis of Eq. (5) that no calibration of the galvanometer is necessary in order to determine e/kT.

2) Explain, why do you have to disconnect the voltmeter if you measure small currents on the galvanometer.

WRITE UP

1) Measure the slope of the log J (U) curves for each temperature value. Determine d/k. Your log paper has a $\log_{10}$ scale, while Eq. (5) contains $\log_e$. Don't forget to use the conversion factor.

2) Compare the different values of e/k obtained, with the accepted value.

3) Taking the accepted value of e/k calculate the temperature values for your curves.

SEMICONDUCTOR TEMPERATURE COEFFICIENT OF RESISTANCE

University of Minnesota, Duluth

Object: to determine the temperature coefficient of resistance and the forbidden energy gap of an intrinsic semiconductor.

Apparatus. A post-office box Wheatstone bridge and a thermocouple connected to a potentiometer are used to make measurements of resistance at different temperatures of a semiconductor[1] over a temperature range of around 20° C to 95° C. The semiconductor is placed in a 2.5 cm by 7 cm test tube containing mineral oil. The thermocouple is soldered directly to the end of the semiconductor. The temperature of the semiconductor is stabilized, while measurements are being made, by insulation about the test tube and the mineral oil which acts as a thermal inertia. A variac is used with an isolation transformer to control the heating current.

Theory. Semiconductors have been defined[2] as electrical conductors with a finite forbidden energy gap between the valence and the conduction bands. In a metal such as copper there is an overlapping of the full and partially filled bands. This makes copper a good conductor because the outermost electrons are free to migrate with an electric field. While in an insulator the valence band is completely filled with electrons and the conduction band is empty.

The electrons of the atoms of which a solid is composed occupy certain allowable energy levels or bands. The band widths are limited and the bands may or may not overlap.

Semiconductors also have a high negative temperature coefficient of resistance. A study of this property of a semiconductor permits the determination of the width of the forbidden energy gap. The temperature coefficient of resistance is defined as:

$$\alpha = \frac{\Delta R}{R_o} \frac{1}{\Delta T}$$

(ΔT) is a change in temperature.

(ΔR) is the corresponding change in resistance.

(R_o) is the resistance at some initial temperature.

The charge carriers in an intrinsic semiconductor are produced by thermal excitation of electrons from the valence band to the conduction band; i.e., across the forbidden gap. The energy-band structure of an intrinsic semiconductor is shown below, Fig. 1. The expression for the resistivity, ρ, of an intrinsic semiconductor is:

$$\rho = A \epsilon^{-\frac{\Delta E}{2kT}} \text{ ohm cm}$$

(A) is a slowly varying function of the temperature.

(ΔE) is the width of the forbidden energy gap.

(k) is the Boltzmann's constant.

(T) is the Kelvin temperature.

This expression may be rewritten with ($R_T = \rho$) and ($A = R_o$). (R_T) is the resistance of the semiconductor at any Kelvin temperature (T). (R_o) is the resistance of the semiconductor at room temperature. Therefore to a fair approximation one may compute the energy gap, ΔE, for an intrinsic semiconductor from the slope of the plot: $\ln(R_T/R_o)$ vrs. $1/T$.

$$\Delta E = T\ (\ln R_T/R_o)\ 2k$$

Procedure. The arrangement of the apparatus is shown below, Fig. 2. Measure the temperature of the mineral oil at the very beginning of the experiment and set the ratio dial of the potentiometer so that 4 units on the 10 turn dial represents a temperature difference of 1° C between the thermocouples. Set the variac for values of heating currents of .05, .1, 12, .3, etc., up to 1 amp. Use 10 to 12 intervals, holding the current constant at a particular value for several minutes. Throw switch, S_1, connecting the Wheatstone bridge into the circuit, and carefully measure the resistance and the temperature simultaneously. BE SURE the circuit is wired as shown, in order to protect the Wheatstone bridge from the A.C. potential supplying the heating current. The heating current must be WATCHED very carefully so that it does not run away as the temperature changes. The change in resistance with temperature is an inverse relationship and it is not linear.

When R_T has been measured at the highest value of T used, transfer the test tube with the mineral oil and the semiconductor to a vessel of boiling water (a no. 2 tin can with a wire frame to hold the test tube). Measure the temperature of the boiling water to .2° C and compute the ratio of units/°C for the ten turn dial on the potentiometer.

Write-up. Compute the temperature coefficient of resistance for this semiconductor and compare the value with the handbook value given for a metal such as copper. Compute the energy gap between the valence band and the conduction band. The energy gap of an intrinsic semiconductor should be around lev .

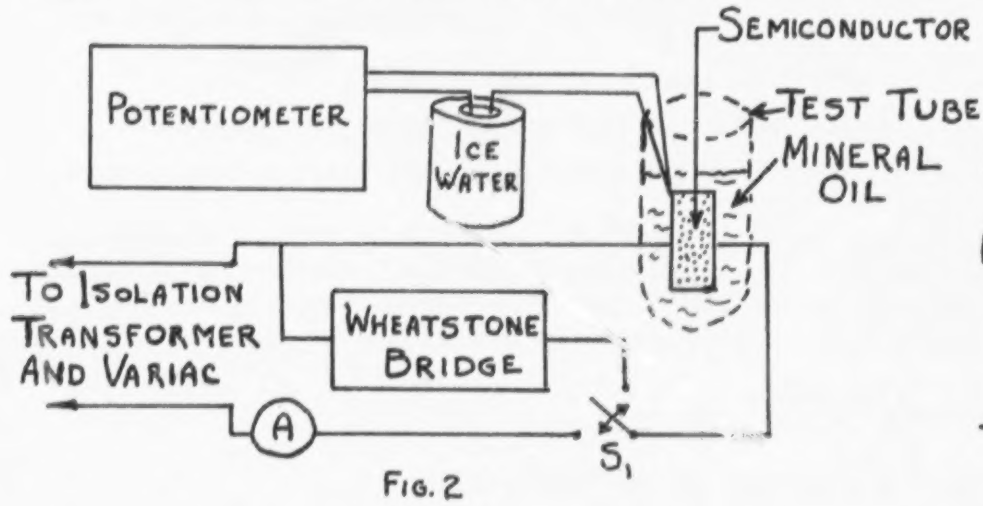


FIG. 2

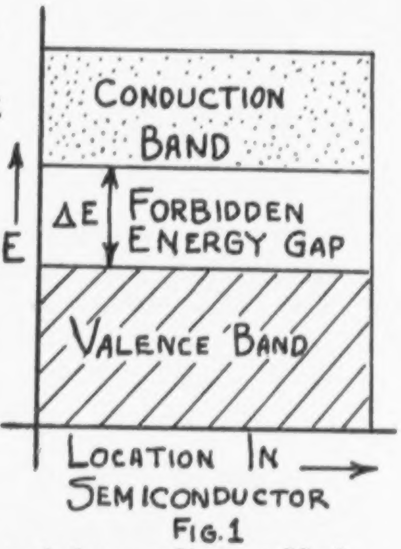


FIG. 1

1. The semiconductor used in this experiment is a globar resistor, often called a surgistor; Model FR9, list $.55 each, manufactured by Workman Electronic Products, Inc., Sarasota, Florida (.6 cm thick and 1.8 cm in diameter).

2. Goldsmid, H. J., Applications of Thermoelectricity; p. 24, John Wiley & Sons, New York, New York, 1960.

DRIFT MOBILITY OF CARRIERS IN GERMANIUM

Purdue University

Purpose

The determination of the drift mobility of minority carriers (electrons or holes), in a germanium sample by a direct measurement of the drift velocity of the minority carriers.*

Introduction

The values for the mobilities of the majority carriers in a semiconductor can be obtained as an indirect result of the measurement of the resistivity and the Hall coefficient (see exp. 3). In the present experiment the drift-velocity of carriers will be measured directly.

Consider a n-type sample of a semiconductor to which two probes A and B, a distance $\underline{d}$ apart, are attached. Suppose a number of electrons is withdrawn from A. An electric field between A and B ($V_B < V_A$) drives this charge pertubation in the original distribution towards B. (See fig. 1)

Injection of electrons at A in the case of a p type sample will result in a transport of the injected negative space charge to B under reversed potential conditions.

On observing the arrival at B and measuring the time interval connected with the travel from A to B the observed drift velocity $\underline{v}_g = \underline{d}/\underline{t}$ leads to the value of a drift mobility given by

$$\underline{\mu}_g = \underline{v}_g/\underline{E} = \underline{d}^2/\underline{Vt} \qquad (1)$$

The conventional but often misleading terminology defines the probe at point A as the emitter, that at point B as the collector.

It is obvious that a single injection of a space charge is difficult to observe. In the actual experiment this injection is repeated with time intervals long compared to the duration of the injection (pulse time) and the value of $\underline{t}$. For example:

Time between repetitions:	10^4 μ sec.
Pulse time:	1 μ sec.
Possible value of t:	10^2 μ sec.

The shape of the pulse is not so important, a square wave pulse is convenient. The shape will change during the transit from A to B, the carriers are subjected to diffusion and normal temperature scattering.[1] A time interval can be measured without error by the observation of the traveling time of the maximum amplitude of the pulse.

The injection of the space charge at A will result in a momentarily change of the potential at B. When B is connected with a scope, the injection at A and the passage at B of the space charge can be observed on the screen, when the time base is properly adjusted.

The measurements are carried out at room temperature and some higher temperatures. A large field between A and B will improve the observation and the measurement of the drift mobility. The reason is, that the traveling distance of the space charge in the observed time should be larger than the diffusion length of a minority carrier during its lifetime.

For example, for E = 0, the diffusion length of a hole in a n-type sample equals

$$L_p = \sqrt{D_p \tau_p}$$

Using $D \simeq 45\ \text{cm}^2\ \text{sec}^{-1}$, $\tau_p \approx 150\ \mu$ sec, than $L_p \approx 0.1$ cm. Taking d = 0.5 cm, than with $\mu_g \approx 1500\ \text{cm}^2/\text{volt sec}$ a field of 3 volt cm^{-1} between A and B is required to observe a reasonable travel time of 100 μ sec.

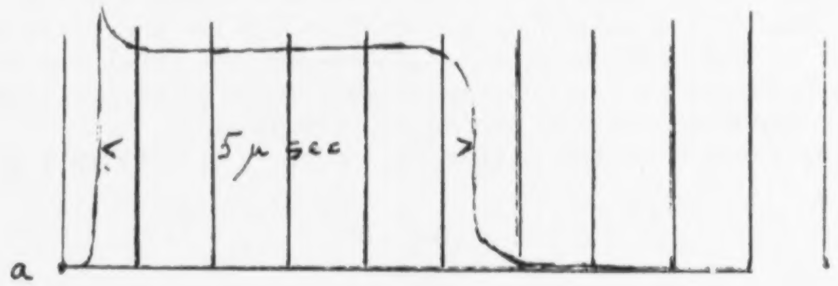
Equipment

A pulse generator (General Radio Model #1217) feeds voltage pulses of a chosen duration (for example, 0.5 μsec) with an adjustable repetition rate into the emitter. For a n-type sample the pulse polarity should be positive, and the direction of the dc current such that $\underline{V}_A > \underline{V}_B$ (positive direction).

The polarity of the pulse and the current are controlled by the adjustment of knobs on the front panel of the control box.
The observation will be made with a scope, that has a highest voltage sensitivity of 50 v/cm, and a pass band in the mega cycle range.

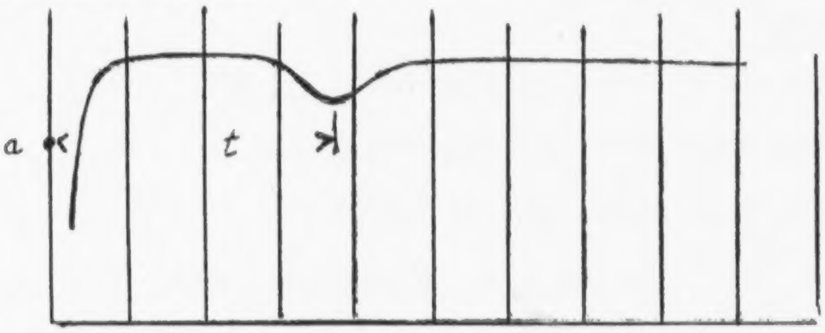
1. Make the following connections for the observation of the operation of the generator. Generator (-) terminal with isolated ground;
Generator (+) terminal with signal input terminal on the scope: generator bare ground terminal with scope ground terminal: generator synchronization terminal with scope trigger input terminal.
Check the operation of the generator by varying the pulse duration and the pulse repetition rate.
Adjust the trigger frequency and amplitude dials on the scope in such a way, that for pulse duration 1000 μ sec and 100 μ sec the full horizontal scale corresponds to the above intervals. In this way the time base of the scope is checked against the oscillator circuits in the pulse generator. Remove all connections between generator and scope, except for the synchronization connection. In particular, remove the jumper connection between the output (-) terminal and the isolated ground terminal on the generator panel.
2. Check the tungsten probes on the probe stand in the furnace. Re-etch, when necessary. Some p-type and n-type samples are provided for the experiment. Choose a n-type sample, etch when its surface looks contaminated. It is also recommended to etch for 10 minutes in a 50% HNO_3 solution after the CP 4 etch. This supposedly removes surface states. Solder two current leads and two potential leads (5 to 10 mm apart) to the sample and place it on the mica strip on the copper plate of the stand in the furnace. Solder the leads to the proper terminals. Lower the tungsten probes gently on the sample, adjust their distance equal to or within the distance betwen the soldered potential leads.
3. Connect a dc voltmeter, having at least 20,000 ohm/volt with the banana plug inserts on the control panel, and adjust the current through the sample so that a 2 volt potential difference between the probes is adjusted.
The circuit is now complete (fig. 1) and the following checks have to be made.

a. Shape of the pulse. Set scope sensitivity range at 1 or 0.5 v/cm; time base at 1 μ sec/cm.
For a pulse time of 5 μ sec, the following trace will be observed:



Note that approximately 0.4 μ sec elapses between the triggering of the rope at the start of the pulse in the generator (t = 0 at a) and the observation of entrance at the emitter by the potential drop over the collector. For travelling times of the order 100 μ sec this difference may be neglected. See that the generator output dial is adjusted in such a way that the pulse is not deformed.

b. Arrival of pulse at collector.
Change time base on scope to 50 μ sec/cm and sensitivity to 50 mv/cm. The trace has to be something like:



The light dot a corresponds again with t = 0, a travel time of 3.7 x 50 μ sec is illustrated.
Always move with the horizontal beam displacement knob spot a at zero: the vertical beam displacement knob is very suitable for finding in the best possible way the minimum at t.
A pulse repetition rate of 60 sec^{-1} will avoid vertical oscillations of the trace due to 60 c interference.

c. Determination of a suitable field range. The current through the sample will warm it up. For a given temperature it follows from (1) that $t = d^2/\mu_g V$ and the upper limit for V can be found by measuring t as a function of V. As long as $t \ \alpha \ V^{-1}$ is satisfied for constant pulse time and pulse amplitude no appreciable heating effects occur. Check this for a 5 μ sec pulse in the voltage range 0 to 3 v/cm. Also the resistance of the sample has to remain constant, check this.

Measurement

The injected charge cloud upsets the field in the sample. One corrects this in first approximation by extrapolating the measured travel times to zero pulse amplitude and zero pulse width.

a. Adjust a pulse of 10 μ sec, maximum amplitude. Measure t. Reduce amplitude (measured in v/cm or scope screen) and measure t. Extrapolate to zero amplitude.

b. Repeat this for several pulse times (for example 6, 4, 2, 1 and 0.5 μ sec). Use the values for zero amplitude to plot and extrapolate to zero pulse time.

4. Proceed in the described manner to take data at least 4 temperatures above room temperature to about 60°C. It is very important to wait for temperature equilibrium. Plot the thermometer readings as a function of time (say every 5 min.)

5. When the experiment on a sample is finished check the influence of light.

6. When time allows and when interested, repeat the experiment for a p-type sample. This is not required, it is better to have the best possible results for a n-type sample.

Evaluation of Results

The carrier mobility μ will be equal to or larger than the drift mobility μ_g (group mobility) derived from the experiment. Read the paper by Prince (ref b) and calculate the correction and the corresponding μ-values. Determine from the value for the resistivity at 300°K the carrier concentration from the sheet attached to the instruction for experiment 3.

Report

7. Attach to the report the extrapolation curves.

8. Plot μ_g and μ versus T in a double logarithmic plot.

9. Determine the slopes and compare with the standard expression $\mu = AT^{-3/2}$.
10. What effect will a transition to the intrinsic range have on the observation of t?
11. Describe the mechanism that enables the observation at the collector of the pulse entrance at the emitter.
12. Explain the effect of illumination in terms of the carrier concentration considering Prince's results.
13. Why should $\mu \geqslant \mu_g$?

References

a. J. R. Haynes and W. Shockley, Phys Rev., 81, 835 (1951)

b. M. B. Prince, Phys. Rev., 91, 271 (1953)

c. M. B. Prince, Phys. Rev., 92, 681 (1953)

1. W. Shockley, Electrons and Holes in Semiconductors, (1953) pp. 66-71, and Section 12.5.

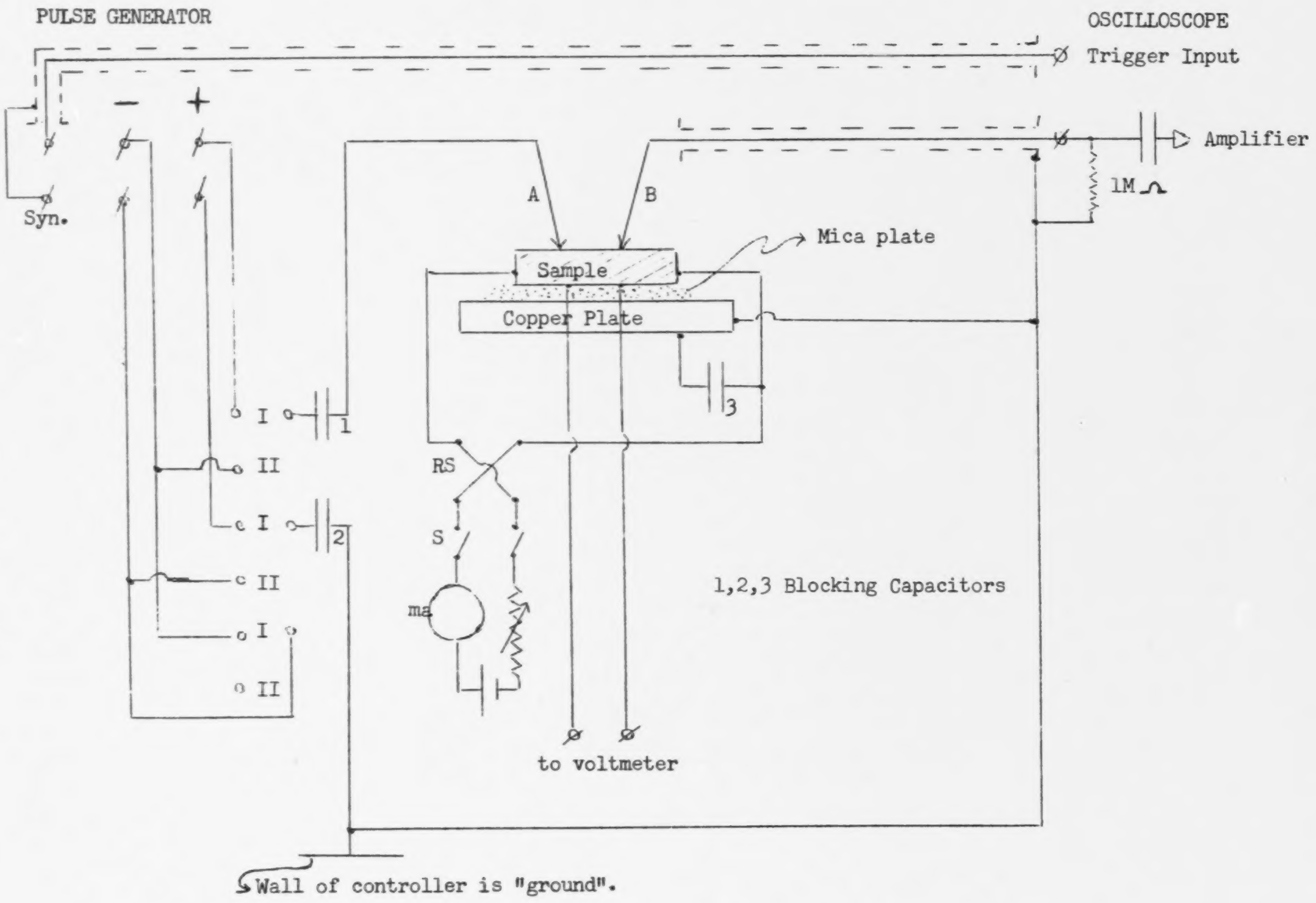

PULSE GENERATOR
OSCILLOSCOPE
Trigger Input
Syn.
−
+
Amplifier
1M Ω
A
B
Mica plate
Sample
Copper Plate
I
II
I
II
I
II
1
2
3
RS
S
ma
1,2,3 Blocking Capacitors
to voltmeter
Wall of controller is "ground".

Figure 1. Measuring Circuit

THE USE OF THE HALL EFFECT TO STUDY THE FORCE ON A CHARGE MOVING IN A MAGNETIC FIELD

University of Colorado

OBJECTIVE: The Hall effect is used to illustrate the force on a charge moving in a magnetic field and to illustrate some phenomena of current conduction.

REFERENCES: Halliday and Resnick, Physics for Students of Science and Engineering, Chapter 33, Sec. 5

THEORY AND PROCEDURE: The Hall Effect, which is investigated in this experiment, can be described in the following way. When a current carrying conductor is placed in a magnetic field which has a component perpendicular to the current, a voltage is developed across the specimen in a direction mutually perpendicular to the current and the magnetic field. This voltage is the Hall voltage and is proportional to the cross product of the current and the magnetic field. Consider the simplified case shown in Figure 1 where the current is in the x direction and the magnetic field B is in the positive z direction. The charge carriers making up this current will experience a force F_m

$$\vec{F}_m = q\,\vec{v} \times \vec{B} \qquad (1)$$

where q is the charge on the carriers and v is their velocity. This force causes the carriers to crowd to one side or the other (depending on the sign of their charge). Thus a net charge accumulates on the top and bottom surfaces of the specimen creating an electric field, $\vec{E}_H$, in the y direction which produces a force $\vec{F}_e = q\vec{E}_H$ to counteract $\vec{F}_m$.

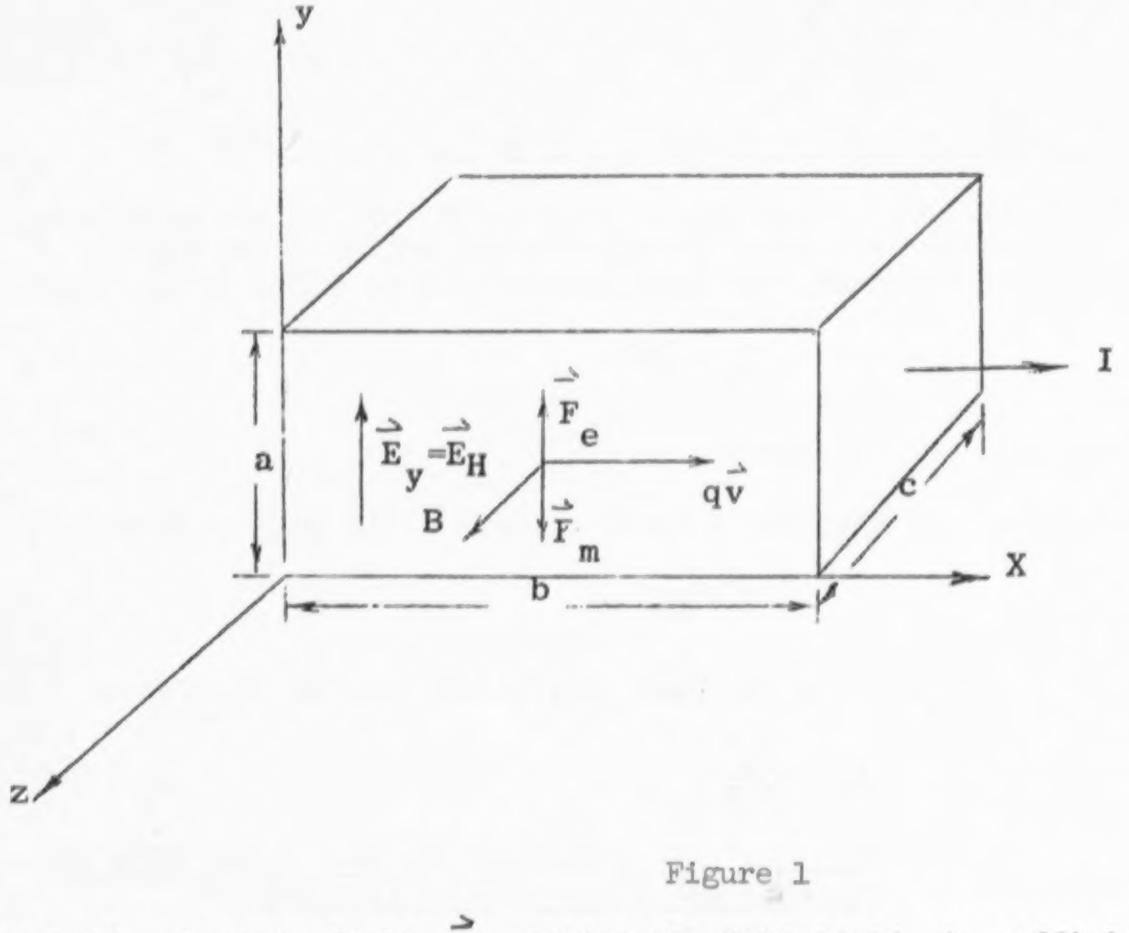


Figure 1

When this electric field, $\vec{E}_H$, called the Hall field, is sufficiently large so that no further charge separation occurs, then $-\vec{F}_m = \vec{F}_e$ and

$$\vec{E}_H = \frac{\vec{F}_e}{q} = -\vec{v} \times \vec{B} \qquad (2)$$

The direction is opposite to the magnetic force.

The current density or current per unit cross sectional area, $\vec{j}$, is given by

$$\vec{j} = n\, q\, \vec{v}$$

where n is the number of charge carriers per unit volume, q the charge per carrier and $\vec{v}$ is the average velocity of the carriers. Then

$$\vec{E}_H = -\frac{\vec{j}}{nq} \times \vec{B} \qquad (3)$$

This is written

$$E_H = R_H\, jB \qquad (4)$$

where R_H is called the Hall coefficient and is equal to $\frac{1}{nq}$. If the current is not perpendicular to the magnetic field, the Hall field is given by:

$$\vec{E}_H = R_H\, \vec{j} \times \vec{B}.$$

When a steady state has been reached, the charge carriers drift in the x direction that is j is in the positive x direction, although the total electric field is in a direction θ off the x axis. This fall angle is usually small and can be given as

$$\theta = \frac{E_H}{E_x} \qquad (5)$$

Is this situation a violation of Ohm's law $j = \sigma E$?

Now from the dimensions of the sample we can see that if two probe contacts were placed opposite each other at the top and bottom faces and the voltage between them is measured, the Hall field could be found by the relation

$$E_H = \frac{V_H}{a}.$$

where a is the thickness of the sample.

Also from dimensions we see that for a total current I the current density j will be

$$j = \frac{I}{ac}$$

but from Ohm's law $j = \sigma E_x$. Thus the Hall coefficient can be found from

$$R_H = \frac{V_H c}{B\ I}\ . \qquad (6)$$

How could you determine the Hall angle from the voltages and dimensions of the sample?

Set up the apparatus.

In lab there is an electro-magnet, a Hall plate of Bi, and current supplies for each. The Hall plate consists of a strip of Bi with known dimensions. Check the apparatus for the value of a, b and c. Contacts are provided to supply current in the long direction and sense the Hall voltage across the width (c). Instead of only two contacts to measure V_H there is one fixed contact probe (probe P_1) on one side and two on the other side, P_2 & P_3, stradling the position opposite the first probe (see Figure 2). Across P_2 and P_3 one may connect a potentiometer R_2 and by adjusting the potentiometer wiper arm A the position of a fictitious probe P_o may be set any where along the line joining P_2&P_3. P_o may be set exactly opposite P_1 by adjusting A so that there is no voltage between A and P_1 when $B = 0$ and $I \neq 0$.

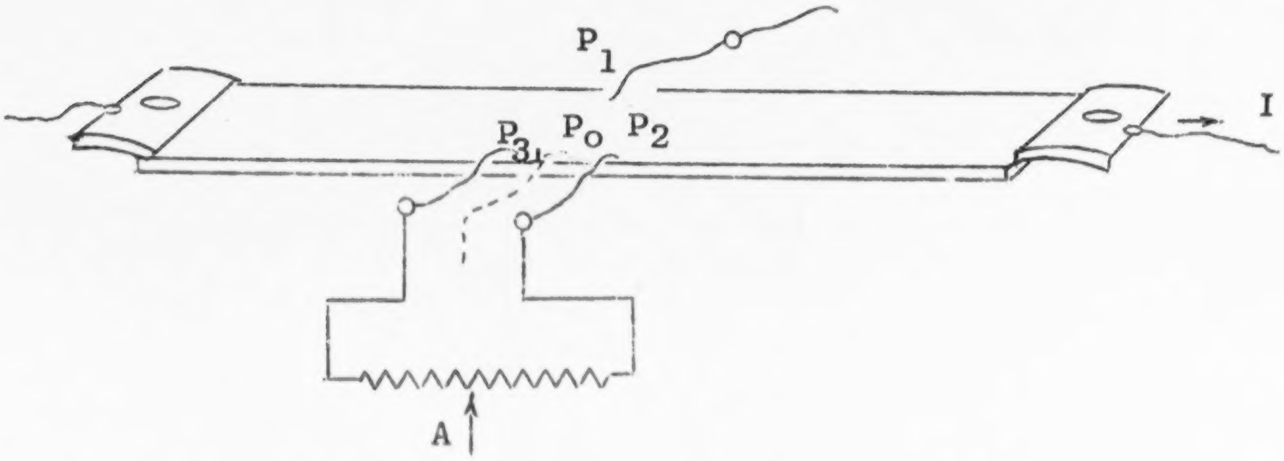


Figure 2

The electromagnet has a coil resistance of about 850 ohms and should be supplied a current which can be varied from 0 to 300 milliampers. DO NOT EXCEED 300 ma. The field B can be measured with a flux meter which is provided. The current through the Bi strip should not exceed 1 amp. To measure V_H a wall galvanometer may be used providing the sensitivity and resistance of the galvanometer are known. (If galvanometer is not marked ask your instructor for the values.)

Measure R_H as a function of both B and j, and plot.
Is R_H constant? Discuss any variations.

The Hall angle may also be measured in another way as a check on our work. Consider the sketch in Figure 3. Let R_o be the fraction of the potentiometer resistance between P_2 and the wiper arm A when the fictitious probe is at P_o directly opposite P_1, thus giving a galvanometer reading of zero when $B = 0$. When the field B, is applied, we adjust the wiper A to give again a zero reading. Let the resistance between P_2 and A be denoted by R' and let the position of the fictitious probe be denoted by P'. Then the angle between the lines P_1P_o and P_1P' will be

$$\beta = \frac{P'-P_o}{d} = \left(\frac{x}{d}\right)\left(\frac{R'-R_o}{R}\right)$$

From the geometry and our knowledge of electric fields and potentials we can see that for small angles

$$\beta = \Theta = \frac{E_H}{E_x}$$

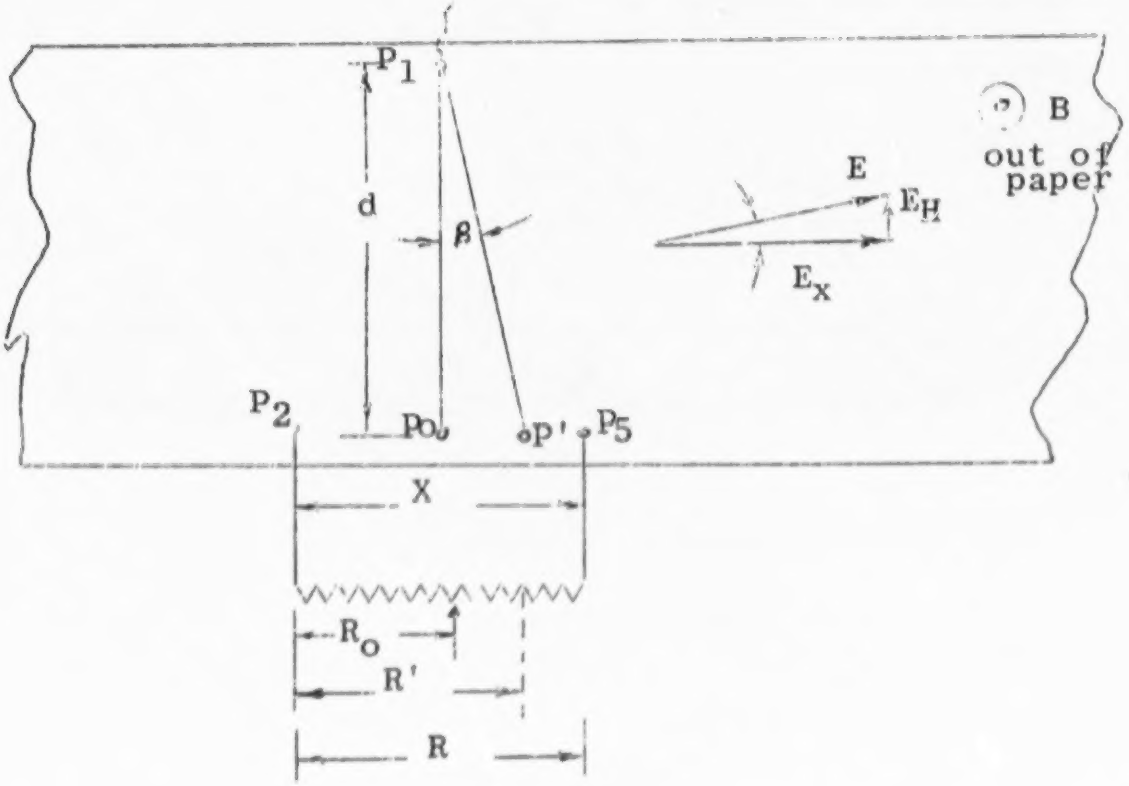


Figure 3

Thus we can measure Θ from the potentiometer readings. Compare this value with that measured earlier for the same value of B. Is it within experimental errors? Which value do you feel is the more accurate? Why? Compare the value of R obtained for bismuth with that for copper assuming there is one conduction electron per atom in normal copper.

Can the Hall effect be used as an analogue multiplier? How?

Can the Hall effect be used as a means to determine the sign of the charge carriers in the bismuth?

Purdue University

A. The measurement of the resistivity and the Hall-voltage of different Germanium samples.

Purpose

This is an exploratory experiment in that its results will demonstrate the differences between the n-type and the p-type samples. Also, when the impurity concentration in the high-resistivity p-type sample is of the right order, one might expect to find a marked difference in the values for the Hall-voltage at room temperature and at the temperature of liquid nitrogen.
The measuring circuit used in this experiment is an introduction to the more precise technique that will be used in experiment 3B.

Experiment

The sample to be investigated has to be mounted on a holder that can be placed between the polefaces of an electromagnet. A small dewar around the sample will protect it against draft during the room temperature measurements or can be filled with liquid nitrogen.
A simple potentiometer (Rubicon Model 2760) is used for the measurement of voltages. The requirements for the electric circuit are twofold. See fig. 1 for the circuit.

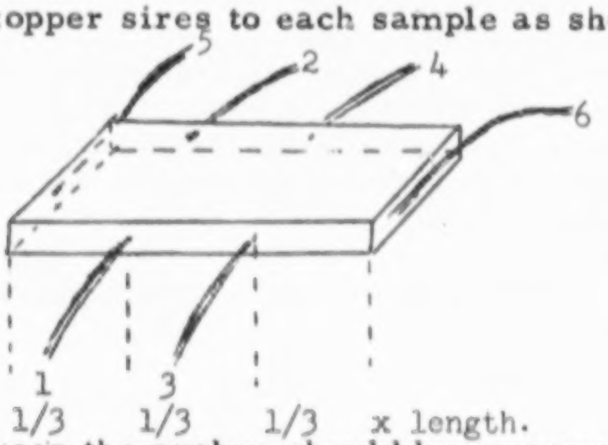
a) One wants to measure the resistance of the sample between two "resistance" probes;

b) One wants to measure the Hall voltage between two "Hall" probes.

For a) the sample resistance will be compared with a properly chosen standard resistor in series with the sample, by a comparison of the voltages over them. As long as the potentiometer is linear, and the currents are constant during this comparison, the absolute values of voltages and currents are of no interest for the computation of the result.
On the other hand, for the measurement of the Hall voltage (b) an accurate calibration of the scale of the potentiometer in volts is required. For this purpose the standard cell is used. The current through the potentiometer should be adjusted so that with dialsetting 1.0193 the galvanometer is at zero.
It is recommended that before doing the experiment the instruction for the Rubicon Model 2760 potentiometer should be read. The resistivity and Hall voltage of the 4 samples should be measured at room temperature and in the liquid nitrogen.

Procedure

a) Solder 6 38 copper sires to each sample as shown in the figure:



The distances between the probes should be measured as accurately as possible with a measuring microscope.

b) Mount each sample on a bakelite sample holder with a small strip of Scotch tape.

c) Solder the leads to the pinson the miniature socket, watch for corresponding numbers. The wiring in the supports is such, that the current lead closest to the socket is supposed to be wire 5.

d) Insert the holder in the socket on the support for the fixed temperatures (room temperature and 77°K). The support is placed on the top of the magnet.

When the electric connections are made, adjust the current through the potentiometer by changing the resistance in the resistance box. Realise that as long as there is no compensation, there is a drain on the standard cell. Therefore, use the sensitivity regulator and the galvanometer key.

With the reversal switch that controls the direction of the sample current in the position marked +, the current in the samples flows from lead 5 through the sample into lead 6.

The direction of the magnetic field can be determined with a small magnetic compass. Remember that the potentiometer circuit reads the voltage V_{12} and V_{34}. Also remember that the north geographic pole of the earth is a magnetic south pole.

You should include in your report a sketch or an explanation of how you determined the proper orientation. Also take one set of Hall coefficient data for a P-type and an N-type sample and determine which Hall probe (1 or 2) is closer to the top of the sample.

When making Hall measurements measure the sample current with the magnet on, since sample magnetoresistance can influence the sample current. Also check the value of the sample current in both current directions. Check regularly the adjustment of the potentiometer current against the standard cell.

After the room temperature measurements on all four samples are finished, repeat them at the temperature of liquid nitrogen.

A few remarks should be made about the use of liquid nitrogen as a coolant. As any liquid, its temperature depends on the pressure of the vapor above it. Determine the barometric pressure in the room and using the supplied P(T) curve for liquid nitrogen find the temperature of the boiling liquid. When an open dewar with liquid nitrogen has been exposed to the air for a long time, oxygen will condense into it. Though not of too great an importance for the present experiment, one should remember that continuous use of an open dewar (that is with free access of the air) will lead to a continuous change of the composition of the liquid, consequently to a continuous increase of its temperature. Also, because of the paramagnetic properties of oxygen, the liquid can no longer be used in a magnetic field.

Report

The evaluation of the results should be included in the report for experiment 3B.

B. The study of resistivity and Hall effect of a high resistance p-sample in the extrinsic and the intrinsic range.

Purpose

By determining the value of the Hall voltage, one might, with a properly chosen p-type sample observe the transition from p-to n-type The combined measurement of Hall-voltage and resistivity results in the determination of the carrier mobility is the extrinsic range.

Theory

First of all, the student should make himself familiar with the different galvanomagnetic and thermomagnetic effects.

See L. L. Campbell Galvanomagmetic and Thermomagnetic Effects
(Results in this monograph are of course far from recent).
See W. Shockley, Electrons and Holes in Semiconductors Ch. 8
F. Seitz, Modern Theory of Solids
A. J. Dehker, Solid State Physics Sect. 13.8
A.H. Wilson, The Theory of Metals. Sect. 8.5

For an intrinsic semiconductor, having n electrons/m^3, p holes/m^3 each electron and each hole having an average mobility μ_n and μ_p, the conductivity and the Hall coefficient R can be calculated under the further assumption that the drift velocities $v_{n,p} = \mu_{n,p}E$ of a carrier have a Maxwell Boltzmann distribution.

One finds:

$$\sigma = e\,n\,\mu_n + e\,p\,\mu_p \qquad (\mathrm{ohm}^{-1}\ \mathrm{m}^{-1}) \qquad (1)$$

$$R = -\frac{3\pi}{8e}\,\frac{n\mu_n^2 - p\mu_p^2}{(n\mu_n + p\mu_p)^2} \qquad (\mathrm{m}^3\ \mathrm{coulomb}^{-1}) \qquad (2)$$

The presence of two kinds of carriers, electrons and holes, implies that the thermal energy is large enough to excite electrons from the valence band into the conduction band, where they can move freely. The holes, left behind in the valence band, contribute also to the current when an electric field is applied.

At lower temperatures the thermal energy is not large enough to excite electrons from the valence band into the conduction band (See figure 4 and 5). The thermal energy is sufficient, however, to excite electrons from the donor states into the conduction band in n-type material. The holes left behind in the donor-states are bound and cannot move and the free charge carriers are only the electrons in the conduction band.

By substituting p = 0 or n = 0, the equations (1) and (2) reduce to:

$$\sigma = e\,n\,\mu_n \qquad\qquad \sigma = e\,p\,\mu_p$$

$$R = -\frac{3\pi}{8e}\,\frac{1}{n} \qquad\qquad R = +\frac{3\pi}{8e}\,\frac{1}{p} \qquad (3)$$

respectively,
for n-type, for p-type
samples in the extrinsic range.

Experiment

The measuring technique is essentially the same as in exp. 3A. The difference is in the use of a more sensitive and more accurate potentiometer (Rubicon Model 2780). Read the instructions of this potentiometer before using it.

The resistivity and Hall voltage of the high resistivity p-sample are to be measured between 77°K and 370°K. For this purpose a cryostat, sketched in the figure 2, is available in which the sample has to be placed. The vacuum chamber isolates the cooling liquid from the inner or sample chamber, that can be heated with the sample to temperatures above the temperature of the bath.

The temperature of the sample is measured by a copper-constantan thermocouple, one end is mounted close to the sample, the other is at room temperature outside the cryostat. Measure the voltage of the couple with the potentiometer. Calibrate the thermocouple when the system is at equilibrium at 77°K. Determine the bath temperature using the vapor pressure relation. Apply the correction pro rata to the standard calibration.

Procedure

1) Mount the sample holder in the cryostat. Check whether the sample and the contacts have survived by checking the resistivity (and eventual Hall voltage) at room temperature. exp. 3A

2) Pump both chambers with the mechanical oil pump. Fill the outer chamber with, say 5 cm helium gas, fill the inner chamber with atmospheric pressure of helium gas.

3) Measure the resistivity and Hall voltage. Compare the results with those obtained with the sample in experiment 3A.

4) Remove the rubber stop on top of the inner chamber and pour 1 plastic funnel full of liquid nitrogen in the chamber. Immediately start filling the outer dewar.

5) Pour another 3 to 4 full plastic funnels of liquid nitrogen in the inner chamber. Close the rubber stop and start pumping the inner chamber down to 1/3 of an atmosphere.

6) Watch the temperature by reading the copper-constantan thermocouple voltage.

7) When the temperature of liquid nitrogen has been reached, measure the resistivity and the Hall voltage. When the results agree and the circuit is in order evacuate the outer chamber, but calibrate the thermocouple first.

8) The outer chamber should be pumped down so far that the properly adjusted Pirani gauge indicates a pressure reading in the middle of the most sensitive scale. The pressure is then around 10^{-3} mm, the gas acts as heat leak of a useful magnitude. Try to maintain this pressure during the experiment.

9) Heat up the sample in about equal steps of 1000/T ($T \approx 78, 100, 120, 160, 250, 300$, degree K, or otherwise when resistivity and Hall coefficient are changing rapidly.

10) When room temperature is reached, remove the Dewar flask with the liquid nitrogen.

11) Continue measurements at intervals of approximately 20°. Do not exceed 100°C since melting of the soldered contacts would follow.

See the laboratory instructor or technician before switching off the apparatus.

Evaluation and Report

(Include the results from experiment 3A.)

1) Calculate the Hall coefficient and the resistivity and plot the logarithm of these two quantities against the reciprocal of the absolute temperature for all four samples.

2) Indicate on your graphs the intrinsic range and the exhaustion range. Define the terms "intrinsic range" and "exhaustion range".

3) Calculate the quantity R/ρ and plot it logarithmically against the logarithm of the absolute temperature. Indicate on your graph the temperature range in which R/ρ is proportional to the temperature.

4) Assume a temperature dependence of the mobility as $\mu = At^{-x}$. Calculate A and x from your curves and compare them with the values published by Morin and Maita in Phys. Rev. 94, 1525 (1954):

$$\mu_n = 4.9 \times 10^3 T^{-1.66} \; (m^2 \; volt^{-1} \; sec^{-1}).$$

$$\mu_p = 1.05 \times 10^5 T^{-2.33} \; (m^2 \; volt^{-1} \; sec^{-1}).$$

5) What value for x is expected from theory? Give a brief and simple derivation for this value. In metals the mobility depends on temperature as $\mu = CT^{-1}$. What is the reason for the difference in the power of T between a metal and a semiconductor?

6) Calculate the concentration of the charge carriers in the exhaustion range. Is the following statement correct: "The concentration of impurity atoms is equal to the concentration of the charge carriers in the exhaustion range"? If not, under what conditions would the above statement be correct, or how would you phrase it so that it is correct? Can you calculate the impurity concentration in the high resistivity p-type sample?

7) Calculate the average drift velocity v_d of the charge carriers in your n-type sample at the temperature of liquid nitrogen using the electric field applied at that temperature. Compare the drift velocity with the average thermal velocity v_{th} of the charge carriers at the same temperature.

$$v_{th} = \sqrt{\frac{2kT}{m^*}}$$

Where $m^* = 1/4 \, m_o$

m^* = effective mass

m_o = mass of electron

k = Boltzmann constant

Is the assumption concerning the magnitudes of v_d and v_{th} justified?

8) From your measurements calculate the intrinsic energy gap.

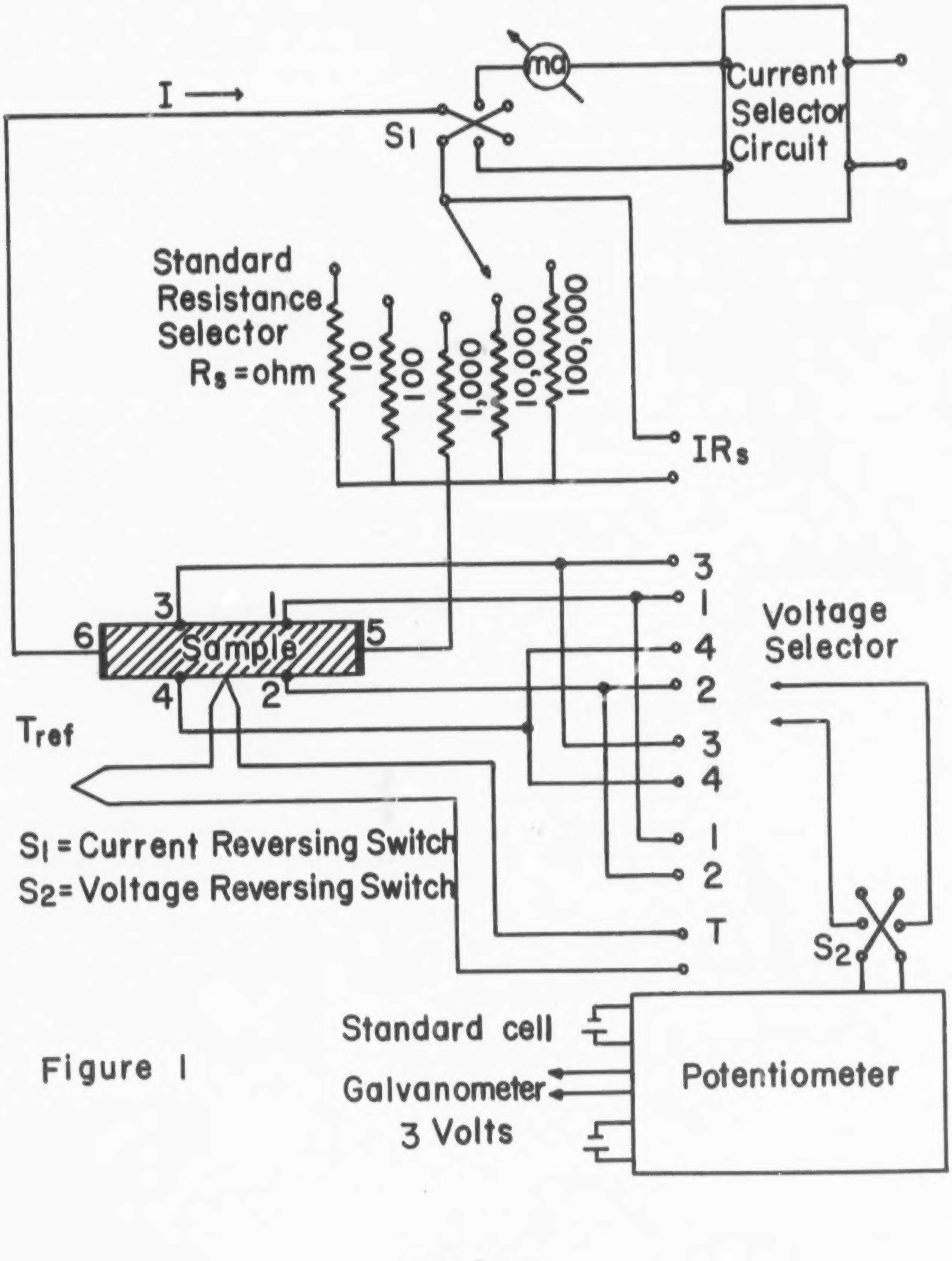
9) Determine the accuracy of your measurements of R, S, and T. How would you increase the accuracy?

10) A Pirani Gauge has good sensitivity only within a limited pressure range. Specify the pressure range and give reasons for its limitations, for the upper limit and the lower limit.

11) Suppose you have reached equilibrium in the cryostat at 200°K. Suddenly the barometric pressure drops from 770 mm to 740 mm. How would this affect your measurement?

12) For what reason will the liquid nitrogen become contaminated with liquid oxygen when the nitrogen is in an dewar open to the air (normal boiling temperatures: nitrogen 77°K, oxygen 90°K).

Electrical Circuit for the Measurement of Resistivity and Hall Coefficient


I ⟶
ma
Current Selector Circuit
S1
Standard Resistance Selector
R_s = ohm
10
100
1,000
10,000
100,000
IR_s
Sample
6
5
3
1
4
2
T_{ref}
3
1
4
2
3
4
1
2
T
Voltage Selector
S2
Standard cell
Galvanometer
3 Volts
Potentiometer

S1 = Current Reversing Switch
S2 = Voltage Reversing Switch

Figure 1

The Cryostat

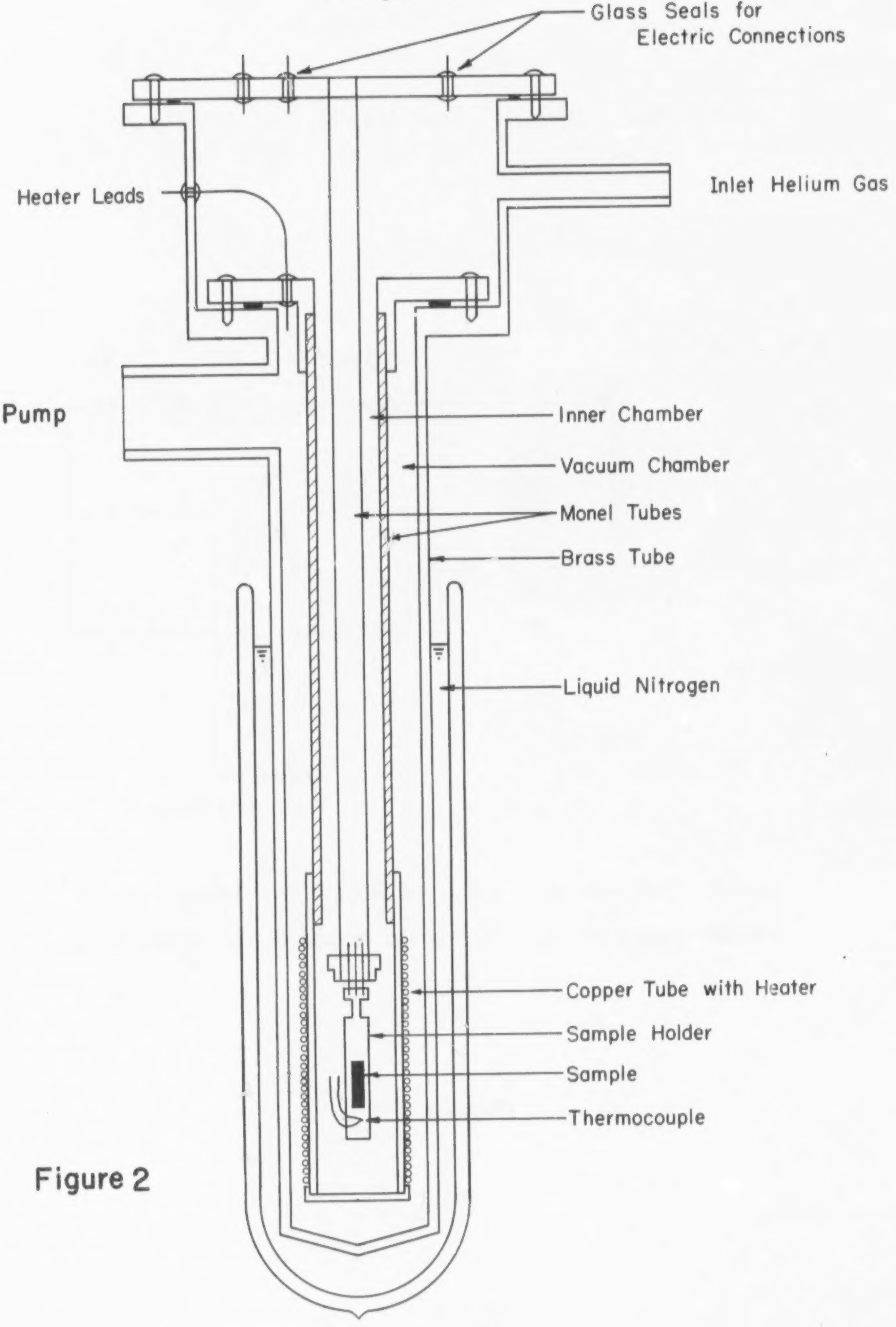
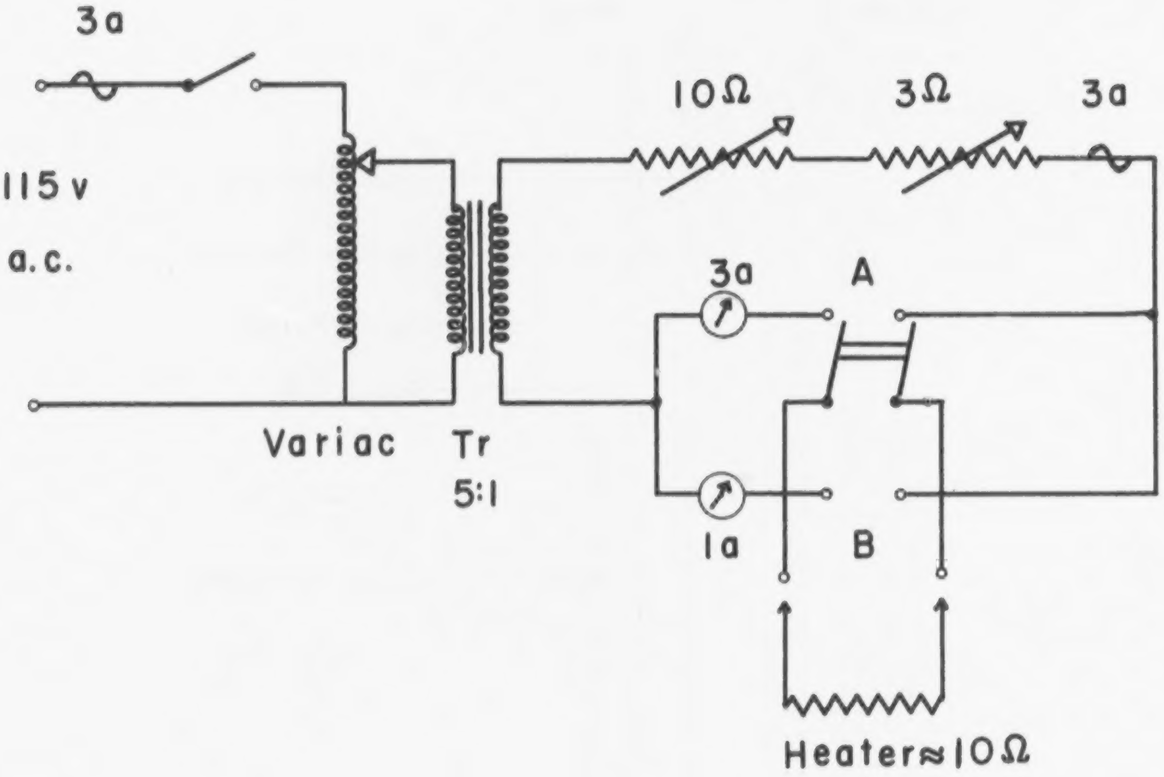

Glass Seals for Electric Connections
Inlet Helium Gas
Heater Leads
Pump
Inner Chamber
Vacuum Chamber
Monel Tubes
Brass Tube
Liquid Nitrogen
Copper Tube with Heater
Sample Holder
Sample
Thermocouple

Figure 2

Electrical Circuit for Heate in Cryostat

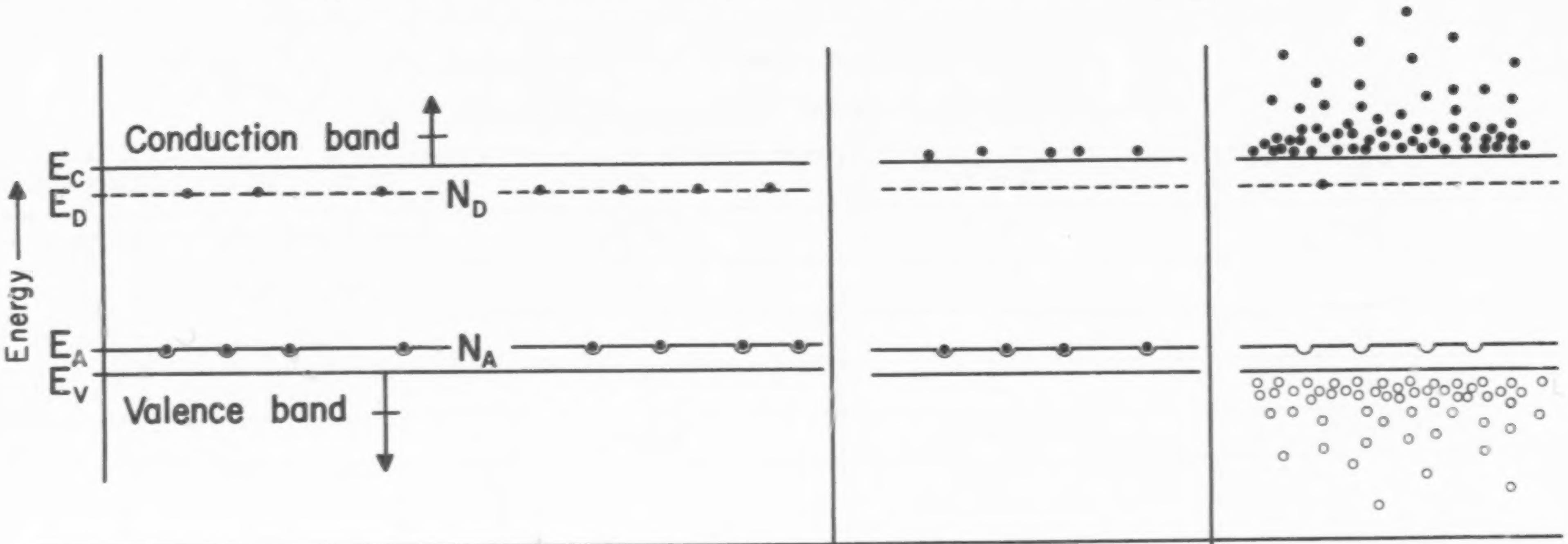

3a
115 v
a.c.
10Ω
3Ω
3a
3a
A
Variac
Tr
5:1
1a
B
Heater≈10Ω

Switch Position A : 1.5 a to Raise the temperature

Switch Position B : 0 to 1a to Maintain Epuilibrium

Figure 3

N-Type: Concentration donors > Concentration acceptors


Conduction band
Energy
E_C
E_D
N_D
E_A
E_V
N_A
Valence band

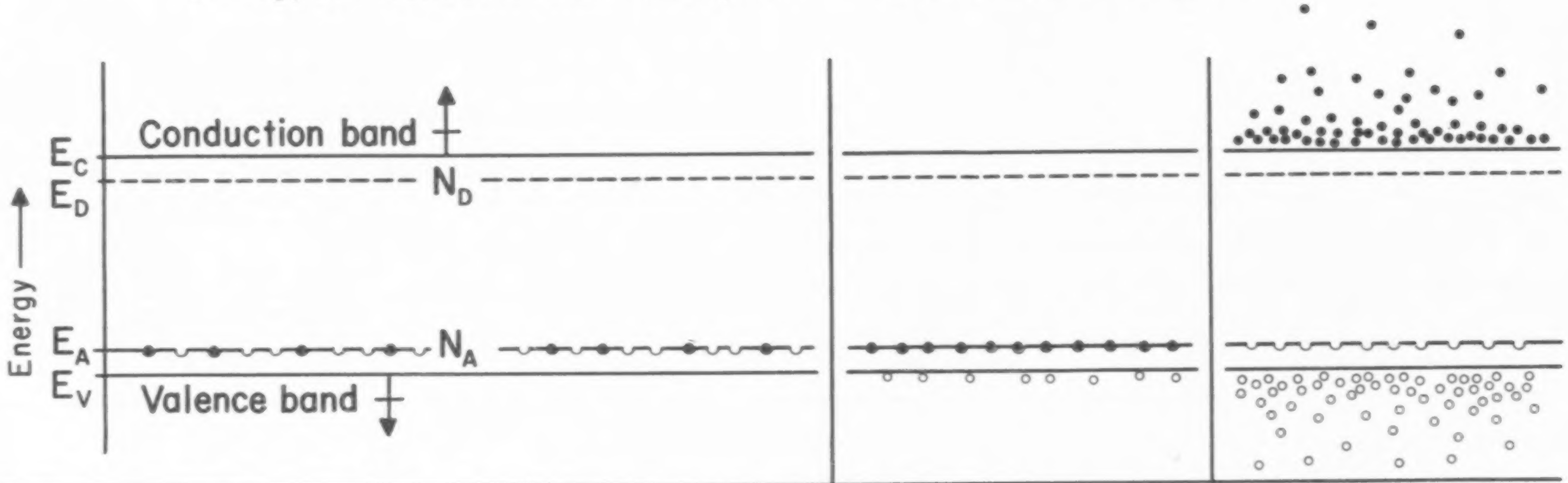
	T = 0°K	exhaustion	intrinsic
Conc. of el. in cond. band	0	$N_D - N_A$	n_i
Conc. of el. in donor states	$N_D - N_A$	0	0
Conc. of el. in acceptor states	N_A	N_A	0
Conc. of holes in valence band	0	0	p_i

N_D = Concentration of donor states
N_A = Concentration of acceptor states

$$n_i = p_i = 2\left(\frac{2\pi kT}{h^3}\right)^{3/2}(m_e^* \times m_h^*)^{3/4} e^{-\frac{E_c - E_v}{2kT}}$$

Figure 4

P—Type: Concentration acceptors > Concentration donors


Energy
E_C
E_D
E_A
E_V
Conduction band
N_D
N_A
Valence band

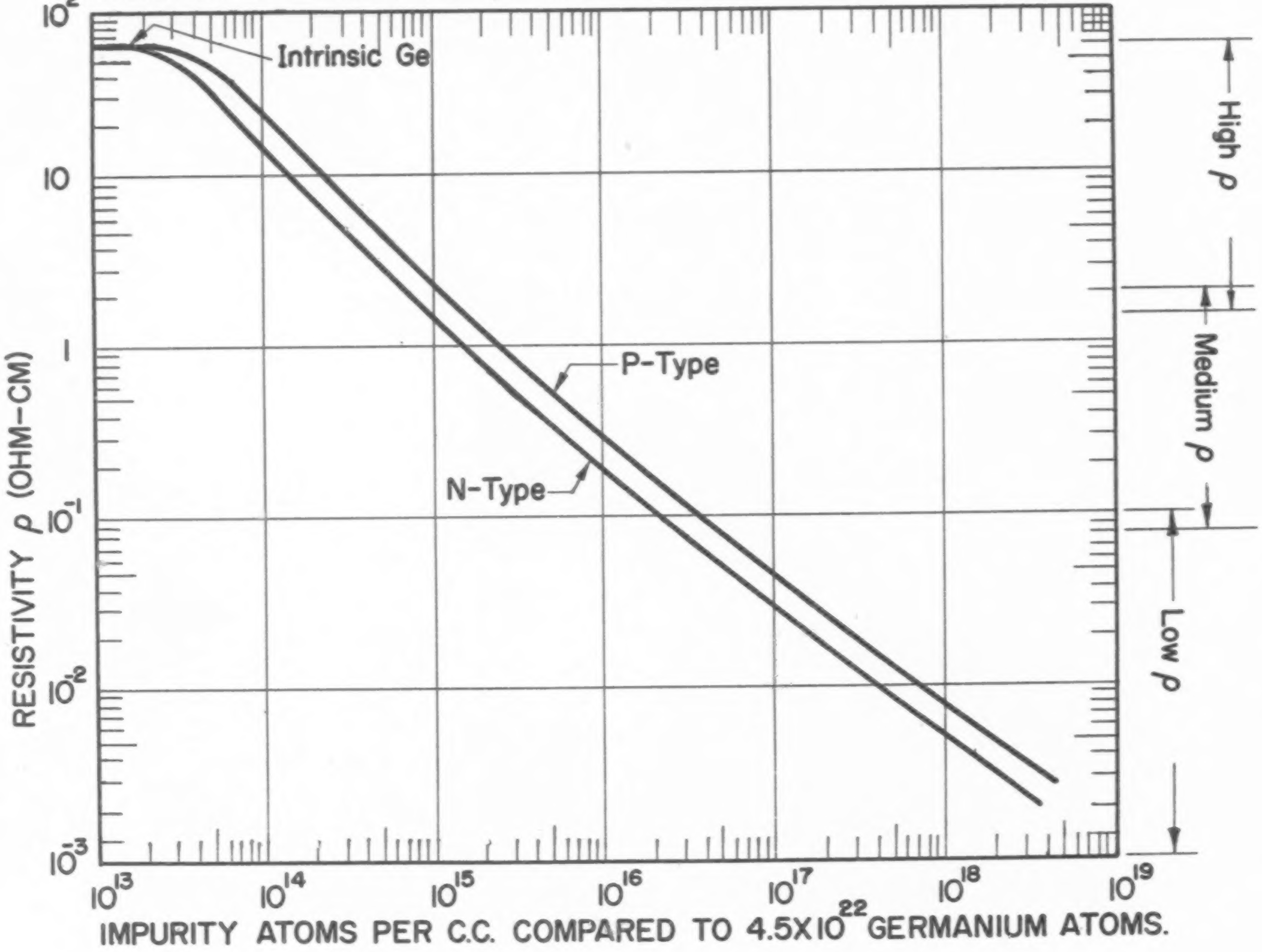
	T=0°K	exhaustion	intrinsic
Conc. of el. in cond. band	0	0	n_i
Conc. of el. in donor states	0	0	0
Conc. of el. in acceptor states	N_D	N_A	0
Conc. of holes in acceptor states	$N_A - N_D$	0	N_A
Conc. of holes in valence band	0	$N_A - N_D$	p_i

N_D = Concentration of donor states
N_A = Concentration of acceptor states

$$n_i = p_i = 2\left(\frac{2\pi kT}{h^3}\right)^{3/2} \cdot (m_e^* \times m_h^*)^{3/4} \cdot e^{-\frac{E_c - E_v}{2kT}}$$

Figure 5

DEPENDENCE OF RESISTIVITY AT 300°K UPON THE IMPURITY CONTENT OF GERMANIUM, SINGLY-DOPED



THERMOELECTRIC POWER OF GERMANIUM AGAINST COPPER

Purdue University

Purpose

1. The determination of the sign of the charge carriers in the sample.
2. The study of the relation, in particular for a p-type sample, between the thermoelectric power and the carrier concentration.

Preparation

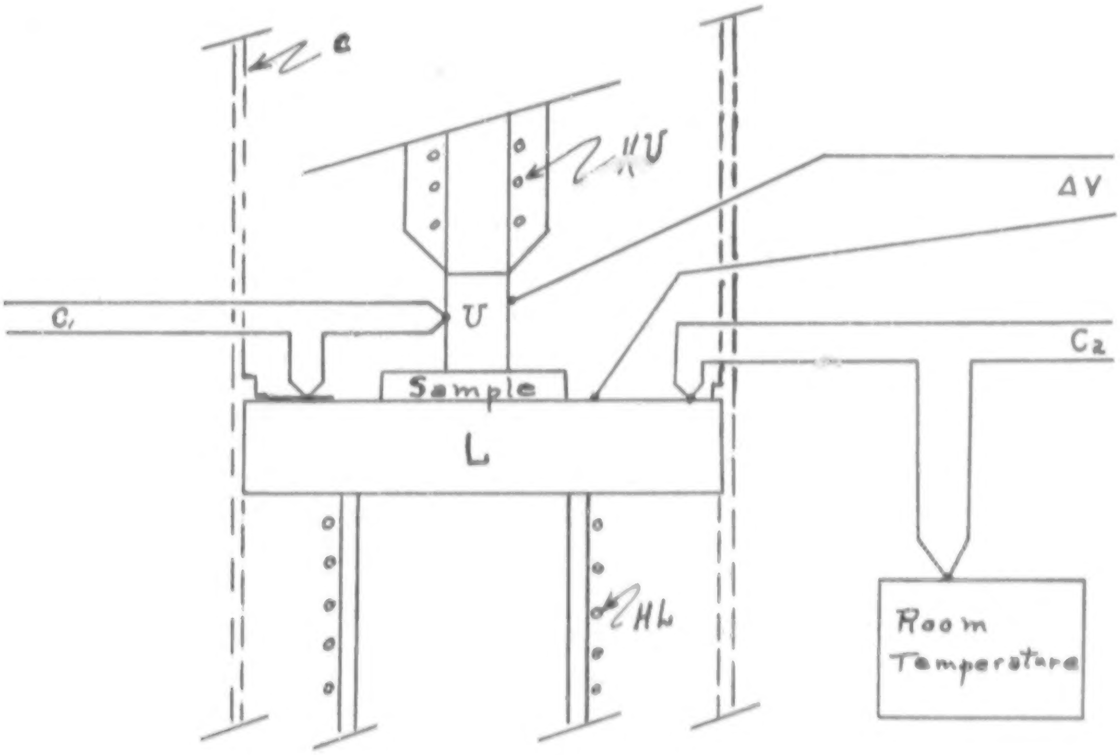
The thermoelectric power at a certain temperature is the slope of the curve of the thermoelectric voltage against temperature:

$$Q = \lim_{\Delta T \to 0} \frac{\Delta V}{\Delta T} \tag{1}$$

For small values of T, or for a slowly varying $V(T)$ curve one is allowed to take for Q the ratio between the voltage measured between the cold and the hot junction and the temperature difference. At room temperature it is in most cases accurate enough to take T of the order 1°C. For electrons $Q < 0$, for holes $Q > 0$. Read the literature, in particular reference 1 in order to get an idea about the behavior of Q that can be expected to exist for your samples. This knowledge will save time in doing the experiment.

Equipment

A stand for mounting a sample between 2 plates having different temperatures is provided. It consists (figure 1) of a 10 m.m. thick red copper disk, L, that can be heated to a certain temperature by the lower plate heater HL. A rod in thermal contact with the heating element HU of a soldering pencil acts as an upper plate. A copper constantan thermocouple C_1 registers the temperature difference between U and L, another couple C_2 registers the temperature difference between L and room temperature. C_1 is electrically insulated from plate L and connected to body U; C_2 is connected with L. Copper leads to L and U are necessary to measure the thermovoltage in the copper-germanium couple. A copper tube C, resting on L, protects the sample against draft. All leads go out at the top, through a hole in a transite cover on the copper tube.



Procedure

Before setting up the measuring circuit make the following preparations:
a. Etch the sample to be in vestigated. Be sure that no solder lumps will prevent the sample to lay flat on a flat surface.
b.. Clean with fine emery paper the copper surfaces that will be in contact with the samples.
c. The copper constantan thermocouples have to be electrically insulated from the plates. Check this.

The circuit to be used is sketched in figure 2. The dotted lines indicate units that are mounted. The voltages to be measured are supplied to the measuring circuit by setting the boxed (thermo voltage free) switch in the positions 1, 2 and 3. Position 0 breaks the contact for all voltages and shorts the measuring circuit. To facilitate a continuous check on the temperature of the lower plate, a millivolt meter is connected with the lower plate thermocouple C_2 in the positions 0, 1 and 2.

The resistors R_1 and R_2 are secondary standard resistors 11 and 1.1 ohm, respectively. Values from 0.1 ohm upward can be connected to the boxed switch terminals within the 1.1 range. Use in the measuring circuit a precision current meter. It is recommended to use this meter for all measurements in one current range and to vary the standard resistance values accordingly.

The voltages to be measured can be rather small. The total resistance in the copper-germanium circuit, on the other hand, may turn out to be rather large. The sensitivity for the voltage measurement then, will turn out to be rather low. Still, with patience and skillfull operation of the circuit good results can be obtained.

Make the following steps:

1. Without a sample in the stand, check the circuit. After observing the true zero of the galvanometer, with the boxed switch in the 0-position, and a shortening lead between the two switch output terminals, check the measuring circuit. Then measure the voltages, if any, in the positions 1, 2 and 3 of the boxed switch. These voltages should be not larger than 1 μv. See whether a small heat surge through H U gives a deflection in the galvanometer.

2. Start with an etched low resistance sample. Put it on L and lower U gently. The samples can stand a rather large pressure, they break under impact. Hold L at room temperature but heat U so high that the temperature difference between U and L in somewhere between 0.5 and 1 degree C. Wait for equilibrium and measure all voltages. Do not forget to note the room temperature.

3. The Q values have to be determined up to 100°C. Besides room temperature, 3 more temperature settings are sufficient for those samples in which the Q-values do not vary more than 20% in this temperature region. When a sample has shown (exp. 3) a transitian to the intrinsic state, spend a laboratory period to measure that transition (to be expected for high resistance p-samples) accurately in steps of 10°C or 15°C ambient temperature. First keep the current for H U the same and heat up the lower plate to about 100°C, selecting the current from the graph supplied for the stand displaying the final temperature of L as a function of heating current. Notice that the temperature difference between U and L plate changes sign. Slowly increase the temperature of the upper plate, aiming at about 1°C difference. Decide for which temperatures below 100°C the sample should be measured.

4. Check on the polarity of all the voltages. Determine the sign of Q for the samples. Note that for copper-constantan couples the copper lead to the cold junction is negative with respect to the copper lead to the hot junction.

Evaluation and Report:

1. Indicate clearly in your report the calculation of the actual thermoelectric voltage, of the sample temperature and the Q values by giving a sample calculation. Summarise the results in tables. Plot Q as a function of the sample temperature. Determine the accuracy of the Q values, considering that the accuracy of the resistors is 0.04% and the accuracy of the meter is given on the instrument.
2. Make a table of the final results of the samples investigated: Code number, the room temperature value for the sign of the carrier the resistivty, the hall coefficient, the thermoelectric power. Indicate the approximate temperature of the transition from impurity to intrinsic conduction.

Remark on the theory.

For a p-type sample the Hall and Q reversals do not occur at the same temperature. Reason: In first approximation the electron density, where reversal occurs is determined by the factor:

$\mu_1^2 n_1 - \mu_2^2 n_2$ for the Hall effect

and

$\mu_1 n_1 - \mu_2 n_2$ for the thermoelectric effect

(subscript 1 for electrons, 2 for holes).

3. Taking the effective mass $m^* = m_o$ (the free-electron mass), neglecting the effect of impurity scattering and using the relation between the carrier concentration and the hall coefficient R for the exhaustion range, then (Ref. 2):

$$Q = \pm \frac{k}{e} \left\{ \ln(R\, T^{3/2}) - 5.32 \right\} \qquad (2)$$

(k is the Boltzmann constant).
Check for those samples, which at room temperature are still in the exhaustion range, how much the measured values of Q differ from those calculated with (2) taking R as measured in experiment 3.
4. Discuss briefly how Q should be effected by light when a sample is photo conductive.

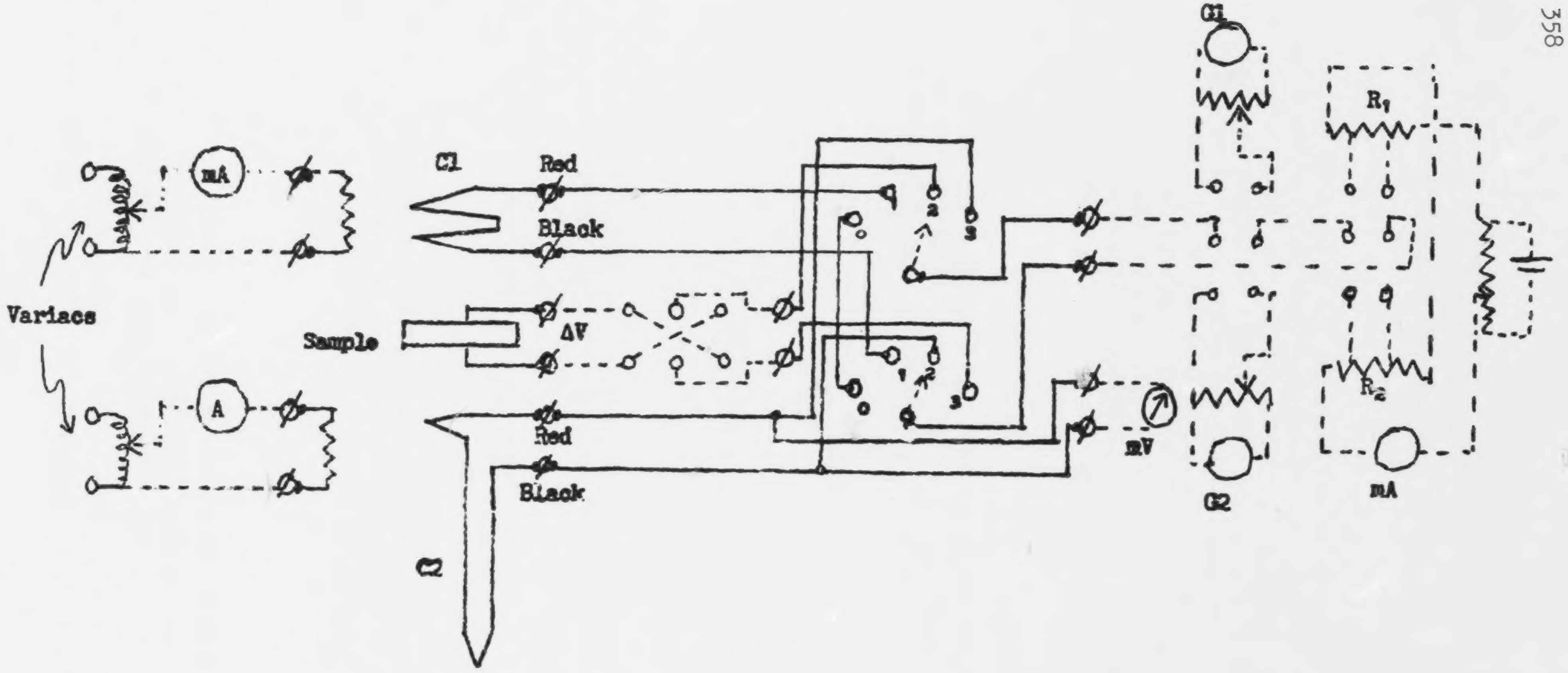
References:

Besides the general textbooks, see in particular:

1. K. Lark-Horovitz "New Electronics" in:
The Present State of Physics, Am. Ass. for the Advancement of Science, 1954.

2. V.A. Johnson and K. Lark Horovitz, Phys. Rev 69 258 (1946)

Expt. 4 Circuit as modified January 1963.
Figure 20.


Variacs
mA
A
C1
Red
Black
Sample
ΔV
Red
Black
C2
0
1
2
3
G1
G2
R1
R2
mV
mA

G_1: Galvanometer, low resistance (≃10 ohm); CDR x ≃50 ohm.

G_2: Galvanometer, medium resistance (≃500ohm); CDR x ≃25,000 ohm.

P: Helipot

For C1 and C2: Red terminal is connected with the hot junction.

COLOR CENTERS

Dartmouth College

When the alkali halides are exposed to x-rays, they turn a deep color which is characteristic of each type of crystal. For example, KCl turns magenta, NaCl yellow, etc. The color of each crystal is determined by a characteristic absorption band whose maximum lies in or near the visible spectrum. This effect is also observed when the alkali halides are subjected to ionizing radiations, or when they are heated in the vapors of their alkali metals, or when an electric current is passed through them while they are heated. The absorption band is called the F-band and the color centers in the crystals F-centers. We will use the last mentioned method (electrolysis) to color our samples.

When an electric current is passed through a sample in such a way that the net effect is to replace some of the negative ions by electrons, color centers are observed. It has been shown that an F-center is, in fact, an electron trapped in a negative ion vacancy, so that under ideal conditions each electron entering the crystal will combine with a vacancy to form one F-center. Therefore, by seeing the diffusion of a cloud of F-centers you are seeing the diffusion of electrons. The reversing switch is provided to see the electrons "sucked" back out of the crystal when the polarity of the electrodes is reversed.

References:

Dekker, Solid State Physics (Prentice Hall, '57), Ch. 15
Seitz, Rev. Mod. Phys. 26, 7 (1954), I, II, V.
Mott and Gurney, Electronic Processes in Ionic Crystals, 2nd ed. (Oxford, 1948), Ch. IV.
G. Heiland, Z. Physik 127, 144 (1950).
W. T. Doyle, Phys. Rev 111, 1067, 1072 (1958)

Apparatus:

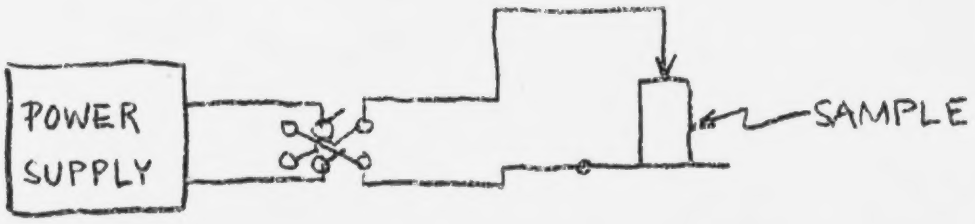
A. For coloration;
furnace, power supply, milliammeter, electrodes, reversing switch, CCl_4

B. For absorption data;
monochromator, light source, Hg and H sources, Densichron with red- and blue-sensitive probes, crystal holder and attachment for the densichron, lenses

C. Miscellaneous;
single edged razor blades, hammer, tongs, sample crystals (KCl, NaCl, and KBr)

Directions:

Since it is difficult to color the samples uniformly by electrolysis, start with somewhat larger crystals and cleave away the sides to fit the sample holders for the Densichron after they are colored. It is a good idea to practice cleaving before you actually start to color the samples.



To color, place the sample on top of the silver electrode in the furnace and place the pointed electrode on top of it with a slight pressure (provided by the brass weight). As the crystal is heated, keep checking for the current (at high potential) but do not turn on the power for good until the current exceeds ½ ma. The sample will be colored well by the time the current increases to 5 ma and should be quickly taken out to be quenched in CCl_4.

Colloids, in addition to F-centers, will form if the sample is not quenched. The formation of the colloids in NaCl can be seen clearly by heating the colored sample a few hundred degrees (say to 300°C).

Calibrate the monochromater after setting the prism at a reproducible position. Recheck the calibration several times during the course of the experiment. Check whether the red and blue probes give the identical readings at the same wave length setting. Make sure that the entrance and the exit slits are paralled. The source and the optical system should be arranged so as to achieve the maximum light output from the monochromater. The intensity should be such that it is possible to get reasonable deflection of the Densichron absorption peaks; i.e. Do not get stuck on the most sensitive scale. Even with this precaution, the absorbance will probably have to be calculated from the %-transmission readings near the absorption peaks rather than being read off directly. (Which may not be a disadvantage.) Plot the sensitivity of the two probes as a function of wavelength to determine the crossover point.

Measure the absorption spectra of the colored samples against the uncolored references. Plot the absorbance against wave length and energy. The absorbance is usually defined to be $\ln(I_o/I_t)$, where I_o is the intensity of transmission without absorption due to color centers and I_t is the intensity of transmission through the sample. Note that the readings on the "optical density" scale of the Densichron differs from this definition in that it is $\log_{10}(I_o/I_t)$.

Note: Avoid touching the samples with your hands at all times. This is for the same reason that the samples are quenched in CCl_4 instead of, say, H_2O.

Questions:

1. Are the charges conserved in the crystal during coloration by electrolysis? What happens to the alkali and the halogen ions?

2. What are colloids?

3. What is the relation between the frequency at the peak of the F-band and the interionic distance for the alkali halides? Can you give a crude explanation for this?

Nuclear and High Energy Physics

GEIGER COUNTERS AND SCALING SYSTEMS

Oak Ridge Institute of Nuclear Studies

OBJECTIVES:

1. To determine the proper operating voltage of a Geiger Counter.
2. To demonstrate the variation of counting rate with distance.
3. To determine the half life of a radioactive isotope.
4. To determine the resolving time of a Geiger Counting system.
5. To determine the half value layer for a gamma emitting isotope.

REFERENCES:

Pulse Counters

Bleuler, E., and Goldsmith, G.J., Experimental Nucleonics, Rinehart and Co., New York, 1952, pp. 16-27; 49-79.

Curran, S. C., and Craggs, J.D., Counting Tubes, Butterworths Scientific Publications, London, 1949, pp. 29-48; 65-78; 94-102.

Friedlander, Gerhart and Kennedy, J.W., Nuclear and Radiochemistry, John Wiley & Sons, New York, 1955, Chapter 8, pp. 231-236.

Lapp, R. E., and Andrews, H.L., Nuclear Radiation Physics, 2nd Edition, Prentice-Hall, New York, 1954, Chapter 9, pp. 225-237.

Price, W. J., Nuclear Radiation Detection, McGraw-Hill Co., New York, 1958, Chapter 5, pp. 115-129.

Overman, R.T, and Clark, H.M., Radioisotope Techniques, McGraw-Hill 1960, Chapter 2.

INTRODUCTORY MATERIAL:

The G-M Counter Tube

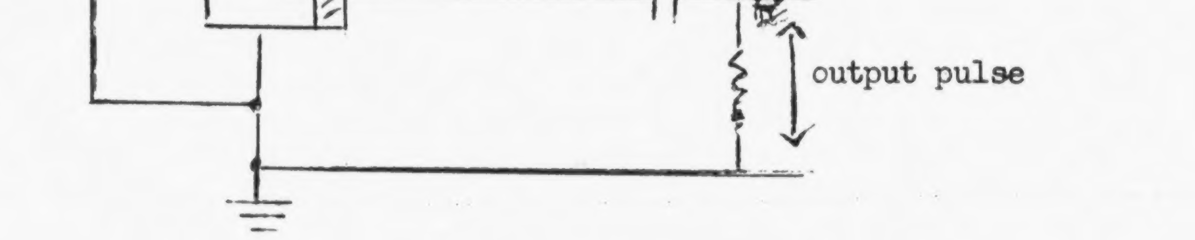
The Geiger-Mueller counter is a sensitive instrument for the detection and measurement of radiation. The counter tube consists of two electrodes, a fine metal wire, the anode, surrounded by a hollow conducting cylinder, the cathode. The two electrodes are enclosed in a glass envelope containing gas at low pressure. A potential difference is maintained between the electrodes only slightly below that which will produce a discharge. The tube normally has a thin wall or window section to permit the entrance of soft radiation into the chamber. By increasing the thickness of the wall or by placing a metal shield around the G-M tube, the softer and weaker radiations can be filtered out.

When ionizing radiation penetrates the G-M detector, it collides with the gas molecules in the tube and ionizes them. The electrons released quickly move toward the anode and due to the high electrical field near the anode wire produce there (by multiple collisions with other molecules) a large number of secondary electrons. When operated in one region of counter tube voltage the multiplication near the wire yields a collected electron charge per pulse which is proportional to the number of electrons released in the initial ionization event. In this voltage region the counter tube is classified as a proportional counter tube.

At higher counter tube voltages, the multiplication near the wire becomes so great that the positive ions produced actually reduce the electrostatic field near the wire and limit the amount of multiplication. This limiting action is in proportion to the number of secondary electrons released near the wire and, therefore, will be more pronounced for large initial ionizing events than for small initial ionizing events. This tends to make all output pulses nearly the same size. As the counter tube voltage is increased, a voltage is reached where all output pulses are limited by the positive ions and all are the same size regardless of the size of initial ionizing event (one ion pair or one hundred thousand). This voltage point is known as the Geiger-Mueller threshold. Above this voltage point for approximately 20 to 30 percent more than the threshold voltage, the pulse height increases linearly with counter tube voltage. This voltage region is known as the Geiger-Mueller region. (This region is characterized by the "plateau" in the curve of counts per minute versus counter tube voltage.) The positive ion cloud which limits the pulse size also prevents formation of a second pulse by a succeeding ionizing event until the cloud moves out a certain distance toward the outer cylindrical electrode. Then, for an additional period of time, it reduces the possible multiplication of a succeeding pulse. The first time interval is known as the dead time of a counter tube and the second time interval the recovery time. (The action of a quenching gas in G-M tubes will not be discussed here since it does not contribute to the detected pulse. It merely prevents the formation of a succeeding pulse if the positive ions from the first pulse are not neutralized in the correct manner.)

The Output Pulse from a Counter Tube

The output pulse from a G-M counter tube may be described as follows: Let us assume that the counter tube is operated at 1100 volts, which is 100 volts above G-M threshold. Further, let us assume that the electrical charge produced in the gas in each pulse is one billion electrons (and a corresponding number of positive ions, of course), and that the electrical capacity of the G-M tube and leads going to the pulse-forming resistor is 100 micro-micro-farads. A typical input circuit of a G-M counter instrument is shown below.

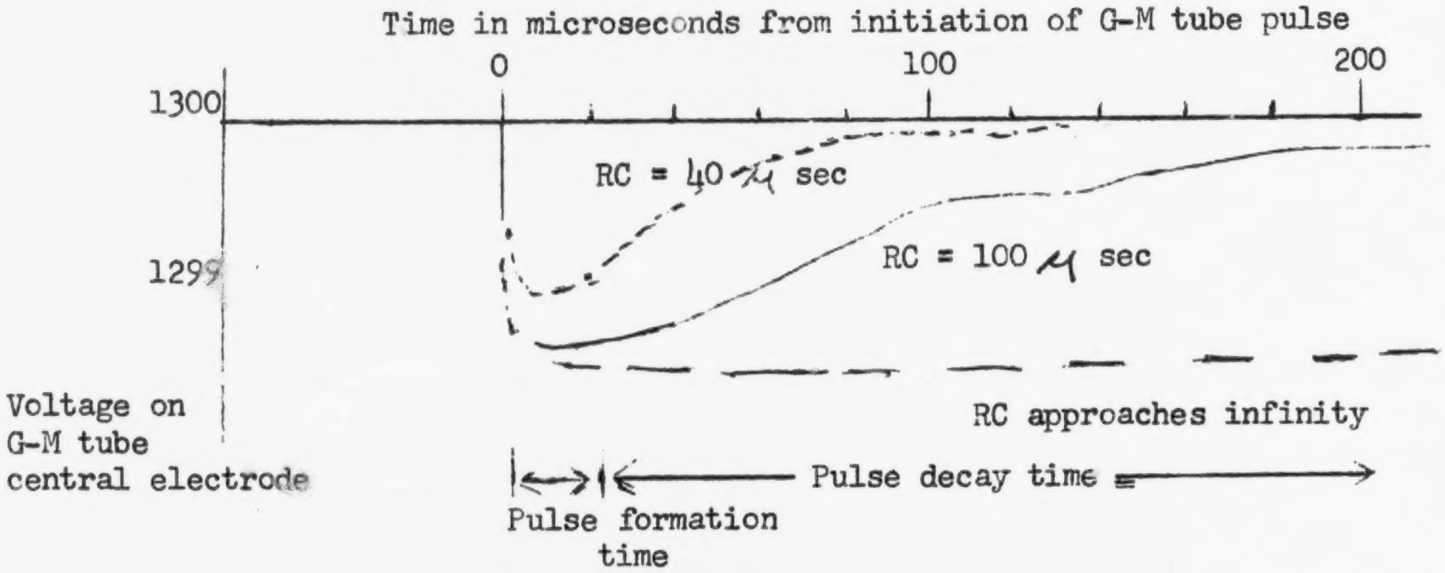


If the G-M tube was charged up to 1100 volts and then the pulse-forming resistor was disconnected from the circuit, each pulse in the G-M tube would reduce the voltage across the G-M tube by an amount $\triangle V = \triangle Q/C$ which applies to condensers. As assumed above, the capacity of the G-M tube and leads is 100 micro-micro-farads and the charge collected per pulse is 10^9 electrons. From this:

$$\triangle V = \frac{\triangle Q}{C} = \frac{(10^9)(1.6 \times 10^{-19}) \text{ coulombs}}{100 \times 10^{-12} \text{ farads}} = 1.6 \text{ volts}$$

Each pulse will change the voltage across the G-M tube 1.6 volts, and 100 pulses will cause a change of 160 volts across the isolated G-M tube. The remaining potential of 940 volts will place the G-M tube below G-M threshold and the G-M tube will have to be connected to the high voltage supply again to return it to the operating voltage.

The pulse-forming resistor (connected between G-M tube and high voltage supply) must be small enough so that the G-M tube voltage does not change too much at high counting rates but not so small as to prevent the pulse reaching its maximum amplitude because of too rapid leakage. With 1 megohm as the pulse-forming resistor and with 100 micro-micro-farads as the G-M tube and lead capacity, the product of the resistance and the capacitance (R x C) will approximate the time in seconds for the charge to drop to 37% of its initial value (100 microseconds in this case). 63% of that remaining after 100 microseconds will leak off in the next 100 microseconds, and so on. If the pulse height builds up to 95% of maximum in 10 microseconds, the pulse height is in effect independent of the RC time constant as long as it is greater than 100 microseconds. This is indicated below for the above conditions.



However, RC time constants of less than 100 microseconds will reduce the pulse height, since charge will leak off at the same time as it is building up.

Let us now consider the input circuit of a G-M counter instrument as shown on the previous page. If the RC time constant of the coupling condenser and the output resistor is larger than the RC constant of the counter tube, leads, and pulse-forming resistor, then the output pulse will be the same shape and size as that shown above. However, the voltage will drop from zero or ground potential to minus 1.6 volts and return to ground rather than between the limits of 1300 to 1298.4 volts on the central wire of the G-M tube. The output pulse size is thus negative 1.6 volts with a build-up time of approximately 10 microseconds and a decay time of approximately 100 microseconds.

The output pulse in numbers of electrons per pulse is characteristic of the G-M counter tube and the voltage at which it is operated. At 100 volts above G-M threshold the Anton Halogen G-M counter tube, Model 210 T, yields about 1.6×10^{10} electrons per pulse. This is about 16 times as many as used in the calculation above. Therefore, with an input capacity of 100 micro-micro-farads and a pulse-forming resistor of 1 megohm, the pulse height will be 1.6 x 16 or 25.6 volts. If now a long connecting cable was used between the G-M tube and the input circuit of the scaler, the capacity of the G-M tube circuit might be increased to 1000 micro-micro-farads. Under this condition the pulse in the above example would be reduced by a factor of ten to 2.56 volts.

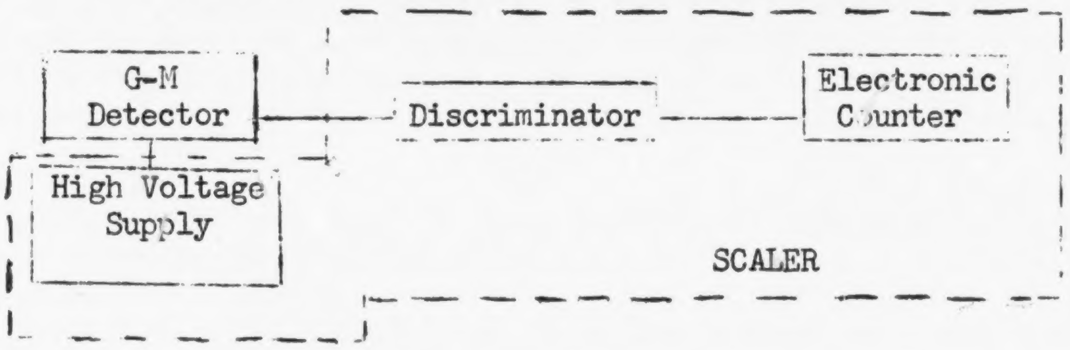
The Anton Model 220 T G-M counter tube has an organic gas as a multiple pulse quenching agent and gives about 1/3 the number of electrons per pulse that the halogen gas quenched tube yields. Other counter tubes may even give pulses one-tenth to one twenty-fifth that of the Anton 210 T Halogen G-M tube.

In the plateau region, on the curve of the counts per minute versus counter tube voltage, the number of electrons collected per pulse varies linearly with the voltage increment above Geiger-Mueller threshold. For example, if a G-M tube has a G-M threshold at 1200 volts, the electrons collected per pulse at 1250 volts will be one-third as many as those collected at 1350 volts. However, just below G-M threshold, the pulse height starts to decrease rapidly with decrease in voltage. The pulse height (proportional to electrons collected per pulse) may decrease more than 100-fold for a one hundred volt change in G-M tube voltage below G-M tube threshold in the tubes discussed above. If the pulse height was 2.5 volts at G-M threshold, it may be 0.025 volts for the largest pulses at 100 volts below G-M threshold.

Nearly all G-M tubes yield pulses greater than 0.25 volts at G-M tube threshold. This, of course, is dependent upon the capacity of the leads connecting the G-M tube to the input of the recording instrument. The high voltage supply for the G-M tube is never an absolutely stable source and, in some cases, gives variations of 0.25 volt or more, but of a slowly changihg nature (not one of 10 to 100 microseconds, but of longer duration). Therefore, the input circuit of a recording instrument usually consists of an electronic device that will respond only to the G-M tube pulses and not to electrical interference pulses. This device is called a discriminator, and its sensitivity is set such that the counter tube pulses must be the size almost equal to that at G-M threshold before they trigger the discriminator.

The Complete Counting System

A radiation counting system is composed of a radiation detector with shielding and a counter (scaler) which electronically records the number of electrical pulses produced in the detector. In this experiment, the radiation detector is a Geiger-Mueller tube of cylindrical shape, filled with a quenching gas mixture. As stated previously, the G-M tube is a gas-filled two element tube or diode which, when operated in the proper voltage range, yields an output pulse of uniform size regardless of the energy or type of radiation which triggers the tube. The output pulses from most G-M tubes when operated in the Geiger region are usually 0.25 to 10 volts high, with a duration of from 100 to 200 microseconds. The counter scaler may be described using the following block diagram:



The high voltage for operating the G-M detector is supplied by a regulated direct current power supply which is normally incorporated in the scaler. This voltage is continuously variable from 500 to 1500 volts and is read on a meter mounted in the panel. In order to count only the pulses coming from the detector, and to reject all other signals, such as electrical noise, a circuit called a discriminator is used to separate these signals. This control circuit sets the input sensitivity of the scaler and allows it to select only signals above a certain predetermined amplitude with the rejection of all others. At the same time, this circuit behaves as a pulse shaper to properly prepare the signal so it can be "counted" by the electronic counter. In the laboratory, there are several types of electronic counters in order to allow the participants to become acquainted with the types of instruments which are commercially available. The original "scale of 2" type of instrument is the simplest and most reliable, but it requires more calculation in order to interpret the results, and errors often arise in these calculations. The vertical read-out scales of 10 are actually four scales of 2 with automatic feed-in of 6 pulses for every 10 pulses from the discriminator or previous scale of 10. These circuits are now well perfected so that they can be reliable, although not as reliable as the scale of 2. However, the ease of read-out usually outweighs the lower reliability. The glow-transfer tube decades merely transfer a gas discharge glow around a ring of ten stages, and can be quite reliable except for driver-tube malfunctioning. An electro-mechanical register is often added after a few stages of electronic counters, since it is cheaper than two stages of a scale of 10.

The main purpose of electronic scaling is to allow for fast counting. Most electro-mechanical registers will record no more than twenty uniformly spaced electrical pulses per second, whereas the electronic counters can respond to 10,000 or 1,000,000 uniformly spaced electrical pulses per second, depending upon the type of electronic counter.

With the discriminator sensitivity preset, the high voltage to the counter tube should be turned on and slowly raised from 500 volts towards 1200 volts. Small electrical pulses will be produced from the counter tube at first because of the lower tube voltage. The amplitude of these pulses increases rapidly with increases in counter tube voltage up to the G-M threshold. However, as long as the counter tube pulses are less than the discriminator setting, which is usually 0.25 volts, no counts will be recorded on the scaling units. As soon as the counter tube pulses are larger than 0.25 volts, a point is reached where some of the larger counter tube pulses are recorded. A further increase in counter tube voltage finally results in G-M threshold operation, and all pulses are the same size. An operating region is then reached where counting rate vs. voltage is relatively constant within a few percent. This region is called the plateau, and extends for a range of about 200 volts. At counter tube voltages of 200 or more volts above G-M threshold, inadequate quenching action allows multiple pulses to be formed from one ionizing event, and the counting rate starts to rise.

A good counter should exhibit a plateau slope of less than 3% per 100 volts on its counting rate. However, for many applications (e.g. relative measurements) counters with relative plateau slopes up to 10% per 100 volts are acceptable, provided the voltage source is sufficiently well stabilized and the counters are periodically checked with a standard source. The length of the plateau should be approximately 30% of the operating voltage.

Counter Resolving Time: The number of counts recorded by a counting system is, in general, less than the number of particles which produced ionization in the detector. The difference is caused by the failure to resolve closely spaced events in (a) the counter tube, (b) the amplifier, discriminator and scaler, and (c) the mechanical register. Electronic scaling circuits usually can respond to pulses spaced two to five microseconds apart. (Some circuits can respond much faster and others much slower, particularly the cheaper glow transfer tube electronic counters.) Although mechanical registers usually cannot respond to uniformly spaced pulses faster than 10 to 20 per second, electronic scaling stages can be added in series to reduce the rate of arrival of pulses at the mechanical register below this rate. It is good practice never to use the register at counting rates where the counting losses caused by the register are greater than 1%. The use of a sufficient number of electronic scaling stages can eliminate the register as a factor in setting the maximum counting rate. On page A2-2 of this experiment, there is a discussion of the dead time and recovery time of counter tube caused by the action of the positive ion cloud or sheath. After the development of a pulse in a G-M counter tube, the space charge of the positive ion sheath depresses the electrostatic field at the wire so as to make any further multiplication by collision impossible. A second particle entering the counter at this time will not produce a pulse. As the space charge moves toward the cathode, the electrostatic field increases again. It becomes sufficiently large so that a second particle may produce a small pulse after a certain time, $t_d + t_r$, t_r being called the recovery time. Following the split sample method as outlined in procedure E of this experiment, the resolving time, τ, of a counter system may be calculated by means of the following relation: (Lapp & Andrews, p.458)

$$\tau = \frac{n_1 + n_2 - \overset{n_{1\&2}}{n_1 \text{ and } 2}}{2n_1 n_2}$$

n_1 = the count rate obtained using source number one
n_2 = the count rate obtained using source number two
~~n_{12}~~= the count rate obtained using sources one and two together
$n_{1\&2}$

and the resolving time, τ, will be in minutes if n_1, n_2, and n_1 and $_2$ are obtained in counts per minute. It will be in seconds if the respective counting rates are obtained in counts per second.

n_1 and n_2 must be at least 20,000 c/min.

A slightly different formula for the resolving time calculation is given in other texts based on the same basic equation used in deriving the above formula. However, slightly different approximations were made in simplifying the formula. The formula suggested above under-estimates the resolving time by 5% to 25%, whereas the formula in other texts over-estimates the resolving time by 11% to 58% for the same counting rates and resolving times fitted into the formulas.

Half Value Layer: The half value layer or the "half thickness" is often used to characterize the energy of x- or gamma radiation. It is the thickness of an absorbing material which reduces the count rate to half its initial value. In principle, absorption measurements may be made with any type of material, although as a practical matter, lead is used with gamma rays because of its high density and high atomic number.

Half Life: The half life of an isotope is the time interval during which the activity is reduced to half of its initial value.

LABORATORY PROCEDURE:

A. Plateau and Operating Voltage

Check to see that the high voltage switch if off and turn the H.V. control as far as it will go in a counter-clockwise direction. Turn the master (power) switch to ON. Insert an ORINS prepared counting sample having a red label, on the second shelf from the top of the counting chamber. (Caution! Do not use the Co^{60} samples marked with the green band. All of the counting samples will be in the plastic boxes located around the counting room or in the wall cabinet. The sources should be in these plastic boxes when not in use.)

Turn the count switch to ON position.

Turn the H.V. switch to ON position.

Slowly turn up the high voltage control (clockwise) until the electronic counter lights just begin recording counts. This is called the "threshold voltage". Starting at the nearest 50 volt mark above which counting begins, take one minute counts every 50 volts up to a maximum of 300 volts above the counting threshold. (Caution! Do not turn the high voltage above the red mark on the glass of the H.V. meter.) The number of counts is read directly from the electronic counter or, on the scale of two counter, it is determined by noting the reading of the register (assuming it is reset to zero before each count is made), multiplying by the scale selector setting, and adding in the values shown on the interpolation lights. When a count has been completed, the reset switch will reset all the scalers to zero and, in some instruments, will reset the mechanical register.

Make a plot of counts/min. vs. voltage. Select the midpoint of the flattest part of this curve. This voltage is called the "operating voltage". The instrument should be set at this particular voltage when a sample is counted.

Each day the "threshold voltage" should be determined by placing a source in the shield and noting the voltage at which counting begins. The source should then be removed from the shield and, starting with the voltage at the threshold value, gradually increase the H.V. by 300 volts. There should be only a slight increase in background counting rate. If there is a considerable increase, another count-rate vs. voltage curve should be prepared.

Counting Rate vs. Distance

Place the same counting sample used above on the lowest shelf in the counting chamber and measure the counting rate at the operating voltage as determined above. How does the counting rate compare with that on the second shelf? Explain.

C. Background Count

Remove the sample from the counting chamber and, with the high voltage set at the proper operating voltage, take a 10 minute count. Calculate the number of counts per minute. This is called the background count, and must be subtracted from the sample count in order to obtain the true sample counting rate. The background count should be checked several times a day.

D. The Half Life of a Gamma-Emitting Isotope

The laboratory supervisor will take some pieces of metal from the exposure chamber of the neutron source located in a corner of the counting room. He will mount the metal pieces on cardboard holders. Each source will be shared by two adjacent groups.

Record the number on the sample which you receive and place the sample on the top shelf of your counting chamber. Make certain that the metal is on the top side of the sample mount. Take a two-minute count on the sample. Note the time at the center of the counting interval.

Remove the sample from your counting chamber and pass it to the neighboring instrument, or keep it until your neighbors ask for it. Take a two minute count at approximately each twenty-minute interval for the next hour, noting the times at the centers of the counting intervals. Make a semi-log plot of log count rate (minus background) versus time on one or two cycle paper. Calculate the half-life of the isotope.

E. Resolving Time of a Counter System

With the Geiger counter set at its proper operating voltage and using a scale of 100 or more, place half of one of the split radioactive sources on the first shelf of the counting chamber. Take a one minute count (the counting rate should be between 20,000 and 40,000 c/min.) Call this n_1. Place the other half of the source on the counting card without altering the position of the first source. Take a one minute count. Call this $n_{1\&2}$. Remove from the counting card the first source without altering the position of the second. Take a one minute count and designate it n_2. Calculate the resolving time using the relation given in the INTRODUCTORY MATERIAL (page A2-7).

F. Gamma Ray Absorption - Calculation of Half Value Layer (HVL)

Place the Co^{60} sample (marked with the green band) on the third shelf from the top of the counting chamber. Place an aluminum absorber, 80 to 150 mg/cm^2 on the next higher shelf, and leave it there for this entire procedure to absorb the beta particles from the source. Determine the counting rates with varying thicknesses of lead. (Use the $2\frac{1}{2}$" by 3" lead sheets 1/16" thick and stack them directly on top of the aluminum absorber.) Prepare a semilog plot (counts/min. minus background on the log axis vs. absorber thickness in inches (or gm/cm^2) on the linear axis), and determine the "half thickness" or "half value layer".

References:

1. Harshaw Scintillation Phosphors by the Harshaw Chemical Co., pp. 8-27. Copies of this reference are available at the office of the Junior Lab. One need not spend too much time on the multitude of details in this reference.
2. Experimental Nucleonics by Bleuler & Goldsmith, pp. 175-178, 305-315. This is required reading.
3. Introductory Nuclear Physics by Halliday, sections 7.10-7.16 This is not required reading.
4. Junior Lab instruction sheet on the use of the Tektronics Scopes, Pulse Height Analyzers, etc. You must be able to handle this equipment properly.
5. Beta and Gamma Ray Spectroscopy by Siegbahn, pp. 24-51, 137-145, 151-155. Chapter 7, sections 1,2,3,4,7. This book is more advanced in its treatment of theory and experiment. It is not required reading, but will be of interest to some students.
6. The Atomic Nucleus by Evans. This has some information on counting statistics.

Introduction:

Since its development in 1944, the scintillation technique for the detection of ionizing radiation has become the most favored method for the investigation of charged particle reactions. This experiment will show the methods utilized in these investigations as well as demonstrate two of the types of experiments possible with scintillation equipment.

In general, scintillation detectors consist of three basic elements: a crystal, plastic or liquid scintillator which emits light when it is traversed by a charged particle, a highly sensitive photomultiplier tube to detect these few photons of light, and an amplifier to amplify the small signals produced by the photomultiplier. The intensity of the light output of the scintillator, and hence the pulse output of the amplifier if the system is linear, will be proportional to the energy of the particle if the particle comes to a stop in the scintillator. Thus it is possible not only to detect but to determine the energies of the charged particles.

Two investigations are to be carried out. In part one, we shall use a single sodium iodide scintillation detector with two single channel pulse height analyzers to study the pulse height spectra of two radioactive sources. In part two, we use two such detectors in conjunction with a coincidence circuit to study the angular correlation of the two gamma rays produced by electron-positron annihilation.

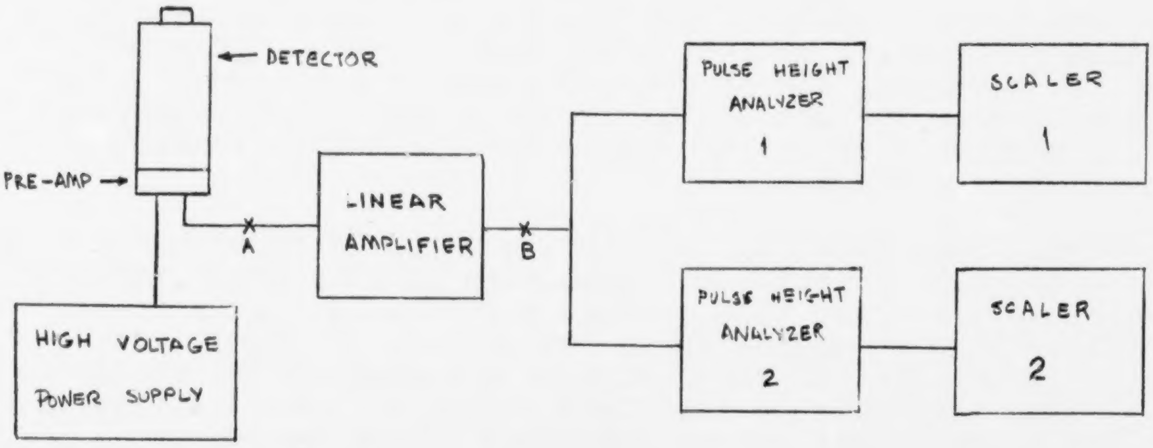
The two parts of this experiment are set up as separate pieces of experimental apparatus. You will be expected to perform each part in one regular laboratory period, while two other students perform the other part. Therefore, you may not do the parts in the order given here. Nevertheless, an adequate understanding of either part will require that you be familiar with the characteristics of both as given in this write-up as well as in the references.

Notes of Caution:

1) High voltages are used to power photomultipliers in this experiment. Do not come into contact with these voltages. Before high voltage is applied to any piece of equipment, be sure it is properly connected.

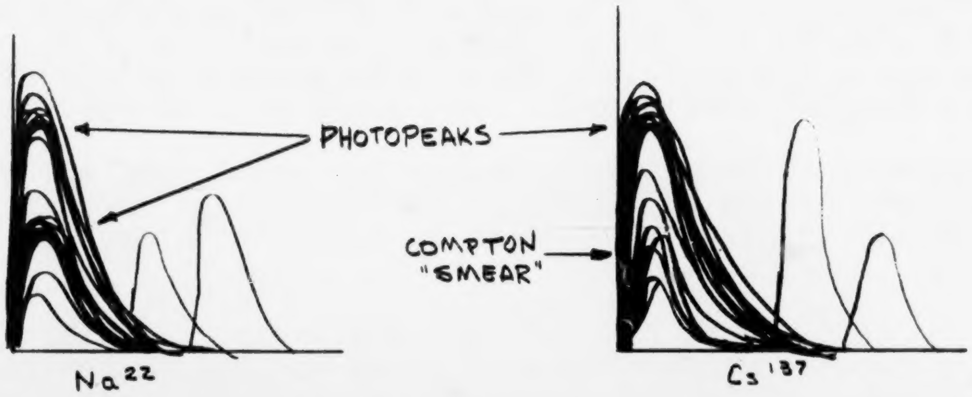
2) High gain photomultipliers of the type used here may be permanently damaged if too great a voltage is applied or if any high voltage is applied while they are exposed to room light. The National Radiac detectors used in parts one and two operate from 900-1200 volts; 1000 volts is recommended.

I. For the determination of pulse height spectra, equipment should be connected as follows:



With a source in place, the output of the detector at point A will be short duration negative pulses of amplitude about 0.1 volts and greater, depending on the energy of the particle detected and the setting of the photomultiplier high voltage.

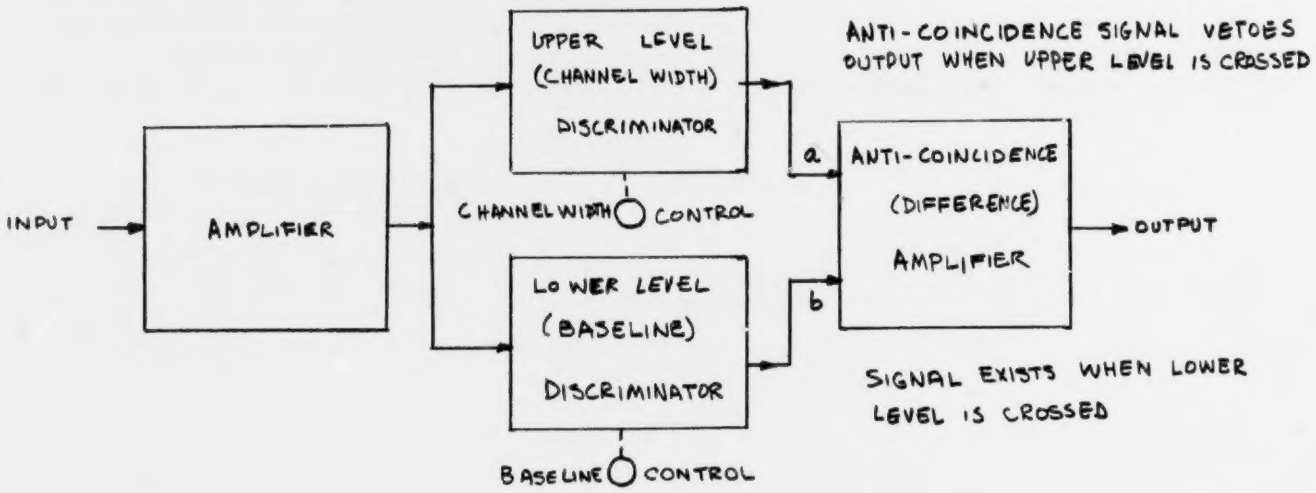
The linear amplifier is a variable gain, non-overloading device. Its output may be adjusted to provide pulses in the range of interest. Pulses which would appear at point B to be over 100 volts are in the amplifier's saturated region and have been limited by the amplifier. Obviously pulses in this saturated region cannot be analyzed, therefore, the gain control must be set so that the output pulses of interest will not be limited. This is best done by observing the output at point B with an oscilloscope. The pulses observed will be positive and will look like:


PHOTOPEAKS
COMPTON "SMEAR"
Na^{22}
Cs^{137}

The left hand trace above is an approximation of the Na^{22} output. It is good practice to adjust the amplifier gain so that the most energetic photo peak of interest produces pulses of about 80 volts amplitude. Reference one contains some sample spectra for the sources available, and some time can be saved in finding spectral shape if the highest photo peak is "normalized" to correspond to the arbitrary units used in these plots. The right hand trace above represents the Cs^{137} output and shows the "smear" of pulses caused by Compton scattered electrons.

Sources may be calibrated against each other for energy determinations by leaving gain settings constant.

The pulse height analyzers work as follows:


INPUT
AMPLIFIER
UPPER LEVEL (CHANNEL WIDTH) DISCRIMINATOR
CHANNEL WIDTH CONTROL
LOWER LEVEL (BASELINE) DISCRIMINATOR
BASELINE CONTROL
a
b
ANTI-COINCIDENCE (DIFFERENCE) AMPLIFIER
OUTPUT
ANTI-COINCIDENCE SIGNAL VETOES OUTPUT WHEN UPPER LEVEL IS CROSSED
SIGNAL EXISTS WHEN LOWER LEVEL IS CROSSED

An input pulse whose amplitude is greater than the voltage setting of the baseline discriminator will cause a pulse to appear at point b. If its amplitude is greater than the baseline setting plus the channel width setting a pulse will also appear at a. The anticoincidence circuit will produce an output if and only if there is a single pulse at point b. Thus if the baseline is set at 20 volts and the channel width setting is one volt, pulses which are in the "window" between 20 and 21 volts will cause output pulses which may be counted by the scalers.

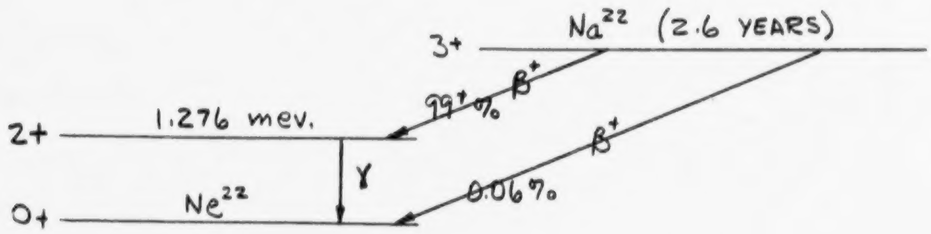
The particles observed (like all radioactive processes) obey Poisson statistics, so that best practice requires taking a counting time and a channel width which will produce points of reasonable statistical precision in your final plotted spectrum.

The output pulses from the analyzers (points C) are about 10 volts negative, so the pulse height discriminators on the scalers may be set at about five volts to eliminate noise and insure accurate counting.

Sources of Cs^{137}, Na^{22} and Co^{60} are available. You should understand the radiations from these sources and the mechanisms of their detection. It is suggested that spectra for the first two sources be obtained first and the Co^{60} spectrum be taken if time allows. Since the two analyzers and scalers are independent, you can measure the pulses at two different "window" settings simultaneously.

Notes: Cs^{137} is a source of a single monoenergetic gamma at 0.661 mev. (The β^- particle is stopped in the detector case.) Na^{22} is the source of an annihilation gamma at 0.51 mev and a 1.276 mev gamma. Measure the Cs^{137} photo peak with the amplifier gain calibrated for this source. Co^{60} is the source of four gamma energies (see attached scheme sheet.)

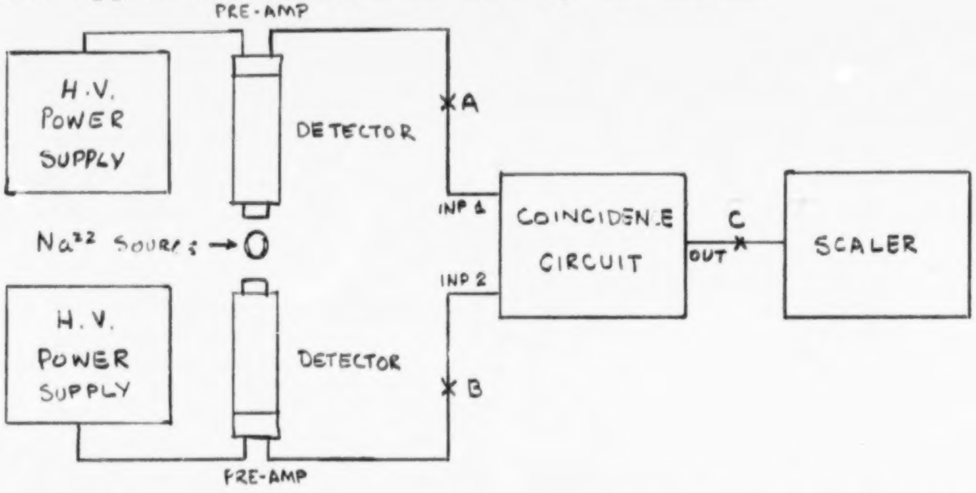
It should be noted that although the appended scheme sheets show the standard format for representing decays, the changes in atomic energy levels shown are accompanied by the particle releases shown on their respective arrows. In other words for Na^{22}:



the β^+ emissions shown represent that the change in level indicated is simultaneous with the release of the positron, but the positron itself is not undergoing that change in level.

II. In this part of the experiment we wish to study the angular correlation or distribution of one with respect to the other of the two annihilation gamma rays which result from positron annihilation. In the center of mass coordinate system of positron and electron, the gamma rays come out at a 180° relative to each other, from the conservation of momenta. The angle may or may not be 180° in the laboratory system depending on the speed of the positron when the annihilation occurs. If the angle is very near 180° in the laboratory system, we know that the positron has been slowed down to a speed small compared to the speed of light. Three quantum annihilation may also occur. With the apparatus set up as shown this type of annihilation will not be differentiated from two quantum annihilation. Three quantum annihilation is improbable in comparison with two quantum annihilation.

The apparatus should be set up as shown:

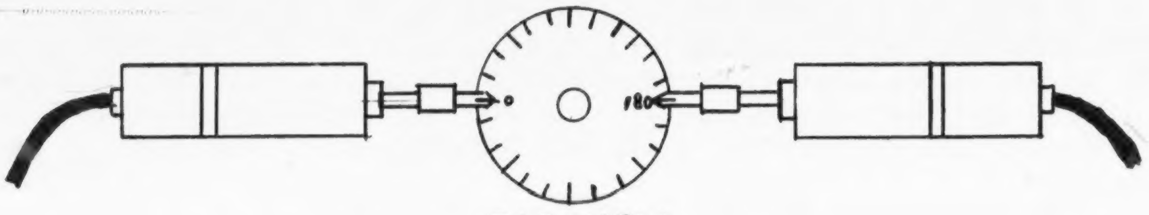


The coincidence circuit is a high-speed, transistorized and battery operated device. It requires no warm-up time but care must be taken to ensure that it is switched off when not in use. Since the coincidence circuit has no pilot light, always make sure it is switched off when not in use and before you leave. The equipment has a triple coincidence capability and may be set by a switch on top to act either as a double or triple coincidence unit. In the apparatus diagram shown above, it is being utilized as a double coincidence unit so that the switch should be set accordingly. When and only when pulses appear simultaneously at all the inputs being used (one and two, or one, two and three) an output pulse of about -3 volts will appear at the coincidence output. Thus the pulse height discriminator on the scaler must be reduced to about two volts to ensure counting these pulses.

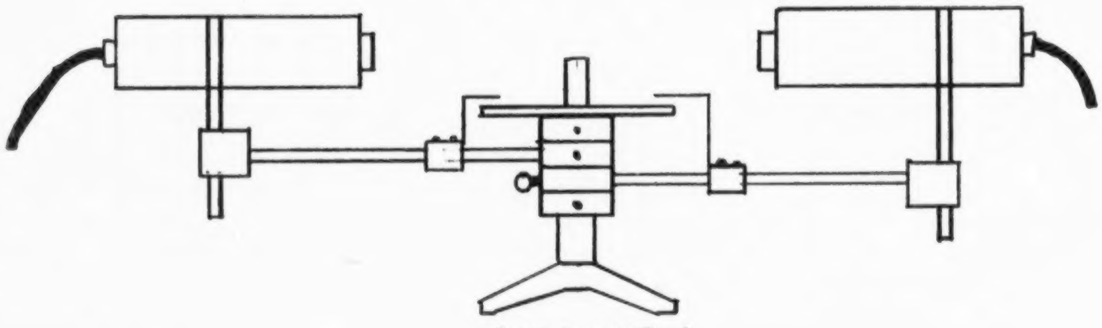
You may wish to demonstrate to yourself that actual coincidences are being counted by counting first with two unshielded detectors then shielding one with lead bricks and counting again.

Enough extra equipment is proveded so that three detectors may be arranged in coincidence. After first investigating the angular correlation with the equipment arranged as above, you may use either the double or triple arrangements to investigate any background or special affects you think would be interesting or pertinent to the experiment.

A table is provided to control the geometry of the experiment. It should be set up as shown:



TOP VIEW



SIDE VIEW

The source to detector distance d is 8 inches.

Determine and plot the angular correlation of the annihilation photons.

Write Up: You should present data and graphed results, including error, for both sections of the experiment. For your write-up select one section and prepare a thorough report on it. Write as if you were going to submit the paper to a journal for publication. In discussing theory you must decide what details are pertinent and present them clearly. It is as serious an error to fill your paper with irrelevant detail as it is to leave out pertinent facts. Error analysis must be rigorous as you must be prepared to vouch for the reproducibility of your results within the error limits you specify.

Questions: The following are some questions which need not be answered explicitly but should aid you in assessing your understanding of the experiment.

1. Since gamma rays are not detected directly, what are the mechanisms of their detection in scintillators?
2. If ionization (light output) is greater for slower moving particles how can we say that pulse height is proportional to energy?
3. How can the annihilation gammas in part two be produced at any angle other than 180°?
4. What do you think the effects of three-gamma annihilations and "accidental" coincidences are in part two? How would you eliminate them?

RANGE OF ALPHA PARTICLES

North Carolina State College

I. References:

Price, Nuclear Radiation Detection, p. 6.
Kaplan, Nuclear Physics (1963), p. 307.
Singh and Saha, Nuc. Inst. and Methods 13 321 (1961).

II. Purpose:

To measure the range of alpha particles in air.

III. Apparatus:

Rosenblum spark counter, alpha particle source, scaler with high voltage (to 5 Kv at 1 Ma).

IV. Theory:

Alpha particles are emitted from certain heavy nuclei at discrete energies which are characteristic of a given radionuclide. For transitions which originate and terminate with the ground state of the parent and daughter, only one alpha particle energy is observed. Transitions which originate with the ground state of the parent but terminate with excited states of the daughter may produce several distinct alpha particle energy groups with gamma radiation associated with the decay of the excited states. In the case of certain alpha emitters of very short half life, alpha decay may originate from the excited state resulting in the emission of the "long range" alpha particles with energies in excess of 10 Mev.

A convenient method of determining the energy of alpha particles is to measure their range in air or some other suitable medium. When an alpha particle traverses matter it interacts electromagnetically with atomic electrons causing excitation and ionization. The energy lost in an individual interaction is relatively small but the process goes on until the alpha particle can no longer ionize. The distance traversed is called the range. The average energy loss per ion pair in air is about 35.5 electron volts. Due to the variation in energy losses per event and the statistical fluctuations in the number of events, there is some spread in the ranges of alpha particles of equal energy. This effect is called "straggling" and for natural alpha particles amounts to about 0.5 per cent of the range. The straggling parameter is defined as the difference between the mean range and the extrapolated range.

These quantities can readily be obtained from a plot of the integral number-distance curve and its derivative.

V. Procedure:

The Rosenblum spark counter consists essentially of a wire anode parallel to a plane or convex cathode. The distance is usually about 2 mm in free air. A positive potential in the range of 3000 to 4000 volts is applied in the manner indicated below. If a highly ionizing particle passes through the region between anode and cathode, a visible and audible spark occurs. The specific ionization required to initiate the spark is much greater than that produced by electrons. The counter is thus virtually insensitive to beta and gamma radiation and therefore lends itself to the counting of alpha particles and heavy ions.

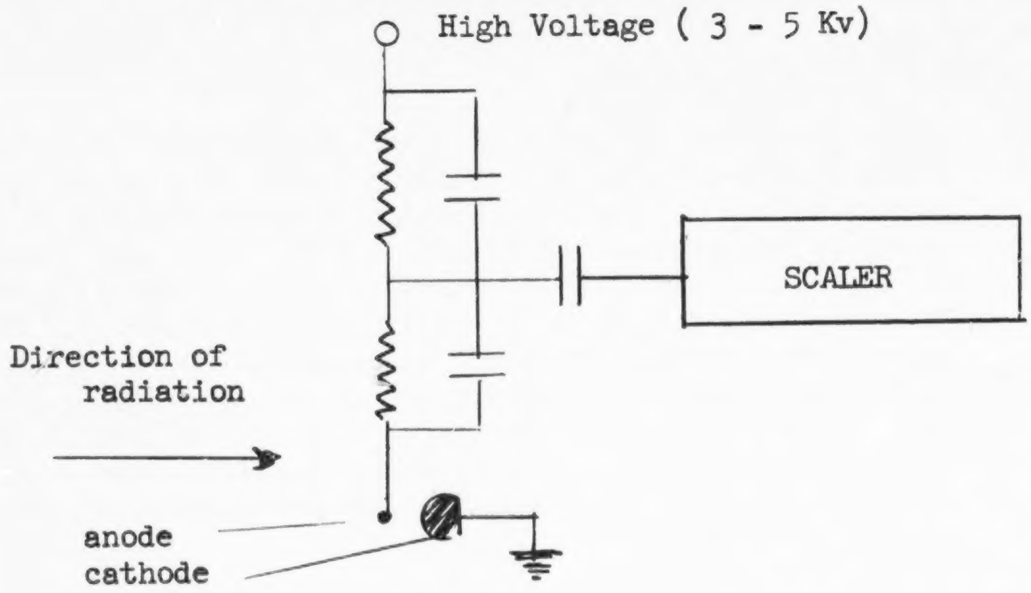


Fig. 1. Spark counter schematic.

There are two methods of using the spark counter in determining the range of alpha particles. In the first method, the counter is operated in free air and the distance between source and counter is varied and the integral number-distance curve is established. The disadvantage of this method is the fact that the change in distance introduces geometry changes which are difficult to evaluate. In the second method, the counter is enclosed and the air pressure is varied. This method was used by Morgan in determining the range of the alpha particles of the $B^{10}(n, \alpha)Li^7$ reaction.

Method A. Air at atmospheric pressure--distance varied.

1. Place the alpha source so that the distance between it and the counter is well within the

range of alpha particles. Determine the characteristic curve for the counter by measuring the counting rate at 50 volt intervals until the counting rate shows an appreciable rise (not over 10 per cent).

2. Choose a suitable operating point from the plot of the data obtained above and measure the counting rate at one millimeter intervals continuing until the counting rate becomes essentially zero.

Method B. Distance fixed--air pressure varied.

1. Obtain the source-counter distance from your instructor. Reduce the pressure until a suitable counting rate is observed (about 3200 volts). Determine the characteristic curve of the counter by measuring the counting rate as a function of voltage in 50 volt intervals continuing until the counting rate shows an appreciable rise (not over 10 per cent).

2. Measure the counting rate as a function of air pressure increasing the pressure in 1 cm Hg intervals until the counting rate becomes essentially zero.

VI. Problems:

1. Plot the characteristic curve for the counter as determined in V-1.

2. Plot the counts vs. distance curve as obtained in V-2. If method B is used, plot the counting rate vs. pressure curve and calculate the equivalent source-counter distance at each pressure. Plot the counting rate against this distance.

3. Determine the mean and extrapolated range. The extrapolation of the curve forVI-2 yields the extrapolated range directly. The first derivative of the same curve yields the mean range.

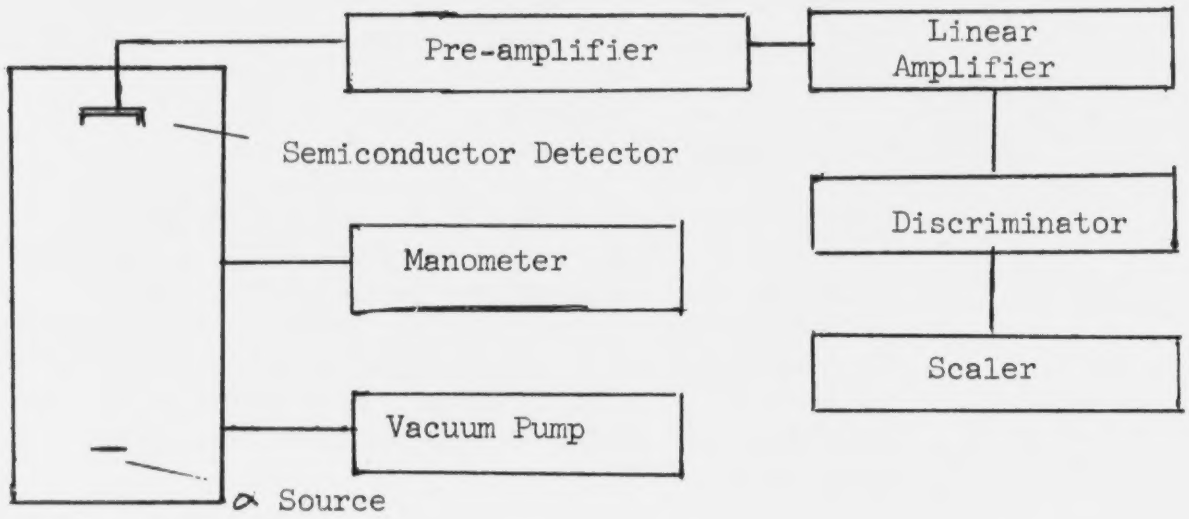
4. Estimate the energy of the alpha particles from the range-energy table in the appendix.

5. Discuss the sources of error in your experiment.

RANGE OF ALPHA PARTICLES

Appendix A

The recent development of semiconductor radiation detectors makes possible the alternate experimental arrangement indicated below. It is, however, necessary in this case to have the experimental chamber light tight inasmuch as the detectors are damaged by exposure to light.


Pre-amplifier
Linear Amplifier
Semiconductor Detector
Discriminator
Manometer
Scaler
Vacuum Pump
α Source

Experimental Arrangement

The following instruments have been used successfully in this experiment:

Pre-amplifier - Hamner N358A
Amplifier - Discriminator - Hamner N-371
Detector - Ortec SBFJ100-60

The other components are not critical.

THE ABSORPTION OF GAMMA RAYS IN LEAD

University of Colorado

The purpose of this experiment is to give the students an opportunity to use techniques which are employed where unknown gamma rays are to be identified as to the relative numbers present and their energies. Experiments such as this are used to identify gamma rays from artificial emitters produced by cyclotrons, nuclear reactors, and other "atom smashing" devices.

Theory

When a parallel beam of monochromatic X or gamma radiation passes through matter, the following relation is observed to hold between the incident intensity in quanta per square centimeter per second (I_o) and the transmitted intensity (I) in the same units, where the radiation is passing through a thickness x of the absorbing material.

$$1)\qquad I = I_o e^{-\mu x}$$

In this equation, μ is the absorption coefficient, and it is a quantity that is characteristic of both the type of material (element and density) doing the absorbing, and of the wavelength or energy of the radiation being absorbed. If x is measured in centimeters, then μ is the linear absorption coefficient, and it has the dimensions of 1/cm. Quite commonly equation 1) is written in this form

$$2)\qquad I = I_o e^{-\sigma t}$$

where $\sigma = \mu/\rho$ and $t = \rho x$. The quantity (ρ) is the density of the absorbing material in grams per cubic centimeter. Then σ is called the mass absorption coefficient and has the dimensions of cm^2/gram, while the thickness t is measured in grams/cm^2. This representation has the advantage of being independent of the physical properties of the absorber. Certainly μ would vary for water depending on whether it was in the form of a liquid, solid, or gas, whereas σ is a constant for water independent of its physical state. If one looks up the density of an absorber such as copper, one will find small variations in the tabulated densities depending on the method of manufacture. For each density there would have to be a corresponding value of μ for that element, because one centimeter of dense copper would do a more effective job of absorbing than one centimeter of less dense copper. Hence it is common and preferable to use mass absorption coefficients and to express thicknesses in grams per square centimeter.

Suppose a gamma ray source emits several monochromatic gamma rays. For instance, suppose three wavelengths λ_1, λ_2, and λ_3 are omitted. The expression for the transmitted intensity (I) then becomes

$$3)\qquad I = I_{10} e^{-\sigma_1 t} + I_{20} e^{-\sigma_2 t} + I_{30} e^{-\sigma_3 t}$$

In equation 3), I_{1o} is the incident intensity of gamma ray number one in quanta per square centimeter per second, and σ is the mass absorption coefficient of this particular wavelength in the absorber under study.

Our experiment is to observe I as a function of t, and the following analysis allows us, in theory at least, to determine all of the I_o's and all of the σ's. Once we know the σ's we may consult tables to determine the corresponding gamma ray energies.

Our analysis will be a graphical analysis in which we start with a large scale semilogarithmic plot of intensity I vs. thickness t. In order to better understand what has to be done, let us examine some of the properties of semilogarithmic plots of exponential functions.

A semilog plot of I vs. t in equation 2) is shown in Fig. 1.

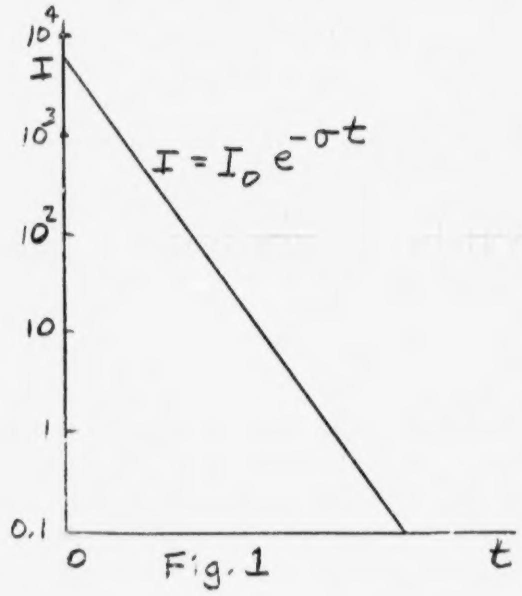


Fig. 1

If now we take the data of Fig. 1 and add a constant to each point we have the following equation, where B is the added constant, and is the constant background count.

$$4) \quad I = I_0 e^{-\sigma t} + B$$

A semilog plot of equation 4) is shown in Fig. 2. Note that no part of this curve is a straight line. At large values of t, the curve becomes asymptotic to the horizontal line I = B, and at small thicknesses, the line is asymptotic to the line $I_0 e^{-\sigma t}$

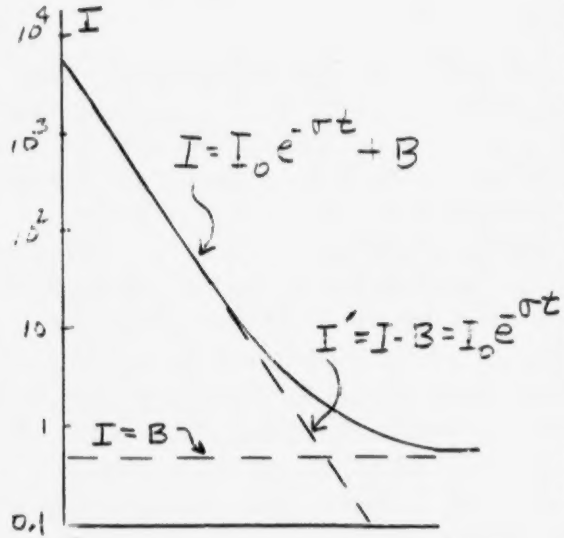


Fig 2

Laboratory data on the absorption of a monochromatic gamma ray beam would resemble Fig. 2. Analysis of Fig. 2 would be as follows. At large values of t, one evaluates B. Then B is subtracted from the data at each thickness t and the difference I' is plotted

$$I' = I - B = I_0 e^{-\sigma t}$$

This is shown as the dotted line. The slope of the dotted line allows one to evaluate , and the intercept I_0 is a measure of the incident intensity of the gamma ray beam. σ is evaluated from equation 2) as follows.

$$\sigma = \frac{1}{t} \ln \frac{I_0}{I}$$

Suppose one had two gamma rays present so that

$$5) \quad I = I_{01} e^{-\sigma_1 t} + I_{02} e^{-\sigma_2 t} + B$$

A plot of equation 5) is shown in Fig. 3. Analysis of Fig. 3 would be as follows

a) When the curve levels off at large values of t, we have I = B.

b) We now plot I' vs. t, where I' is

$$I' = I - B = I_{01} e^{-\sigma_1 t} + I_{02} e^{-\sigma_2 t}$$

This plot is shown in Fig. 4., and suppose we designate the more easily absorbed gamma ray as gamma ray No. 1. At large thicknesses of absorber, the transmitted intensity of gamma ray No. 1 is reduced to a very small value, and the only gamma rays coming through the absorber in appreciable quantities are the No. 2 gamma rays. Thus at large values of t, the plot of I' vs. t should approach the straight line

$$I_{02} e^{-\sigma_2 t}$$

just as if gamma ray No. 1 was not present at all.

c) On Fig. 4, one draws a straight line (shown dotted) whose slope is determined by the curve of I' vs. t at large values of t. This straight line has the equation

$$I_{02} e^{-\sigma_2 t}$$

and from it we may evaluate both σ_2 and I_{20}.

d) From Fig. 4 we may read off values of I''

$$I'' = I' - I_{02} e^{-\sigma_2 t} = I_{01} e^{-\sigma_1 t}$$

and these may be plotted vs. t as shown in Fig. 5, from which both I_{10} and σ_1 may be evaluated.

The theory of this process may be extended to the determination of the I_o's and σ's of any number of gamma rays in a beam. Practical difficulties limit it to the separation of only 4 or 5 gamma rays.

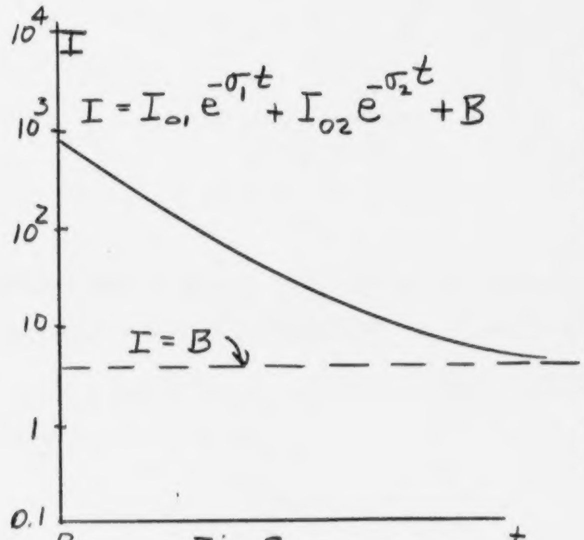


Fig 3

We have assumed that I_{10}, I_{20}, and I_{30} etc. are in incident intensities of the gamma rays of the monochromatic groups 1, 2, 3, etc. This would be true if our detector (geiger counter tube) was 100 percent efficient, giving one count for every gamma ray that passed through it. Unfortunately this is not the case, the geiger counter may only give one count for every 100 or so gamma ray quanta that pass through it. The geiger tube is not only very inefficient, but more important, the efficiency of the counter for detecting gamma rays is a function of the gamma ray energy. The student should understand the reason for this variation of geiger counter efficiency, because it is closely related to the reasons for this experiment: Thus we can say,

$$I_{1\circ} = I_1 \epsilon_1$$

6) $$I_{2\circ} = I_2 \epsilon_2$$

$$I_{3\circ} = I_3 \epsilon_3$$

where I_1, and I_2, and I_3 are the true gamma ray incident intensities in quanta per square centimeter per second, and ϵ_1, ϵ_2, and ϵ_3 are the efficiencies of the geiger counter for the numbered gamma rays. The work of two groups of investigators, (references 1, and 2) indicates that in the range from 0.3 Mev. to 2.5 Mev. the efficiency of a geiger counter with a brass or copper cathode

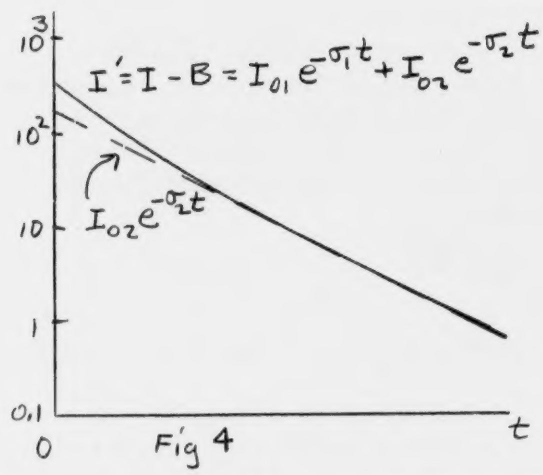


Fig 4

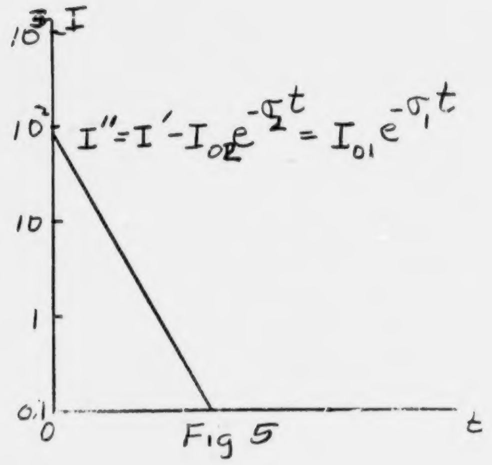
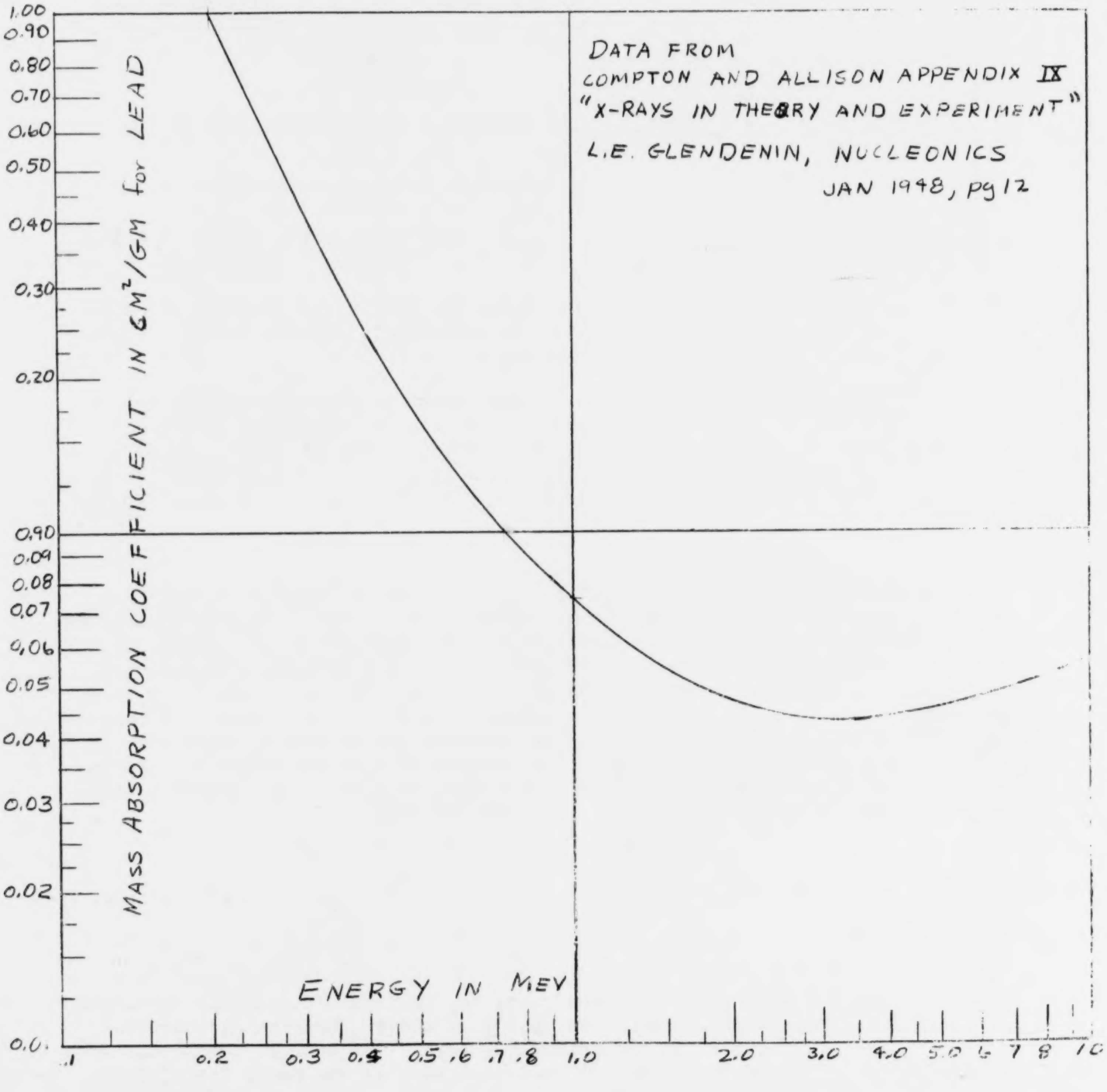


Fig 5



DATA FROM
COMPTON AND ALLISON APPENDIX IX
"X-RAYS IN THEORY AND EXPERIMENT"
L.E. GLENDENIN, NUCLEONICS
JAN 1948, pg 12

is approximately proportional to the photon energy. We may assume from the data in (2) that the efficiency of the geiger counter ϵ is approximately proportional (to within about plus or minus 10 percent) to the energy of the gamma rays, as given in the following equation.

7) $$\epsilon = 0.0054\ E$$

where E is the photon energy in Mev. Our data plus equations 6 and 7 allow an evaluation of the I_o's. The relative intensities so determined will be more accurate than the absolute intensities, because we are not using a brass or copper cathode counter tube.

Laboratory Instructions, General

The gamma ray source used in this experiment is 4.26 milligrams of radium in equilibrium with its disintegration products. The source is in a platinum needle of 0.5 mm. wall thickness, and is sealed in a plastic plug at the lower end of a thin wall brass tube. This tube is kept inside a cylindrical lead safe and should not be removed for any purpose whatsoever without permission of the instructor. The axial hole in the lead safe provides a collimated beam of gamma rays with which the experiment is performed. Two boxes of lead absorbers are provided. (handle these with care, careless dropping or otherwise damaging them is the unforgivable sin!!) All the absorbers are of the same area, 91.1 cm.^{2}, and they are numbered and their weight in grams is stamped on each one. Also provided are some thin lead foils about 0.002 inch thick. These should be weighed, their area measured, and an average grams per square centimeter determined for them. You may assume that all are of the same thickness.

The geiger counter is placed axially in a cylindrical shield that is in line with the collimated beam of gamma rays from the source. The lead shield serves to reduce the random background count from other sources around the room, and it helps make certain that only photons which have passed through the absorbers can reach the tube.

Laboratory Instructions

a) Read the instructions for operating the scaling circuit you will use.
b) Take the plateau of the geiger tube you will use, plotting each counting rate before you take the next higher rate, being very careful not to let the tube go into a continuous discharge. Check your plateau graph with your instructor, and plan to operate the tube in the middle of the plateau.
c) Remove the lead plug that fills the collimating hole in the cylindrical lead safe, and insert the plug in the brass tube that is provided for its protection. Put the plug away where it will not be damaged.
d) The absorbers will stand on edge on a flat board on which graph paper has been pasted. The source shield and the counter shield should be separated by just the width of this board.
e) The counter tube should be carefully placed in its 1/8 inch wall cylindrical lead shield and this assembly should then be placed in the large lead shield to be clamped in place with the tube about midway in the length of the shield. THE COUNTER TUBES HAVE A FRAGILE THIN WALL AND CARELESS HANDLING COULD CAUSE THESE TUBES TO CAVE OR FOLD INWARD. BE CAREFUL.
f) Make a quick determination of the counting rate of the tube with no absorbers in the gamma ray path. The chances are that the rate will be quite high, so high in fact, that the finite dead time of the geiger tube will cause some counts to be lost. The true count will then be higher than the observed count. Review the instructions of experiment No. 40 on the characteristics of geiger tubes and scaling circuits, and proceed to determine the dead time τ of this counter tube by using two of the cobalt sources that

are mounted in the brass rods. Bring two sources to approximately equal distances from the tube, the distances being determined so that the counting rates from the two sources individually are approximately equal, and the counting rate with the two sources together is approximately equal to the counting rate you just observed. Determine the dead time τ of the geiger counter tube just as you did in Experiment No. 40.

g) In all your counting rate observations, make a dead time correction to get the true rate from the observed rate whenever the magnitude of the correction would be larger than half of the probable error of the rate.

h) Proceed to take data of counting rate vs. lead thickness in grams/cm.^{2}. For the first few points, use the thin lead foils in groups of three or four. When you have used all 18 foils, remove them and use the 1/16th inch sheet from the calibrated set, adding additional sheets until the entire two boxes are in use. Data should be taken in 1/16th inch steps at the start, up to 1/2 inch intervals in the middle of the range, and down to 1/4 inch intervals near the end of the two boxes. This data taking is tedious, and only one partner need be in the laboratory at a time. You may take turns watching the apparatus, and you may want to let it run over night, or through morning class periods. Make arrangements with the instructor for this.

i) The probable error of each point should be calculated, and in general each point should be taken to a probable error of less than two percent. When the counting rates are high it is easy to take data to a very much smaller probable error, and this is quite profitable.

j) The important parts of the curve are the start, near zero thickness; the region around 130 to 170 gm./cm.^{2}; and the background rate at large lead thicknesses.

k) After all the lead of the two boxes has been used, remove this lead carefully, measure the length of the lead plug for the source, and insert this plug over the source. Take a data point for this lead thickness, add about half the stack of lead plates, take a point, and finally put the second plug in over the counter tube, and add the rest of the lead plates, and take a rate which will be the background rate. Here it would be convenient to let the apparatus run for a period of several hours to get better accuracy on the background rate. (turn a blower on the scaler tubes to help keep them cool)

The Report:

1) Your data sheet should show in tabular form,
   The number or other designation of the lead foils in place.
   The grams/cm.^{2} of lead over the source.
   The number of counts observed.
   The time elapsed for this count.
   The calculated counting rate.
   The counting rate corrected for the dead time loss (the true rate).
   The probable error of each rate.
   The rate corrected by substraction of the background (see below)

2) With a large expanse of semi-log graph paper, a sharp pencil, and drafting equipment (available in the laboratory), you should plot a large scale graph of
   a) Your data of corrected counting rate vs. lead thickness. Show the background level on this graph.
   b) On this graph plot the corrected rate minus the background.
   c) Extrapolate the straight line portion of this graph back to zero thickness of lead.
   d) Proceed with the analysis discussed in the theory to determine the absorption coefficients, energies, and relative intensities of as many gamma rays as you can separate. You should tabulate each absorption coefficient, each relative intensity with a clear statement of exactly what it is, and then look up the corresponding energies on a graph of absorption coefficient vs. energy.
   e) Compare this with the tabulated results for radium, and comment on the discrepancies, if any.

Note Each graph point should be a point, a circle should be drawn around it to show where it is, and a vertical bar with cross bars at the ends of the vertical bar would show the magnitude of the probable error.
Also Note: You should show your calculations clearly with enough explanation so that the person reading the report can tell that you understand what you are doing.

Optional Part B.
Determine an upper limit to the efficiency of the geiger tube. Have the instructor give you a burned out or damaged tube of the same type that you were using to detect the gamma rays. Place this tube in the beam of gamma rays with its axis at right angles to the beam and determine by what percent the tube reduces the intensity of the transmitted beam. Write a paragraph in your report to explain why this gives an upper limit to the sensitivity rather than giving the actual sensitivity. To do this you must understand the processes by which a gamma ray may discharge a geiger tube.

Optional Part C
Determine the variation of absorption coefficient with atomic number.

In addition to the lead sheets, there are available some absorbers of carbon (graphite), aluminum, copper, iron, an alloy of copper and aluminum, and perhaps others. These should be measured with the vernier calipers and weighed in a balance to determine the grams/cm.^{2} for each absorber. They are of different sizes.

One may work in the direct beam from the source, or if one is interested in having a more monochromatic radiation, one can place one half to one inch of lead over the source to absorb the softer radiation. Now carefully determine the counting rates with the various thicknesses of each absorber in place over the bare source, or over the lead filter. Plot your results in the form of an absorption curve, determine the mass absorption coefficient for each element, and plot these coefficients as a function of the atomic number and comment on the results.

ALL STUDENTS NOTE. When you have completed the experiment the long lead plug should be reinserted over the source, the lead sheets should be carefully put back in their case, and the geiger tube should be returned to the shelf.

ARTIFICIAL RADIOACTIVITY

University of Pennsylvania

When a nucleus is bombarded by a particle such as a proton, neutron, deuteron, or alpha particle, there is the possibility that the target nucleus may capture the bombarding particle to form a compound nucleus. This compound nucleus immediately breaks up into a light particle or a photon plus the appropriate heavier nucleus necessary for the conservation of charge and the conservation of mass plus energy. These heavier nuclei may be stable, but usually are not in which case they are known as artificially radioactive nuclei.

Neutrons, because they possess no electric charge, have proved to be very effective in penetrating the positively charged nuclei, thereby producing nuclear transformations. In this experiment we shall use slow neutrons as bombarding particles. The source consists of radium which gives off alpha particles and beryllium which captures the alpha particles and gives off fast neutrons. A nuclear reaction can be represented by an equation analogous to an equation for a chemical reaction. The equation for this reaction is

$$_4Be^9 + {}_2He^4 \longrightarrow ({}_6C^{13}) \longrightarrow {}_6C^{12} + {}_0n^1 \qquad (1)$$

where $({}_6C^{13})$ is the compound nucleus. Note that the upper index is the mass, the lower one is the charge. The neutrons pass out of the lead container into the paraffin surrounding it. Here the neutrons collide with hydrogen nuclei losing large portions of their energy with each collision. After many such collisions, they have an average energy which is equal to the energy of thermal agitation equivalent to $\frac{1}{40}$ electron volt at room temperature.

$$_{49}In^{115} + {}_0n^1 \longrightarrow {}_{49}In^{116} \qquad (2)$$

$$_{49}In^{116} \longrightarrow {}_{50}Sn^{116} + {}_{-1}e^0 \qquad (3)$$

${}_{-1}e^0$ represents an electron.

The rate at which a particular radioactive material disintegrates is a constant independent of all physical and chemical conditions. Given a large number of atoms of any one radioactive element, the number dN that will disintegrate in a small time interval dt is found to be proportional to the number of atoms N present at the time t; that is

$$-dN = \lambda N dt \qquad (4)$$

where λ is a constant for the particular radioactive element known as the decay constant. This equation may be integrated at once to give

$$N = N_o e^{-\lambda t} \qquad (5)$$

where No represents the number of atoms present at the time t = 0. When N = $\frac{1}{2}$ No, half of the original number of atoms present will have disintegrated. The time in which this occurs is called the half-life T. We use Eqn. (5) to find T

$$\frac{N}{No} = \frac{1}{2} = e^{-\lambda T} \qquad \ell n\tfrac{1}{2} = -\lambda T$$

$$T = \frac{.693}{\lambda} \tag{6}$$

The half-life of $_{49}In^{116}$ is about 54 minutes. The average lifetime T_a of a single atom may be computed by multiplying dN the number of atoms disintegrating by the time t during which they existed, summing these products over all the atoms and then dividing by the total number of atoms at the start No

$$T_a = \frac{\int_0^{No} t\,dN}{No} = \frac{\int_0^{No} t\,(-No\,\lambda\,e^{-\lambda t}dt)}{No} = \frac{1}{\lambda} \tag{7}$$

The reciprocal of the decay constant λ is thus the average lifetime of a radioactive atom. If the half-life is known from experimental data, then the average lifetime $\frac{1}{\lambda}$ can be computed from Eqn. (6).

The presence of the radioactive indium is detected by means of a Geiger counter which is sensitive to gamma rays and high speed electrons and positrons. The counter consists of a wire mounted co-axially within a metal cylinder. This assembly is placed in a glass envelope which is filled with a combination of gases at a pressure of 0.10 m of Hg. One end of the glass envelope is made very thin to allow the high speed electrons to penetrate into the tube. The electrical connections are shown in Fig. 1.

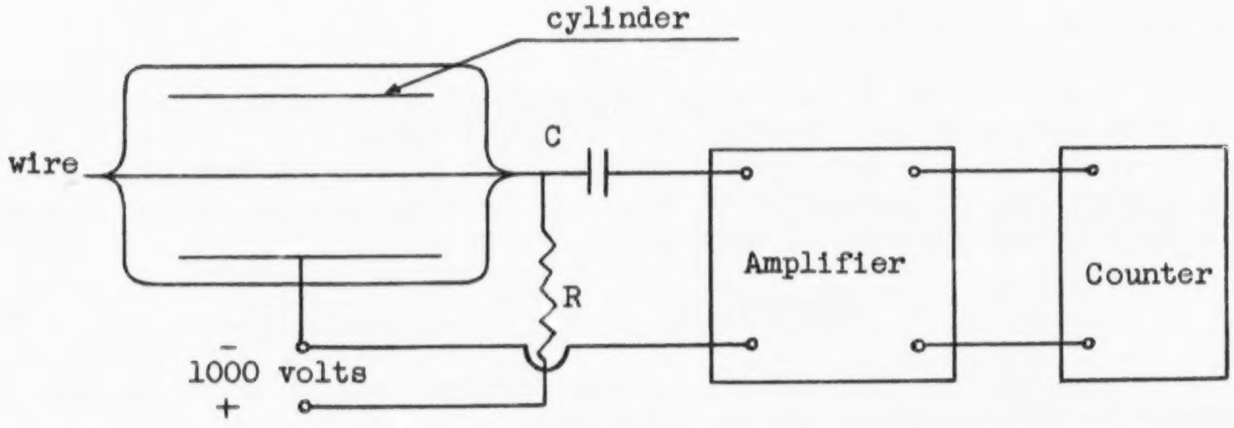


Fig. 1

The voltage between wire and cylinder is normally 1000 volts and this voltage is not quite high enough to produce an arc. However, if some ions are produced between the cylinder and the wire they will be accelerated in the strong electric field and gain enough energy so that they in turn can form more ions and the process cascades until soon a large current is flowing through the resistance R. The resulting voltage drop from the current flowing through the resistor lowers the potential difference between the wire and cylinder until the arc can no longer sustain itself. Once the arc is extinguished the voltage will rise again until it reaches its maximum value. The tube will remain in this static condition until some more ions are formed, and it goes through the cycle again. When a photon or high speed electron passes through the tube it will produce enough ions to form the cascade which produces the arc. Every time this arc is formed the voltage of the point C changes greatly and this voltage pulse is amplified by vacuum tube circuits until it has sufficient energy to operate a counter. The number of counts per unit time is proportional to the number of ${}_{49}\mathrm{In}^{116}$ atoms present.

The number of counts per minute is not very large and since this is a purely random matter large statistical fluctuations are to be expected. The fluctuation to be expected is about $\sqrt{N}$ where N is the number of counts. This means that if we repeat the measurement of the number of counts under identical conditions for the same length of time, the probability is 2 out of 3 that the two N's will disagree by less than the square root of either one. (In probability theory this is called the "standard deviation".) If the first result were 25 counts, many repetitions of the same experiment would give us a number somewhere between 20 and 30 about two-thirds of the time. Thus, considerable scattering of the experimental points may be expected.

Procedure:

(a) Place several indium foils in the center of the paraffin slabs surrounding the source of neutrons.

(b) Have the instructor demonstrate the equipment. The first particle detected by the Geiger tube causes a small light to flash on. The second particle causes this light to go off and a second to go on. The third particle detected causes the first light to go on. The fourth particle causes both lights to be turned off and a third light to go on. Five lights are able to count up to 64. At the 64th count a mechanical counter is activated. Some counters also operate on a scale of 10.

(c) Even with no radioactive sample near the Geiger tube, counts are recorded. This is known as "background count" and is caused by cosmic rays, residual ions from the X-ray machines or radioactive sources around the laboratory. Any "measurement" of a radioactive sample then includes this background which must be subtracted from the measurement to give a true value of the radioactivity of the sample. Determine this background by finding the time required to make 500 counts. Repeat this determination and use the average as the background count in counts per second.

(d) Remove the indium foils from the paraffin and place it under the Geiger counter. Record the readings of the counter for two minute runs waiting one minute between runs. Thus you will start runs at t = 0, 3, 6, 9 ... and stop runs at 2, 5, 8, 11 ... Follow the counting rate for one hour.

Report:

(1) Tabulate the counts per two minute intervals after background is subtracted. Consider this the true value of the counting rate at the mid-point of the interval.

(2) Plot the log of these numbers against time or use semilog graph paper. Indicate the expected spread of values by drawing a vertical line from $N + \sqrt{N}$ to $N - \sqrt{N}$ and short horizontal lines at the end. Draw the straight line that is the "best" fit to the experimental points. This line should cut about half of the vertical lines of the graph.

(3) Find the time at which the counting rate falls to half value. This value of the time is known as the half-life. Determine the slope of the line. Compute the half-life from the slope using equation (6). Estimate the error from your uncertainty in determining the slope of the line. Which method of finding the half-life is better? Why?

(4) How much time is required with a source of neutrons of constant intensity to form 90% of the maximum possible amount of ${}_{49}In^{116}$? Take the half-life to be 54 minutes.

(5) Find and explain the meaning of the average lifetime of ${}_{49}I^{116}$. What is the value of the decay constant?

(6) How can the accuracy of this experiment be improved?

THE ACTIVATION OF COPPER

North Carolina State College

I. References:

Friedlander and Kennedy, Introduction to Radiochemistry, Ch. 5, p. 127.
Evans, The Atomic Nucleus, Ch. 15, p. 470.
Bleuler and Goldsmith, Experimental Nucleonics, Ch. 4, p. 105.
Price, Nuclear Radiation Detection, Ch. 9, p. 281.

II. Purpose:

To measure the build-up of Cu^{66} and the decay of Cu^{66} and Cu^{64}.

III. Apparatus:

Thin wall GM counter (Victoreen 1B85 or equivalent), scaler with high voltage, timer, neutron source (yield of about 10^6 n/sec) with paraffin moderator fitted with central port for source and sample, Cu cylinders with inner diameter of about 25/32 inches to fit over GM tube and a length of 3 inches.

IV. Theory:

The build-up of activity of a given isotope as the result of a nuclear reaction offers, if the reaction rate is constant, a good illustration of secular equilibrium. The equation describing the process is

$$\frac{dN}{dt} = p - N\lambda \qquad (1)$$

where p is the production rate, N is the number of atoms present and λ is the decay constant. If the isotope is produced by neutron bombardment

$$p = \Sigma V \varphi \qquad (2)$$

where Σ is the macroscopic activation cross section, V the volume of the sample, and φ the neutron flux. We may write equation (1)

$$\frac{dN}{dt} = \Sigma V \varphi - N\lambda . \qquad (3)$$

The solution of the above equation is

$$N = \frac{\Sigma V \varphi}{\lambda} (1 - e^{-\lambda t}) \qquad (4)$$

where t is the irradiation time. This may be written as

$$A = \Sigma V\phi (1 - e^{-\lambda t}) \qquad (5)$$

where A represents the activity of the sample.

In the case of copper, there are two stable isotopes having mass numbers of 63 and 65. The irradiation of copper with thermal neutrons produces the following reactions:

$$Cu^{63} (n, \gamma) Cu^{64} \qquad (6)$$

$$Cu^{65} (n, \gamma) Cu^{66} \qquad (7)$$

The decay of these isotopes is as follows:

$Cu^{66} (\beta^-) Zn^{66}$ $T_{\frac{1}{2}}$ 5.1 m

$Cu^{64} (\beta^-) Zn^{64}$ (39%) $T_{\frac{1}{2}}$ 12.8 h

or $Cu^{64} (\beta^+) Ni^{64}$ (19%)

or Cu^{64} (e.c.)Ni^{64}(42%) (9)

Each of these isotopes will build up according to equation (5) until a saturation value is reached at which time the decay rate equals the production rate. It is possible, by using short irradiation times, to make the Cu^{66} activity the dominant one. If, on the other hand, irradiation is prolonged, both isotopes are readily detectable from the decay curve.

V. Procedure:

1. Expose copper cylinders near the neutron source in the paraffin moderator for periods of 1, 2, 5, 10, and 20 minutes. Measure the relative activity of these samples by counting each sample for 100 seconds. The time between the end of the irradiation and the start of the counting must be kept constant.

2. Measure the decay of the copper cylinder which has been activated for about 10 minutes. Take a 25 second count every 30 seconds. Continue until the counting rate approaches a constant value.

3. Measure the decay of a copper cylinder which has been irradiated for about 24 hours. Take readings for about 30 minutes or until the counting rate approaches a constant value. Record the total activation time and decay times.

VI. Problems:

1. Plot the build-up curve for Cu^{66} together with the theoretical curve, based on a half-life of 5.1 minutes.

2. Plot the decay curve and determine the half-life of Cu^{66} and Cu^{64}.

3. Determine the relative saturation counting rates due to the Cu^{66} and Cu^{64} activity.

4. Calculate the relative activity of Cu^{66} and Cu^{64} from the activation cross sections and the irradiation time. Does this confirm the results of (3)? Explain. What factors need to be considered in the calculation?

COINCIDENCE MEASUREMENTS

North Carolina State College

I. References:

Siegbahn, Beta and Gamma Ray Spectroscopy, Ch. 7, p. 201.
Bleuler and Goldsmith, Experimental Nucleonics, Ch. 22, p. 320.

II. Purpose:

1. To measure the resolving time of a coincidence circuit.
2. To use the coincidence method to obtain information on decay schemes.

III. Apparatus:

Two scintillation detectors with associated preamplifiers, amplifiers, and high voltage supply; coincidence circuit; adjustable support for scintillation counters; gamma ray sources. (Detail in block form under Section VI.)

IV. Theory:

The coincidence method is useful in determining the decay scheme of a radionuclide and in determining absolute source strengths. Basically, the coincidence circuit is one which delivers a response only when pulses appear at two or more inputs within a time, τ, called the resolving time. In the subsequent discussion, we shall assume two-fold coincidence systems. For purposes of discussion, let us assume two detectors viewing a source--one being a beta counter and the other a gamma counter. The counting rate at each counter will be given by the relations

$$N_\beta = N_0 \omega_\beta \epsilon_\beta \qquad (1)$$

$$N_\gamma = N_0 \omega_\gamma \epsilon_\gamma K \qquad (2)$$

where ω_β and ω_γ represent the solid angles subtended by the counters ϵ_β and ϵ_γ the efficiencies of the counters. K represents the number of gamma rays per disintegration and N_0 the number of disintegrations per second. The true coincidence rate is given by the relation

$$N_{\beta\gamma} = N_0 \omega_\beta \omega_\gamma \epsilon_\beta \epsilon_\gamma K \qquad (3)$$

The chance or accidental coincidence rate is given by

$$\begin{aligned} N_{ch} &= 2\tau N_\beta N_\gamma \\ &= 2\tau N_0^2 \omega_\beta \omega_\gamma \epsilon_\beta \epsilon_\gamma K \qquad (4) \end{aligned}$$

Combining equation (3) and (4) we obtain

$$\frac{N_{ch}}{N_{\beta\gamma}} = 2\tau N_0. \qquad (5)$$

The resolving time may be measured by means of equation (4).

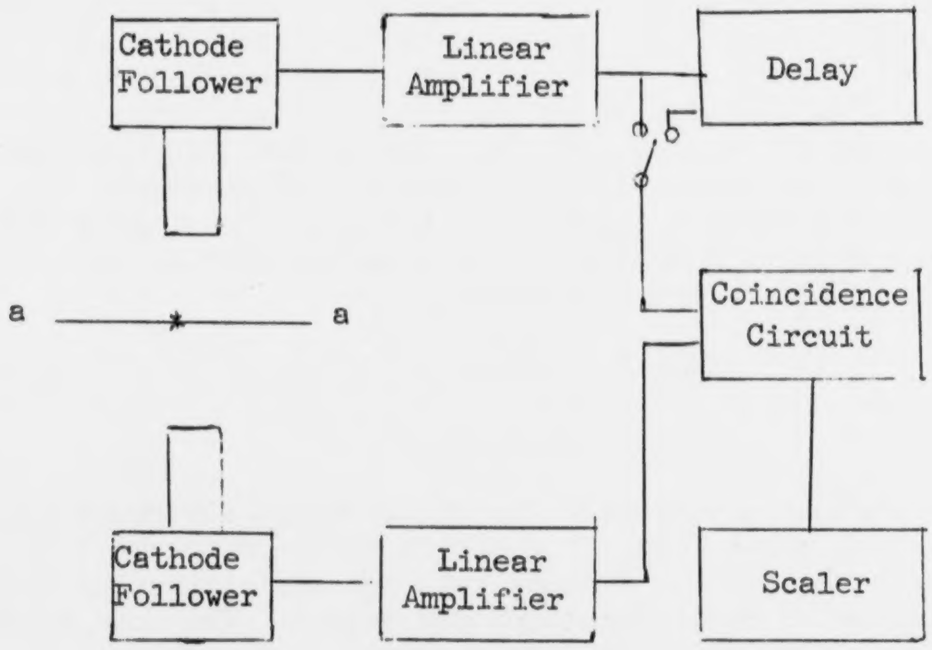
The two detectors should be separated at some distance and independent sources used near each detector. The single and coincidence rates are measured and τ is thus obtained. A somewhat more convenient method involves the use of an artificial delay in one of the two inputs. If the delay exceeds the resolving time the only coincidences observed will be chance coincidences.

Once the resolving time has been determined the absolute strength of a source may be calculated from equation (5).

Much information on the decay scheme of nuclides emitting complex gamma ray spectra may be obtained from γ-γ and β-γ coincidences. Various cases are discussed in the first reference.

V. Procedure:

Arrange the two scintillation counters in the support with gamma counters in each channel.



Experimental arrangement

Delay and coincidence circuit are incorporated in Beva 201 Pulse Analyzer. Source is supported on optical bench and is free to move along line a - a.

1. Measure the resolving time of the coincidence circuit by determining the singles and chance coincidence rates. Obtain the chance coincidence rate by inserting a delay in one of the channels.

2. Place a Na^{22} source at a point midway between two NaI scintillation counters. Measure the total and chance coincidence rates. Move the source along a line perpendicular to the axis of the crystal. Repeat the measurement at 2 mm intervals, continuing until the source lies well beyond the cylinder defined by the NaI crystals.

3. Repeat the measurements using a Co^{60} source.

4. Repeat the measurements using a Cs^{137} source.

5. Determine the absolute strength of Au^{198} source by the method described above. Use a stilbene scintillation counter for the beta detector.

VI. Problems:

1. Calculate the resolving time of the coincidence circuit. Include its standard deviation.

2. Plot the results of (2), (3) and (4) on the same graph and give a qualitative discussion of the results.

3. Discuss the origin of the coincidences produced when the Cs^{137} source is used.

4. Calculate the strength of the Au^{198} source giving the standard deviation of your result.

NUCLEAR INDUCTION

University of California, Berkeley

References:

Fretter, Introduction to Experimental Physics, pp 266-276.

Harnwell, Principles of Electricity and Electromagnetism, 2nd ed., pp 366-371.

Pake, Nuclear Magnetic Resonance, Amer. Journal of Physics, 18, 438, 473, (1950).

Bloch, Hansen and Packard, Phys. Rev., 70, 474 (1946; Bloch, Phys.Rev.70, 460(1946).

Apparatus:

Large permanent magnet

Nuclear induction apparatus and power supply

Frequency meter (BC 221)

Oscilloscope

Sweep coil transformer, Variac, and phase shifter.

Purpose:

To observe the nuclear induction signals from protons in water, to study relaxation times, and to compare the gyromagnetic ratios of H^1 and F^{19}.

Method:

In this experiment the phenomenon of nuclear magnetic resonance is observed using the technique of nuclear induction. A sample of the nuclei under investigation (e.g.,the protons in water contained in a test tube) is placed in a D.C. magnetic field H_o, which is perpendicular to the paper of Figure 1. If the nuclei have an angular momentum or "spin" of $I\hbar$, and a magnetic moment μ, they will individually precess in this field at a frequency given by the gyroscopic equation

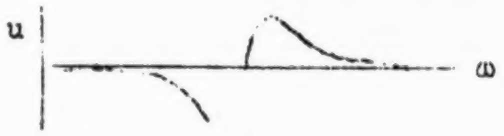
$$\omega_o = \frac{\text{torque}}{\text{angular momentum}} = \frac{\mu H_o}{I \hbar} = \gamma H_o \quad \text{-- ----------} \quad (1)$$

This frequency ω_o is called the Larmor frequency or the nuclear resonance frequency of the nuclei in the field H_o; γ is the gyromagnetic ratio. The nuclear spins are also in thermal contact with the water at temperatures T and this establishes in a characteristic time T_1 a net magnetization given by the Curie formula:

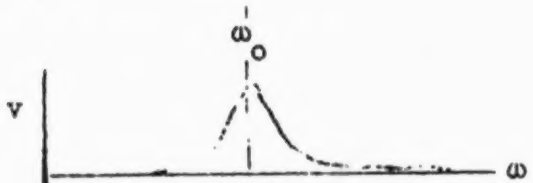
$$\vec{M}_o = \chi_o \vec{H}_o = \frac{N\mu^2}{3kT} \frac{I+1}{I} H_o ,$$ where N is the number of nuclei/cc .

We aim to observe the voltage induced by this macroscopic magnetization. This is done by applying with an oscillator a radio-frequency field $H_1 \cos \omega t$ to the sample along an axis mutually perpendicular to both H_o and to the receiver coil axis (see Fig. 1). If ω is made approximately equal to ω_o, the r-f field will effectively produce a rotating component of M in the plane of the receiver coil and there will be induced a measurable voltage at the frequency ω_o. Bloch has solved this problem in detail and he finds the receiver coil V_n to be of the following form for the case of H_1 very small:

$$V_n \propto u \sin \omega t + v \cos \omega t \text{ -(2)}$$



$$u = \frac{\gamma H_1 T_2^2 (\omega - \omega_o) M_o}{1 + T_2^2 (\omega - \omega_o)^2} \text{ - - - - - -(3)}$$



$$v = \frac{\gamma H_1 T_2 H_o}{1 + T_2^2 (\omega - \omega_o)^2} \text{ - - - - - - (4)}$$

Here, T_2 is the "total" relaxation time (or the inverse line width) due to all processes which broaden the resonance line: thermal relaxation processes; spin-spin effects, i.e., the effects of neighboring nuclei on each other via their magnetic moments; and non-uniformities in the field H. The component of the receiver coil voltage in phase with the rf field has the shape of a classical dispersion curve; the out-of-phase component has the shape of an absorption curve. This should seem reasonable when we recall the analogy between the nuclear spin system and the classical driven damped oscillator. In the latter there are two components of the displacement (the dispersion and the absorption) which vary in amplitude as one goes through resonance like the u and v components or "modes" of the receiver coil voltage. Instead of varying the frequency to go through resonance we can periodically vary the field H_o by superimposing on the D.C. field an A.C. field from the sweep coils.

In the present apparatus it is possible to observe either or both modes by adjusting the magnitude and phase of the"leakage" voltage V_L which is induced into the receiver coil by the r-f field H_1. This may be done by the paddles A and B (Figure 1). Usually $V_L \gg V_n$. If $V_L = k \sin \omega t$, then the resultant of $V_n + V_L$ is essentially u + k; and if $V_L = k \cos \omega t$, the resultant is essentially v + k. The magnitude of the leakage k is measured by the meter tube circuit, which simply measures the D.C. voltage developed by the diode detector. The u and v components of the diode voltage vary with time at the sweep frequency rate, and these are amplified by the audio amplifier and applied to an oscilloscope which thus writes a picture of either a nuclear magnetic absorption or dispersion curve.

Procedure:

Before operating, take off the cover of the r-f head and examine the construction. Paddle A is a loop of copper wire which couples the oscillator and receiver coils by an amount dependent on the angle of rotation, which may be 360°. Paddle B is limited in rotation to a few degrees (caution, don't force it) and gives out-of-phase coupling by virtue of the resistance of the nichrome wire. For protons $I = 1/2$, $\mu = 2.7925\ \mu$, where

$$\mu_o = \frac{eh}{4\pi Mc} = \text{nuclear magneton.}$$

The magnet field H_o = 4000 gauss. Tune the oscillator and transmitter to the resonance frequency and observe the signals for protons in water plus one molar $MnCl_2$. ($MnCl_2$ is added so that the paramagnetic M_n++ ions will shorten T_1 to a value much shorter than the sweep period, which is assumed in equations 2, 3, 4.) A communication receiver is a convenient frequency meter. Observe both the dispersion and absorption modes by adjusting the paddles. Observe the effect of varying the sweep amplitude and the phase of the scope sweep. Observe the effect of moving the head in the magnet gap. The field is very uniform near the center, but becomes non-uniform off-center; this broadens the resonance. Calibrate the x-axis of the oscilloscope picture in terms of gauss by making a measured increment in the oscillator frequency and observing the shift in the position of the resonance. Measure the width of the resonance absorption curve in gauss. Observe the signals from the other H_2O samples containing less $MnC\ell_2$; the rapid oscillations (the "wiggles") are due to the inability of the spins to follow the r-f field because of the longer thermal relaxation time; a beating effect results (Phys. Rev. 73, 942, 1948). Use the sample containing HF (Caution) to measure precisely the resonance frequencies of both H^1 and F^{19} in the same magnetic field H_o. If the frequency meter sensitivity is inconveniently low, observe the zero beat via the nuclear induction receiver, which is extremely sensitive.

Report:

1. Make a simple derivation of equation (1).
2. From your measurements of the line width in the H_2O + 1 molar $M_nC\ell_2$, calculate the total relaxation time T_2.
3. Calculate exactly the field H_o.
4. Calculate exactly the magnetic moment of F^{19} in units of the nuclear magneton; compare to published values.

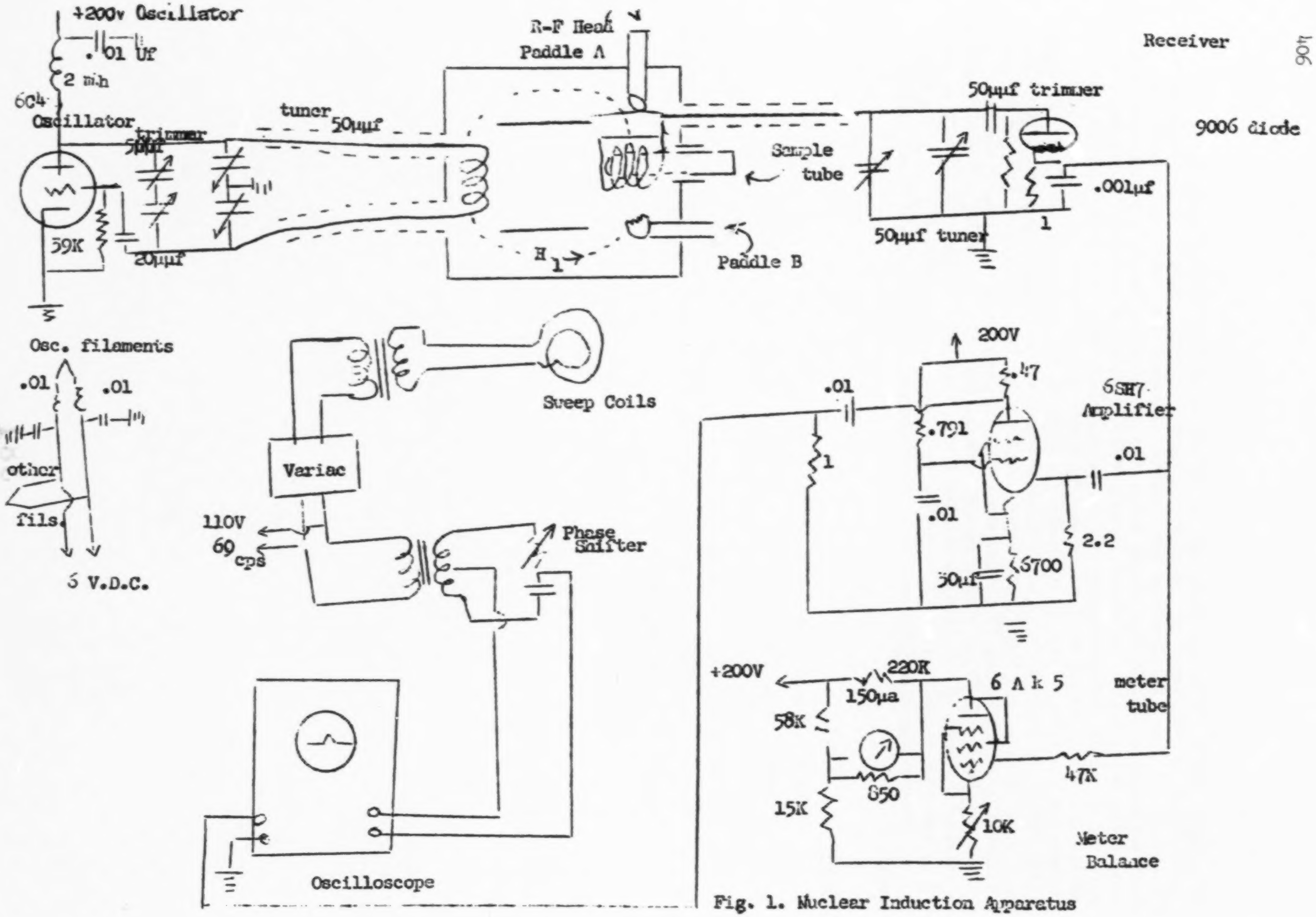

+200v Oscillator
.01 Uf
2 mh
6C4
Oscillator
trimmer
50μμf
20μμf
39K
tuner
50μμf
R-F Head
Paddle A
Sample
tube
Paddle B
H
1
Receiver
50μμf trimmer
9006 diode
.001μf
1
50μμf tuner
Osc. filaments
.01
.01
other
fils.
5 V.D.C.
Sweep Coils
Variac
110V
69 cps
Phase
Shifter
Oscilloscope
200V
.47
.01
.791
6SH7
Amplifier
.01
1
.01
2.2
50μf
5700
+200V
220K
150μa
58K
850
15K
6 A k 5
meter
tube
47K
10K
Meter
Balance

Fig. 1. Nuclear Induction Apparatus

MÖSSBAUER EFFECT

California Institute of Technology

PURPOSE: To measure the line width in the recoilless resonance absorption of 14.4 kev gamma rays from Fe^{57*}.

PRINCIPLES:

The emission and absorption of electromagnetic radiation by classical oscillators, such as bound electrons, is the basis of the electron theory of the optical properties of dielectrics. The absorption is a maximum at the "natural" frequency ω_o, and the "natural line width" $\Delta\omega$ is intimately related to the mean decay time τ of the oscillator damped by radiation loss only.

Atomic oscillators are quantized so that radiation can only occur for discrete frequencies defined by $\hbar\omega_{mn} = E_n - E_m$. However, due to the uncertainty principle the "discrete" energies are not precisely defined, so that the emitted radiation still has a spread in energy (or frequency) $\Gamma = \hbar\Delta\omega$. This spread, or "natural line width" is related to the lifetime τ of the excited (upper) state by

$$\Gamma \cdot \tau = \hbar = 0.66 \times 10^{-15} \text{ e.v.-sec.} \qquad (1)$$

Thus, for a typical atomic state lifetime of 10^{-8} seconds, the natural linewidth is 0.66×10^{-7} e.v. .... a very narrow energy spread indeed! In actual practice "lines" for atomic spectra are broadened due to thermal motion (Doppler effect) of the emitting sources, pressure broadening (collisions between atoms), etc.

Nuclei also have quantized levels and emit electromagnetic radiation (gamma rays) at discrete energies. In general gamma-ray energies (a few kev to several Mev) are greater than optical or X-ray energies, and the lifetimes of excited nuclear states decaying by gamma emission may be very short (thus the levels may be many e.v. wide). However, due to selection rules, some states decay slowly; the 14.4 kev level in Fe^{57}, for example, has a lifetime of $\tau = 1.4 \times 10^{-7}$ sec. Prior to 1958 the very narrow natural line widths of such nuclear radiations were considered unobservable due to the Doppler broadening caused by the recoil motion of the emitting nucleus. Although the velocity of the recoil nucleus is small, the kinetic energy E_r absorbed is much greater than Γ; i.e.

$$E_r = \frac{p^2}{2M} = \frac{E_\gamma^2}{2M} \qquad (2)$$

For $E_\gamma = 14.4$ kev and Fe^{57} nucleus, $E_r = .002$ e.v.

In 1958 Mössbauer showed that for atoms bound in solids (lattice structures such as crystals), the entire lattice absorbs the recoil momentum. Thus "M" is many orders of magnitudes larger (number of atoms in a microcrystal) and E_r becomes correspondingly smaller. The fraction of atoms thus bound is not 100 per cent, and the strength of the lattice bonds relative to the recoil forces is important, but a large number of radioactive substances are now known to exhibit the "Mössbauer effect".

The extreme sensitivity of resonance emission and absorption is dramatically shown by Doppler-shifting the gamma-ray frequency by moving the source (or absorber). For example, the entire "tuning curve" for Fe^{57} spans a relative velocity range of less than 1 mm/second. (It is fairly difficult to construct a drive which maintains constant velocity to within 5 percent of this figure.) The Doppler shift of photon energy is

$$E' = E[(1-\beta)/)1+\beta)]^{1/2} \qquad (3)$$

where $\beta = v/c$; if $\beta << 1$, $(E-E')/E \cong \beta$. Thus v = 0.3 mm/sec. corresponds to $\Delta E/E = 10^{-12}$.

Such energy resolution makes it possible to utilize the Mössbauer effect for investigations of very small shifts in photon energies, such as the gravitational "red shift", nuclear Zeeman effect, internal magnetic fields, and many other problems in general relativity, nuclear physics, and solid state physics. (See References).

In this experiment the influence of the environment (e.g., magnetic field at the Fe^{57} nucleus produced by neighboring electron current density) is seen by the shift of the absorption peak when a copper backing is used instead of a stainless steel backing for the source. In addition, use of a pure iron absorber (in place of stainless steel) shifts the resonance peak. Application of a moderately strong external magnetic field to the iron foil will split the absorption line into (6) Zeeman lines (a possible further extension of the experiment).

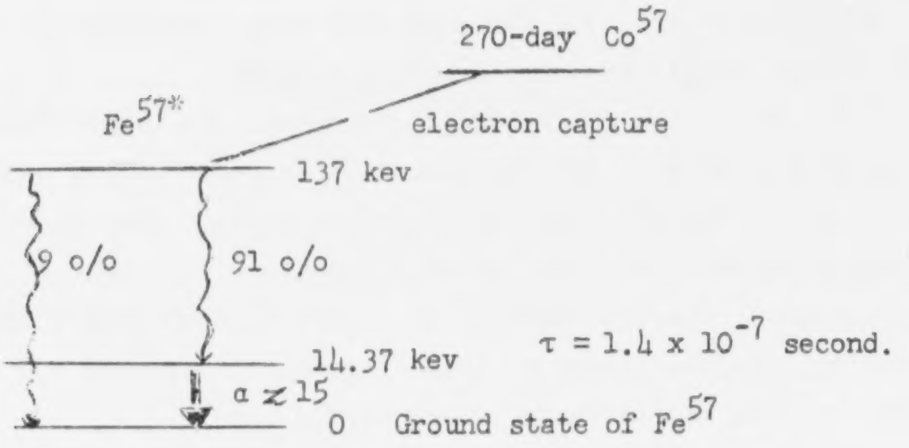
References:

(a) The Mössbauer Effect, by Hans Fraunfelder. (Benjamin, Inc. "Frontiers in Physics" lecture notes and reprints).

(b) S. De Benedetti, "The Mössbauer Effect", Scientific American, 202, 72 (April 1960)

Decay Scheme of Fe^{57}

The radioactive source used here is Co^{57*}, which decays by electron capture (the nuclear net charge Z decreasing by one) to the 137 kev excited state of Fe^{57}:



Only 9 percent of the excited state atoms decay directly to the ground state by emitting a 137 kev gamma-ray; 91 percent decay first to the 14.37 kev level (emitting 123 kev photons) and then to the ground state. Subsequent decay from the 14.37 kev level does not always yield a gamma-ray; the gamma-ray is "internally-converted" (i.e. knocks out an electron, from the K-shell) approximately 15/16 of the time (α = internal conversion coefficient). A 6-kev X-ray follows each internal conversion. Thus there are 15 6-kev photons for each 14.4-kev photon, and 15 conversion electrons.

PROCEDURE:

(1) Understand the electronics block diagram and the function of each item of equipment. (See notes attached to the equipment.)

(2) Turn on power switches to equipment.

(NOTE: turn on filament power to pre-amp first, then B+; finally HV to counter. Set to 2500 volts.)

(3) Test that the pulses counted are primarily due to the source.

(4) With source stationary, observe pulses on oscilloscope. (Use positive pulses at "low level" output of linear amplifier, and trigger scope internally.) You should be able to see a large number of 6-kev pulses, a noticeable grouping of the 14.4 kev pulses, and a sprinkling of other pulses due to interactions of the 140 kev gamma-ray in the counter.

(5) Now trigger the 'scope externally with the pulse-height analyzer (PHA) output. Adjust the minimum acceptable pulse height ("baseline") to cut

out the 6-kev pulses, but leave the 14.4 kev pulses. Adjust the "window" aperture to accept just above 14.4 kev. This will reduce unwanted pulses. If this is satisfactory, you are ready to run.

(6) Make sure that the lead shield is centered on the counter window (thin beryllium -- care!). The stainless steel absorbers (4 to 6 sheets each .00016" thick) should be mounted over the window.

(7) Mount one of the Co^{57*} sources on the Brush "Surfindicator" constant-velocity piston drive. Set the amplitude to about 10 mm, and the "near" position of the source a few mm. from the absorbing foils. The entire absorption peak is less than 1 mm/sec. in width, so that the drive will have at least 10 seconds in each direction. Measure velocity by timing a known amplitude. It is best to keep the amplitude constant to avoid changes in average solid angle. The "zero velocity" points should be very long period runs.

(8) Measure counts/second as a function of velocity, collecting data for + ("toward") and - ("away") velocity separately. (The Mössbauer peak may be shifted from zero velocity if the atomic environment -- due to the backing material -- is not identical for source and absorber.)

(9) Plot your data as you go in order to foresee the best points for additional data. The sources available are:

Co^{57*} in stainless steel backing
Co^{57*} in copper backing.

Each shows ~ 20 percent change in intensity at the peak absorption.

A thin iron absorber is also available to compare effects of changing absorbers.

(10) Summarize your results as follows:

Absorption peaks observed:

Percent absorption (max)
Peak velocity (mm/sec) = $\bar{v}$
Width of peak (full width at half max) in mm/sec. (Δv)

Using the Doppler shift in energy of the photon (14.4 kev γ-ray) express the observed results (for $\bar{v}$ and Δv) in electron-volts.

How does the observed width compare with the natural line width of the 14.4 kev. transition? (Ask your instructor how to calculate natural line width.)

Oak Ridge Institute of Nuclear Studies

Scintillation counting has come into a wide spread use for gamma ray analysis in the last several years. Gross counting of low energy betas dissolved in suitable liquid scintillation mixtures have also been widely used. The use of scintillators for beta ray spectrometry has, however, been somewhat neglected.

When electrons or beta particles pass through a scintillator, a number of low energy photons (in the blue and ultra-violet regions) are emitted which is proportional to the energy absorbed. Sodium iodide is not a useful crystal for beta spectrometers because (1) its high efficiency for gamma ray interactions results in a masking of beta spectra when gammas are also present, and (2) beta particles which enter the crystal are likely to be scattered out due to the high atomic number of iodine. To make a good crystal beta ray spectrometer one should choose a scintillator of low atomic number. Anthracene is one of the best choices for several reasons; (1) it is readily available in a number of sizes, and is fairly inexpensive, (2) the quantum efficiency is relatively high (the quantum efficiency of a crystal is the ratio of the total energy emitted as light to the total energy absorbed), and (3) its low atomic number minimizes the scattering out of beta particles and affords a relatively low efficiency for gamma ray interactions.

Chapter 5 of "Beta- and Gamma-Ray Spectroscopy" by Kai Siegbahn has a discussion of the scintillation methods utilizing anthracene in the determination of beta spectra. Although this chapter describes various shapes and mountings of crystals, we have found that a flat piece of anthracene mounted directly on a photomultiplier tube gives useful results. A major disadvantage of all scintillation detectors is their rather poor energy resolution. An analysis of the operation of the scintillation-photomultiplier tube combination shows that this is largely due to the relatively small number of photo-electrons which are ejected from the photocathode of the tube as the photons produced in the scintillator pass through to the photocathode. That is to say, the quantum efficiency of the photocathode is low. Since the quantum efficiency of anthracene is of the order of 0.02 and that of the photocathode is of the order of 0.05 to 0.1, it can be seen that a very small percentage of the total energy absorbed is used in producing a pulse.

The photocathode surface is not always uniformly deposited. Thus, the quantum efficiency of one section of the surface may be different from that of another section. The quantum efficiency may vary by a factor of 2 or 3 from the highest to the lowest value. Obviously, the beta spectrometer would have the best resolution if the region of highest quantum efficiency could be chosen as a site on which to place the detector crystal. In order to determine the location of such a region, a source of monoenergetic electrons is required. A "mapping" of the photocathode surface can be done with a point source of Cs^{137} mounted on a $\frac{1}{4}$ inch cube of anthracene. The mapping crystal need only be large enough to completely stop the 0.624 Mev conversion electrons of the Cs^{137} source. See Figure 1. The anthracene may be mounted directly on the photomultiplier tube face using Dow Corning stop-cock grease as an optical joint between the anthracene and the tube face. A small amount of the grease should be used, and the anthracene should be pressed firmly down on the tube face in order to squeeze out the excess grease. Care should be taken that no air bubbles or other foreign matter are trapped under the crystal.

The photomultiplier tube can be mounted in the place of that used with an ordinary gamma ray spectrometer. However, care must be taken that the photomultiplier tube and crystal are completely shielded from light whenever the high voltage is applied to the tube. Exposure to even low levels of light may permanently damage or destroy the phototube. One way of keeping the phototube in the dark is to tape it from the base up to the end with a double layer of black electrical tape. A 2 inch cylindrical box is useful to cover the scintillator. See Figure 2. But the joint between the box and phototube should also be sealed with black tape. With the high voltage and amplifier gain fixed, a series of spectra can be obtained for different positions on the phototube face. It is only necessary to run the conversion line portion of the Cs^{137} beta spectrum for this part of the experiment. A "map" consists of a plot of pulse height of the conversion line versus position in polar coordinates, positions of equal pulse height being connected by contour lines. The location of the mapping crystal which yields the highest pulse height is the one at which a full-size crystal should be permanently mounted, provided that the site also displays uniformity of quantum efficiency as indicated by the contour lines of the map. If the quantum efficiency or its uniformity is too poor, it may be necessary to select another phototube.

Once the scintillator has been mounted as directed above, obtaining the spectra is quite simple. Counts are made as with a gamma spectrum to obtain data from which a count rate versus pulse height curve may be plotted. Should beta energy measurement only, be required, it is not necessary to construct a Kurie plot. For this purpose, one can plot the square root of count rate versus pulse height. Typical spectra obtained in this manner are shown in Figure 3. Such measurements are possible to a fair degree of accuracy since the count rate appears in a Kurie plot to the $\frac{1}{2}$ power. It will be noted that there is on each of the spectra a straight line portion on the high energy end. If this straight line is extrapolated to the pulse height axis, a pulse height corresponding to the maximum beta energy may be obtained. If one then plots the maximum energy versus pulse height corresponding to that energy for two or more differing energies, a straight line is obtained at the high energy end as shown in the figure. Unknown beta energies can then be obtained from such a calibration curve.

If beta energies must be measured with greater precision a Kurie plot must be made. This process requires that the daughter product be known for the isotope whose beta energy is to be determined. In addition, the calibration of the spectrometer energy dial must be accurately known, and arriving at this accuracy is perhaps the most difficult part of the process. This difficulty arises from the variation of resolution with beta energy and the fact that the resolution is never very good (for example, the resolution of the 0.624 Mev conversion line, of Cs^{137}, full width at half maximum, is of the order of 12 to 15 percent under the best of conditions.)

The Kurie plot is derived from the theory of beta decay, for which there are many references. The discussion contained in Siegbahn, Chapters

9, 10 and 11, together with the Appendix in this reference will be found to be most useful. From Chapter 9 it can be seen that a description of a beta spectrum is given by the expression

$$\left[\frac{N(W)}{F(Z,W)PW}\right]^{\frac{1}{2}} = K(W_o - W) \qquad (1)$$

N = counts
F(Z,W) = Fermi function
P = electronic momentum
W = total energy of particulate beta
W_o = maximum energy
K = constant independent of energy

If a plot is made of the quantity on the left of the above expression versus W, an allowed spectrum will yield a straight line which may be extrapolated to the W axis and W_o may be obtained from the intersection. For ease of calculation, a modified Fermi function, G(Z,W) where G is equal to $\frac{P}{W}F$ may be used. When the latter expression is inserted in Equation 1, one obtains

$$\frac{1}{W}\left[\frac{N(W)}{G(Z,W)}\right]^{\frac{1}{2}} = K(W_o - W) \qquad (2)$$

It should be noted that W = E + 1 where E is kinetic energy in units of mc^2. Combining this expression for W with Equation 2 gives

$$\frac{1}{W}\left[\frac{N(E)}{G(Z,W)}\right]^{\frac{1}{2}} = K(E_o - E) \qquad (3)$$

E = kinetic energy
E_o = maximum energy

After obtaining the spectrum data and plotting a smoothed curve of count rate versus pulse height, the following steps may be followed in order to construct a Kurie plot:

1. Convert pulse height to kinetic energy from calibration curves.*

2. Divide each point on the kinetic energy axis by mc^2 to yield it dimensionless.

3. Calculate the total energy W for each point above from W = E + 1.

4. Calculate the momentum P from the expression $P = (W^2 - 1)^{\frac{1}{2}}$. This momentum will be in dimensional units.

5. From Appendix 2 of Siegbahn obtain the modified Fermi function G from the momentum and atomic number of the daughter of the beta decay. For example, the daughter of Cs^{137} is Ba^{137} which has an atomic number of 56. If a P of 0.4 were obtained, the corresponding G value would be (0.6778) (10) or 6.778.

*If one is interested only in the maximum beta energy and not necessarily the shape of the lower energy portion of the spectrum, it is necessary only to plot the last ten or fifteen points of the spectrum.

6. Plot the quantity $\frac{1}{W}\left[\frac{N(E)}{G(Z,W)}\right]^{\frac{1}{2}}$ versus E.

The straight line portion extrapolated to the E axis then yields E_o, the maximum kinetic energy of the beta emitter. See Figure 4.

Massachusetts Institute of Technology

Introduction

When we observe with our eyes the size and shape of an object, we are performing a scattering experiment with light. Our eyes serve as detectors and our conditioned minds interpret the data and register such features as size, surface character, and color. As freshmen you performed an experiment where the size of a cylinder could be determined by the angular distribution of the scattering of steel pellets. From the character of this distribution one could also have examined the shape of the cylinder; that is, whether it was round, rectangular, etc.

Much of our information in physics about the size and shape of molecules, atoms, and nuclei comes from such scattering experiments where the projectiles have known interactions with the objects under study and thus predictable scatter patterns. In other cases the character of the scattering is used to show the nature of the force between particles. In this experiment we use this technique to determine the spatial distribution of matter and charge in an atom. The projectiles used in this experiment are α-particles (doubly ionized helium atoms) and the atom we study is gold. The α-particles are obtained from the radioactive decay of Po^{210} and have about 5 Mev (5×10^6 ev) of kinetic energy.

The essential idea of Rutherford's theory is to consider the α-particle as a charged mass travelling according to the classical equations of motion in the coulomb field of a nucleus. The dimensions of both the α-particle and nucleus are assumed to be small compared to atomic dimensions (10^{-5} of the atomic diameter). The nucleus was assumed to contain most of the atomic mass and a charge Ze. On this picture the Z-electrons which make an atom neutral would not contribute much to the deflection of an impinging α-particle because of their small mass.

Other models had been proposed for atoms at this time (~1911) to account for features such as optical spectra. One of these (Thomson's Model) pictured the atom as a continuous distribution of positive charge and mass with the electrons imbedded throughout. This model predicts a very small amount of scattering at large angles compared to the Rutherford theory since the α's traversing this atom rarely see much charge concentrated in a large mass.

A derivation of the predictions of the Rutherford theory as well as discussions of other atomic models may be found in the references below.

References

1. Bitter, Currents, Fields, and Particles, Chapter 3.

2. E. Rutherford, The Scattering of α- and β-Particles by Matter and the Structure of the Atom, Phil Mag 6, 21 (1911).

References (continued)

3. Semat, Introduction to Atomic Physics.

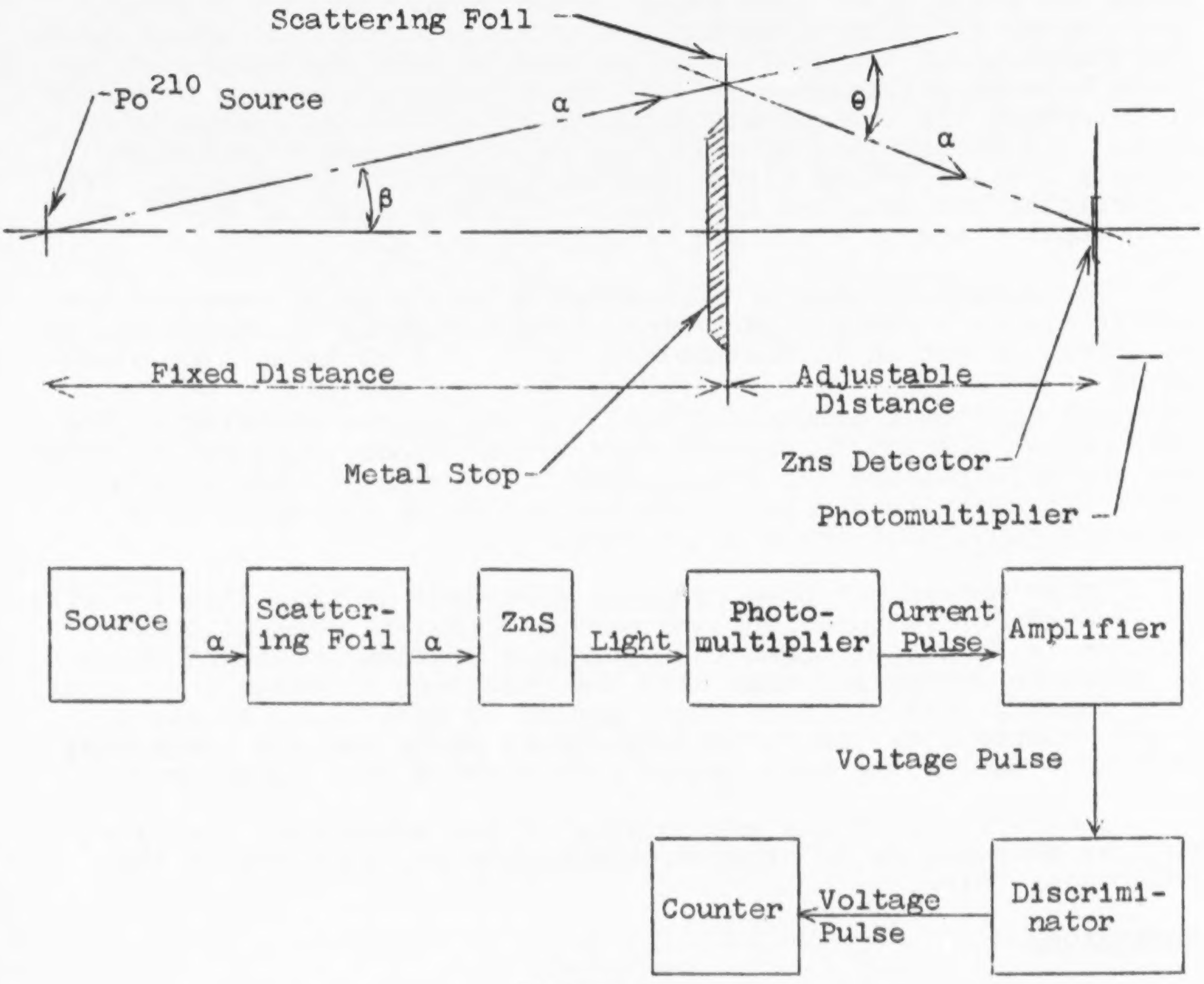
4. Richtmyer, Kennard, and Lauritsen, Introduction to Modern Physics.

5. Born, Atomic Physics.

6. Bleuler and Goldsmith, Experimental Nucleonics.

Description of Experimental Apparatus

Figure 1 is a simplified cross section of the scattering geometry located in the vacuum system, and a block diagram of the apparatus.



Scattering Foil
Po^{210} Source
α
θ
α
β
Fixed Distance
Adjustable Distance
Metal Stop
Zns Detector
Photomultiplier
Source
α
Scattering Foil
α
ZnS
Light
Photo-multiplier
Current Pulse
Amplifier
Voltage Pulse
Discriminator
Voltage Pulse
Counter

Figure 1

The α-particle source is a circular foil of 1/4" diameter and the detector is a 3/8" diameter circle of ZnS powder located on the face of a 5819 photomultiplier. The scattering foil is an annulus located coaxially with the α-source and detector with inner and outer diameters, 4.6 and 5.4 cm respectively. The angle β is determined by the fixed distance from source to scattering foil. The scattering angle θ is varied by changing the distance from the scattering plane to the plane of the detector. The metal stop prevents the direct beam of α-particles from striking the detector; however, it can be removed to perform intensity and range measurements.

A scintillation counter is used to detect the α-particles. In this type of counter the detecting area is covered with a thin coating of zinc sulfide. When an α-particle strikes a crystal of zinc sulfide, it emits a flash of light. The number of light quanta emitted is approximately proportional to the energy lost by the α-particles in the crystal. If the initial energies of all the α-particles were the same, the number of light quanta emitted for each incident α-particle would be nearly constant. However, small variations in the kinetic energy of the α-particles, variations in the efficiency with which the emitted light is converted into electrical signals and variations in the energy of the α-particles lost in the ZnS crystals give rise to a fairly broad distribution in the size of the pulses from the detector.

The light quanta emitted from the ZnS impinge upon the cathode of the photomultiplier and electrons are emitted (the photoelectric effect). The number of electrons emitted by this process is not sufficient for direct electronic detection so that they are first put through an electron multiplier mounted within the tube itself. In the multiplier the process of secondary emission is used to generate a cascade of electrons which increases from electrode to electrode. The photoelectrons are accelerated to a dynode maintained at a higher potential than the cathode. The dynode emits three or four times as many electrons as are incident on it, and the emitted electrons are accelerated to the next dynode. This process is repeated ten times, whereupon an amplification of from 10,000 to a million is achieved (depending upon the voltage between dynodes). Since the number of electrons emitted in the original photoelectric process depends upon the number of incident light quanta, the number of electrons collected at the final anode of the phototube will be proportional to the amount of scintillation light collected. The scintillation light is emitted by the zinc sulfide crystal in about 10^{-4} seconds. The transit time of an electron cascade through the multiplier is less than 10^{-7} seconds. A pulse of current may therefore be observed at the output of the phototube for each α-particle incident upon the zinc sulfide, although the α-particles may arrive in rapid succession.

The pulse of current from the photomultiplier charges a capacitor at the input to the amplifier. The amplifier further increases the size of the pulse and modifies its shape, so that it may be detected and counted by the counter. The proper settings of the amplifier and of the high voltage meter for the phototube will be given in the laboratory. They must be chosen so as to give adequate amplification, but not so much that the desired pulses are too large for the amplifier to handle properly.

In any electronic circuit electrical "noise" (unwanted signals) is always present due to the random motions of the electrons in the circuit. In this particular circuit a more important source of noise is the thermionic emission of electrons from the photocathode which gives rise to signals at the anode that look like small scintillation pulses. A discriminator is used at the input to the counter to prevent these unwanted signals from being counted. The variable potentiometer on the discriminator controls the bias voltage on the grid of a vacuum tube. The bias voltage is adjusted so that the tube does not conduct unless a pulse of a certain minimum amplitude is applied to the grid. When such a pulse does occur, this tube conducts and the discriminator circuit generates a pulse that is recorded by the binary scaling circuit of the counter.

If the pulses were all approximately of the same amplitude, the counting rate would be constant over a range of discriminator settings. When the discriminator is set too low the counting rate will increase rapidly because of the noise. If the discriminator is set too high, most of the desired pulses will not be counted. In practice, the desired pulses have a range of sizes so that no perfectly constant "plateau" can be found. However, a range of settings can be found where the counting rate changes only slowly with discriminator setting, and the proper position for the discriminator during the experiment is at the high counting rate end of the plateau (see Figure 2). With this setting almost all of the desired pulses will be counted. The discriminator setting may be adjusted by turning the calibrated dial located on the upper left-hand corner of the counter.

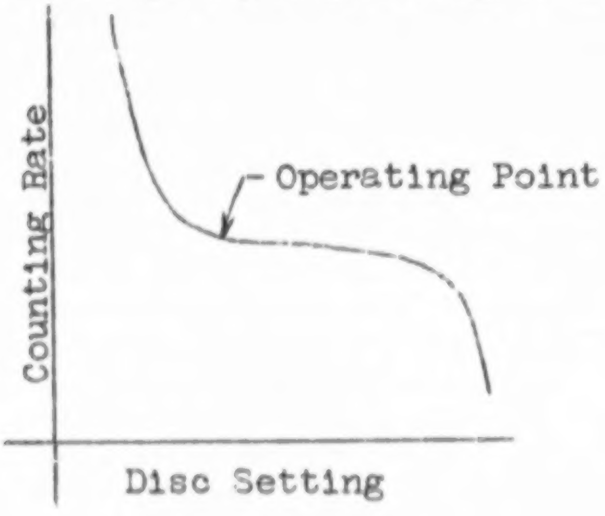


Figure 2

In air the α-particles lose energy rapidly by ionizing collisions with air atoms. In order to avoid this loss, provision has been made for evacuating the air from around the scattering apparatus. To keep out stray light, the photomultiplier is covered with black tape.

The radioactive decay of Po^{210} and the scattering of the α-particles at some angle are random phenomena. This means that having measured a counting rate of n counts/min at some angle, we have measured a relative probability. If we were to make this measurement many times, we would see that n would be different each time. It may be shown that, if we were to repeat such an experiment in which we had previously obtained N counts, we have about a 68% chance of obtaining a number of counts which fall in range $N \pm \sqrt{N}$. Obviously, the larger the number of counts, the better the percentage accuracy. This subject of statistical fluctuations is discussed in many texts. A possible reference is: Evans, The Atomic Nucleus.

Precautions

1. Always turn off the high voltage before releasing vacuum or exposing the photomultiplier to room light. Exposure to even low levels of room light will destroy these photomultipliers which are very expensive.

2. Do not open the vacuum system to air without consulting your instructor.

3. Treat the scattering foil and mounting with care to avoid breaking the delicate foil.

4. Do not touch the Po^{210} source with anything.

Come to the laboratory with a brief outline of the experiment you intend to perform.

The student may assume that the following quantities are known independent of the experiment:

$\frac{1}{2}mu^2$ = the energy of the α-particle

Ne = total nuclear charge of each scattering atom

E = the charge of the α-particle

The thickness of the foil will be given in the laboratory.

Supplementary Notes - Rutherford Scattering Experiment

Relation between the differential cross section and the expected counting rate

Definition of differential scattering cross section:

$$\sigma(\theta)\,d\Omega = \frac{\text{number of particles scattered at angle } \theta \text{ in a solid angle, } d\Omega \text{ per unit time}}{\text{number of particles incident per unit area per unit time (incident flux)}}$$

Note the dimension of $\sigma(\theta)d\Omega$ is area. From the literature

$$\sigma(\theta) = \frac{Z^2e^2z^2e^2}{16E^2}\ \text{cosec}^4\ \theta/2$$

where Ze = charge of the scattering center
ze = charge of the incident particle
E = kinetic energy of the incident particle

and θ is the angle between the incident and scattered beams. The coefficient of $\text{cosec}^4\ \theta/2$ should be calculated before the experiment.

Geometry (Chadwick geometry)

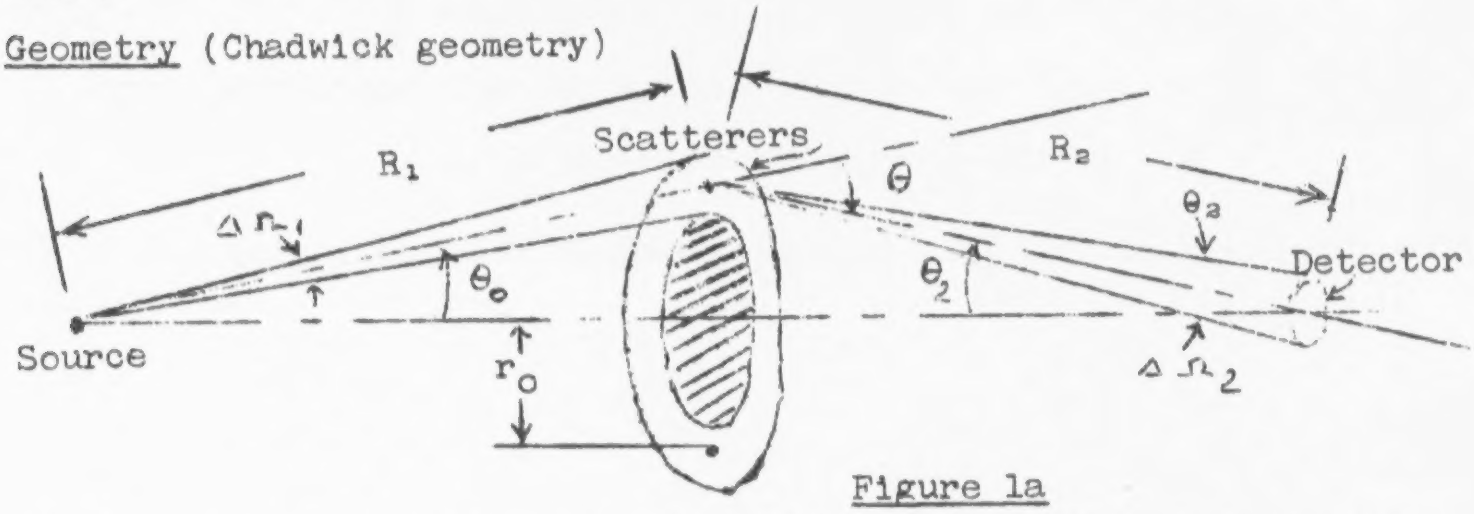


Figure 1a

$\Delta\Omega_1$ = Average solid angle subtended by the gold annulus at the source.

$\Delta\Omega_2$ = Average solid angle subtended by the detector at the scatterer. ($\Delta\Omega_2 = d\Omega$ above in the definition of $\sigma(\theta)$.)

Approximately
$$\left.\begin{aligned}\Delta\Omega_1 &\approx \frac{A_1\cos\theta_0}{R_1^2}\\ \Delta\Omega_2 &\approx \frac{A_2\cos\theta_2}{R_2^2}\end{aligned}\right\}$$
Check to see how good this approximation is for various angles.

Let N_0 = total number of particles emitted by the source per unit solid angle per unit time
A_2 = area of the detector
A_1 = area of the scattering (gold foil) material
ρ = density of the scattering material
t = thickness of the scattering material
L = Avogadro's number
A_G = atomic weight of scattering material

$$n = \frac{\rho L}{A_G} A_1 t = \text{total number of scattering centers}$$

N_1 = total number of particles reaching A_1 per unit time
$= N_0 \, \Delta\Omega_1$

$$\text{Incident flux} = \frac{N_1}{A_1\cos\theta_0} = \frac{N_0\Delta\Omega_1}{A_1\cos\theta_0} \approx \frac{N_0}{R_1^2}$$

N_2 = total number of particles reaching A_2 per unit time (expected rate)

$$N_2 = (\text{incident flux}) \times \sigma(\theta)\Delta\Omega_2 \times n$$

$$= \frac{nN_0}{R_1^2}\sigma(\theta)\Delta\Omega_2 = \frac{nN_0A_2\cos\theta_2}{R_1^2R_2^2}\sigma(\theta)$$

$$\theta_2 = \theta - \theta_0, \quad \theta = \theta_2 + \theta_0$$

$$R_2 = \frac{r_0}{\sin\theta_2}$$

$$\text{Expected rate} = \frac{nN_0A_2\cos\theta_2\sin^2\theta_2}{R_1^2r_0^2}\sigma(\theta) = N_2$$

The easiest parameter to measure for different values of the angle θ is the perpendicular distance between the gold annulus and the detector. Let it be y.

$$y = R_2\cos\theta_2 = r_0\cot\theta_2$$

$$N_2 = \underbrace{\left[\frac{nN_0A_2}{R_1^2r_0^2}\,\frac{Z^2z^2e^4}{16E^2}\right]}_{N_0G}\;\underbrace{\left[\frac{\cot\theta_2\sin^3\theta_2}{\sin^4\theta/2}\right]}_{f(y)}$$

Exercises and Preparation

1. Using the values of the parameters given in the notes, estimate G, assuming $t = 10^{-4}$ inches (thickness of gold); fixed distance in Figure 1 = 3 inches; E_{avg} = 2.5 Mev (why so low?).

2. f(y) should be plotted versus y before the experiment.

3. Having found G, estimate what N_0 should be for this experiment to give minimum precision (20%) within the time allotted (2½ hours). Remember you will need several points to establish the angular distribution. The curve f(y) vs y will give you an idea of how to space your points.

4. Why is this geometry advantageous even though it introduces extra angular dependence into the expected rate?

5. Think about how you are going to measure N_0. You are provided with a brass shield that blocks α-particles scattered from the gold. There is a hole in the center of this shield, 0.016 inch in diameter, which allows an unscattered beam to impinge on the detector.

6. What are the possible sources of systematic error?

Procedure

1. (30 minutes) The student should test for a plateau setting on the discriminator by counting at several different settings. It is suggested that the solid metal plug be used for this process. The height of the voltage pulse is a function of the energy and the discriminator should be adjusted so as to count those α-particles which have lost considerable energy by traversing the gold foil and still discriminate against noise.

2. (60 minutes) The dependence of the differential cross section on θ (the scattering angle) can now be found by adjusting the relative position of the foil with respect to the detector. The student should plot the expected shape of the intensity vs y (the perpendicular distance from the scintillator to the plane of the gold foil) assuming Rutherford scattering to be valid. It is suggested that in the laboratory the student select 6 or 8 positions covering the entire range of y. For best results, the student should take about 500 counts (~ 5% statistics) for each position. (For most apparatus, this should take about 5 minutes per position.) If time permits, irregularities in the plot of intensity vs y can be investigated.

3. (15 minutes) The strength of the source should be determined, using the brass blocking plate.

4. Optional. (45 minutes) Distribution of Counts. Take a group of 15-second counts and plot the number of times each specific count is obtained. In 30 minutes you can get about 100 countings. Check and see how many counts lie within $N_{avg} \pm \sqrt{N_{avg}}$.

Precautions and Directions on Rutherford Scattering

1. (a) H.V. OFF when opening phototube to light. (Switch for this is on the back of the phototube box.)

(b) Keep fingers off Po^{210} source.

2. Treat gold foil with care:

(a) Put plug in from inside.
(b) Let air in slowly (make sure system is closed off from pump).

3. Don't touch face of phototube with anything.

(a) Keep rulers, etc, away from the tube.
(b) Make sure plug will not hit tube.

4. Keep away from the following hot spots:

(a) H.V. Supply
(b) Internal part of phototube box
(c) Terminals on isolation transformer

5. Be sure that (-) negative terminal of battery is hooked up to the proper terminal on the box.

Tests for Proper Operation

1. Letting air in (slowly) should stop counts.

2. Raising H.V. a little should not increase nor decrease counting rate.

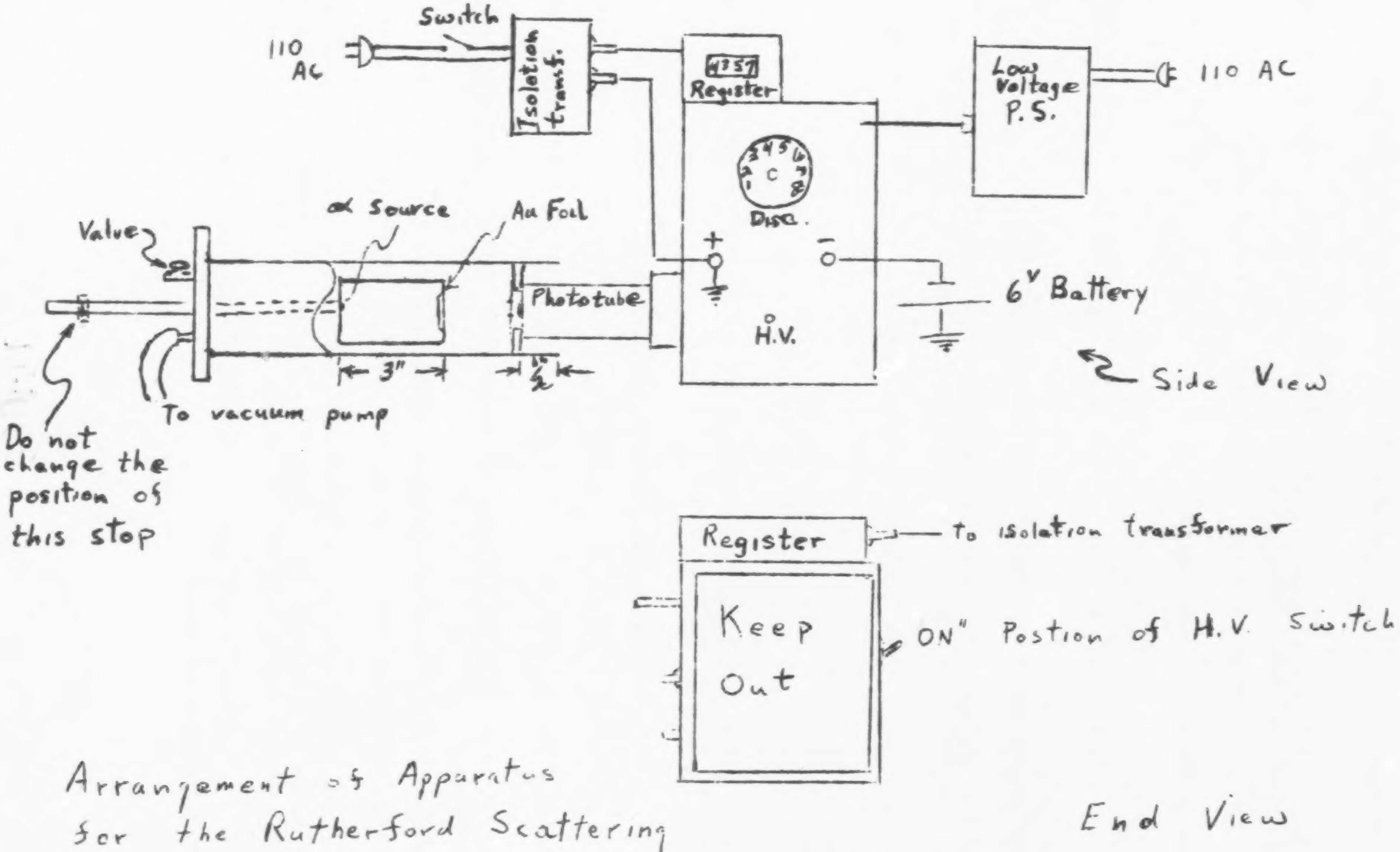
3. Raising bias discriminator (clockwise) should not decrease counting rate.

4. Shutting H.V. off should stop counting rate.

5. If discriminator bias run gives no flat portion, raise H.V.

NOTES: Run phototube H.V. at about 40 on the H.V. supply meter for all units except 11 and 12 which are to be run at about 60-65. Some units might need higher voltage. If so, notify instructor.

Make all geometrical measurements on sample unit on front table, except for length of steel rod.


Switch
110 AC
Isolation transf.
Register
4757
Low Voltage P.S.
110 AC
Disc.
C
+
−
H.V.
6V Battery
Side View
Valve
α Source
Au Foil
Phototube
3"
½"
To vacuum pump
Do not change the position of this stop
Register
To isolation transformer
Keep Out
ON" Postion of H.V. Switch
End View

Arrangement of Apparatus for the Rutherford Scattering Experiment

SCATTERING OF ALPHA PARTICLES

Stanford University

References

Rutherford, Chadwich, and Ellis, Radiation from Radioactive Substances (The MacMillan Company, London, 1930), p. 191 ff.

Geiger and Marsden, Phil. Mag. 25, 604 (1913)

Object

To measure the absolute differential cross section for Rutherford scattering of 6.11-Mev Cm^{242} alpha particles by palladium, and to compare with theory.

Comments

1. The Palladium foil is VERY FRAGILE. Handle with great care; let the vacuum chamber down to air gently to avoid possible wind damage.
2. Set up to count all scattered alphas and minimize background rate.
3. From the angular resolution of your geometry and multiple scattering in the foil, estimate the way in which the counting rates will vary with θ, where the data should measure Rutherford scattering and where deviations set in. Plan your data-taking accordingly.
4. Show that setting the plane of the foil and slit so as to bisect the angle between incident and scattered particles simplifies the data reduction and otherwise improves the experiment.
5. Check your slignment by counting with θ on both sides of 0 and checking for symmetry.
6. The source is VERY DANGEROUS in case it is spilled. It is mounted in its holder and fixed by fusing into a ceramic backing. The radiation is not dangerous, but in case of a spill the danger lies in possible ingestion. Before and after the experiment check the bench and equipment for contamination, with the monitor provided.

COLLISION OF PARTICLES

Dartmouth College

Introduction

The principle of conservation of momentum is essential in the analysis of all scattering problems. If the condition that no external force acts on the system of particles is satisfied, no exception to this conservation law is known. To use the principle, some momenta (or energies) must be known. One way of obtaining these quantities is from "range-energy" curves. If, outside the region of the collision itself, a particle loses energy by interaction with the matter through which it is passing, measurement of its range in the matter will yield its initial energy, provided the relation between range and energy is known by some means. A nucleus or other charged particle, in passing through a photographic emulsion loses energy by ionizing the atoms of the matter through which it is passing. This ionization "exposes" the grains of the emulsion, so that the path of the particle is visible (under magnification) as a "track" after the emulsion is developed. Such emulsions are an extremely important way of observing and studying nuclear events. An exact analog on the macroscopic level of the nuclear collisions in an emulsion exists in collisions between coins sliding on a table top. The collisions obey the same energy - and momentum - conservation laws, and the energy loss in the "medium" outside the collision region is due to friction with the table top.

This experiment is divided into two parts. In the first part of the experiment collisions between pennies are investigated. Since only two-body collisions are studied, the fact that pennies slide on a table top is not restrictive. The bombarding coin has known energy and the energy of the scattered coin and recoil coin is obtained from experimentally determined range-energy curves. The collisions are endoergic[1], and the Q of the reaction can be obtained. The statistical uncertainty in the energy obtained from a range measurement is large, because of range "straggling" but is small enough to obtain a rough value for Q.

The second part involves analysing an actual nuclear particle scattering event from a vastly enlarged photograph of a nuclear emulsion.

Apparatus

The "accelerator" is a hammer, pivoted at its upper end. The hammer is displaced and released so as to strike a penny placed at the edge of the table. The hammer is suspended so that when hanging freely the center of the face of the hammer-head just touches the upper edge of the table top. If the hammer is initially drawn back an angle θ_o, its angular velocity at the bottom of the swing is given by conservation of energy:

$$\dot{\theta}^2 = 2\,(mgd/I)\,(1 - \cos\,\theta_o),$$

---

[1] R. T. Weidner and R. L. Sells, Elementary Modern Physics (Allyn and Bacon, Boston, 1960), p. 373 ff.
Blanchard et al., Introduction to Modern Physics (Prentice-Hall, Englewood Cliffs, N.J. 1958). P. 326.

where m is the mass of the hammer, d is the distance from its center of gravity to the pivot, and I its moment of inertia. From this, the linear velocity of the head can be obtained if the length is measured. The quantity (mgd/I) is simply the square of the angular frequency of small oscillations.[2] The velocity imparted to the penny will be just twice the velocity of the hammer head, if the approximation is made that the inertia of the hammer is infinite and that the blow is perfectly elastic.

Procedure

First, the experimental range-energy curve is obtained. For a given initial velocity (angular displacement of the hammer), the coin's range (about 45 cm) should be measured about 50 times. This can be done quickly, as it is necessary to measure the range only to the nearest centimeter. This procedure should then be repeated for another initial velocity (range about 15 cm). Plot histograms[3] of number versus range for the events belonging to the two different energy groups; these give a picture of the spread in the data. Take an additional (but smaller) set of data at an intermediate range (about 30 cm). The mean range is found for each angular displacement, and converted to energy using the above equation. Next, from the mean ranges and calculated energies, the range-energy curve is plotted. Derive a theoretical relation between range and energy, and compare with your experimental curve.

The experiment to be performed is the inelastic scattering of pennies by pennies. Show that for this case where all the masses involved are equal that

$$Q = 2\,(E_1\,E_2)^{\frac{1}{2}}\cos\,\phi\ ,$$

where E_1 and E_2 are the energies of the reaction products and ϕ is the angle between their paths. Q is to be found experimentally from a measurement of E_1, E_2 and ϕ. The bombarding coin is accelerated toward a target coin at rest a few centimeters away. The positions of both coins at the instant of collision and the final positions of the pennies are marked on the table, and these points are joined by lines, whose lengths and included angle are measured with meter stick and protractor. In measuring the angle ϕ, it is important to note that the positions of the two coins are not the same at the instant of collision (Fig. 1). The energy E_o which the incident coin brings into the reaction must also be found, and the result expressed as the ratio Q/E_o.

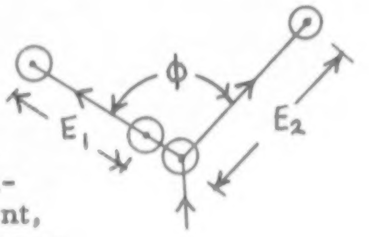


Fig. 1

Emulsion Analysis

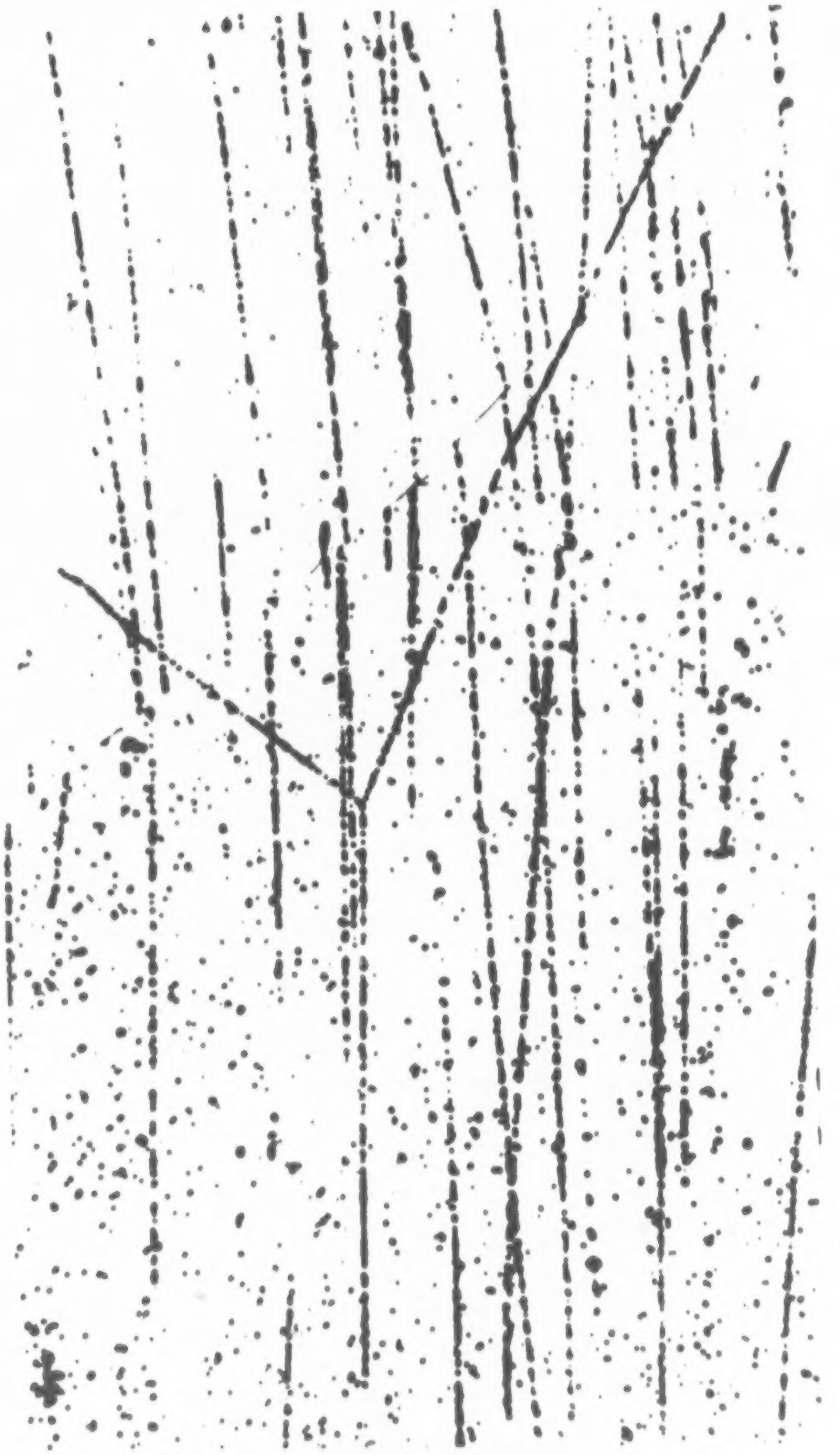
At the conclusion of the laboratory period you will be given an enlargement of a nuclear emulsion[4] which contains among other things a nuclear particle scattering event, similar to that shown in Fig. 2. Be sure that before you leave the laboratory you fully understand just which tracks in your photograph are important. Assume that the scattering is

---

[2] R. J. Stephenson, Mechanics and Properties of Matter (Wiley, N. Y., 1952) pp. 191-3.

[3] C. M. G. Lattes, P. H. Fowler, and P. Cuer, Proc. Phys. Soc. A59, 883 (1947).

[4] Weidner and Sells, op. cit., pp. 278-80.
Blanchard et al., op. cit., p. 306 ff, p. 326 ff.



Scattering of a deuteron by a proton

Fig. 2

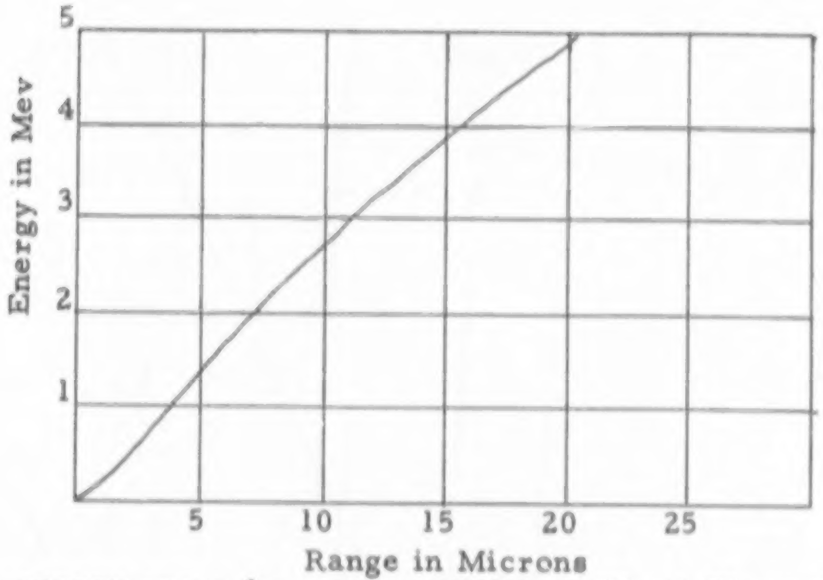


Fig. 3 Range-energy curve for α-particles in Ilford G5 emulsions.

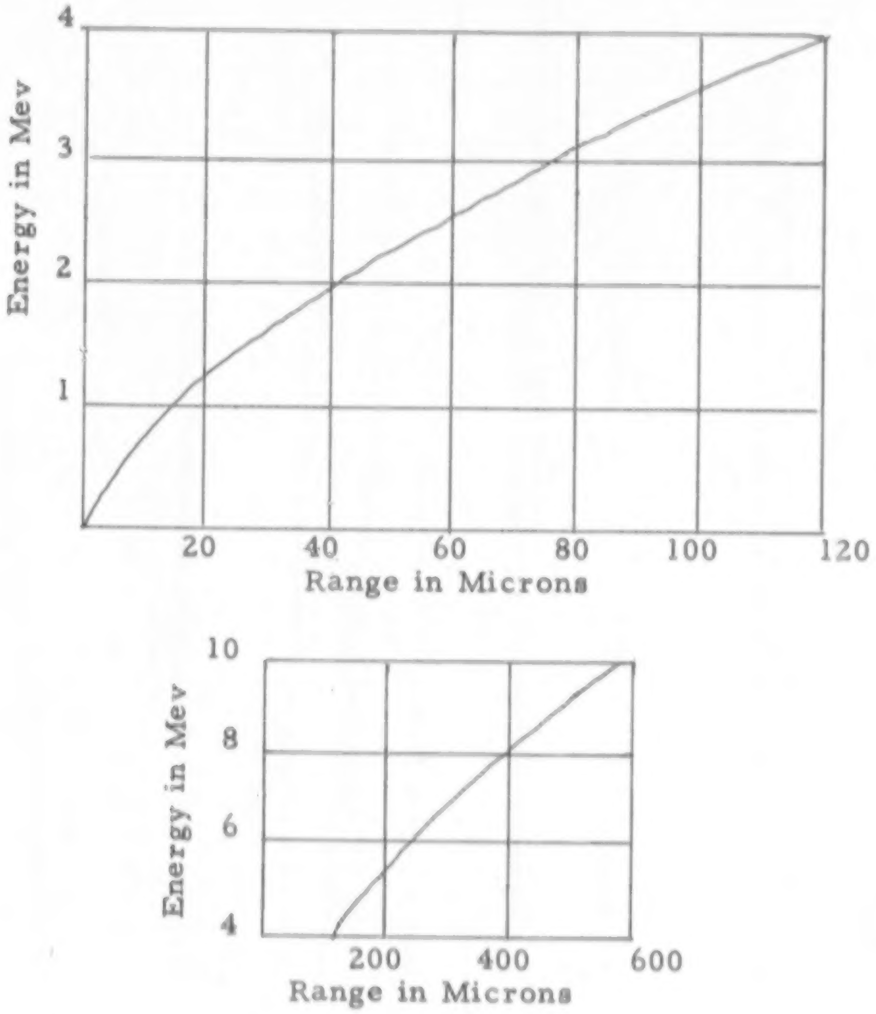


Fig. 4 Range-energy curves for protons in Ilford G5 emulsions.

431/32

elastic and be sure to include in your analysis such things as initial energy of the incoming particle in ev, agreement of the included angle measurement with the theoretical value, etc. Examples[5] of such analyses can be found in abundance in the Physical Review and other scientific periodicals.

Range-energy curves[6] for protons and α-particles in Ilford G5 emulsion are to be found in Figures 3 and 4.

Energy loss of charged particles in a given length of trajectory depends only on charge and speed. The energy of a proton is half the energy of a deuteron of the same speed. The particles carry the same charge so that the loss in energy of the deuteron will be twice that of the proton for a given reduction in speed. Therefore, the range of a deuteron is twice that of a proton of the same speed, i.e. twice that of a proton of half its energy.

---

[5] See for example Schneps, Fry, and Swami, Phys. Rev. 106, 1062 (1957)

[6] Powell and Occhialini, Nuclear Physics in Photographs (Clarendon Press, Oxford, 1947).

LIFETIME OF THE MU MESON

Massachusetts Institute of Technology

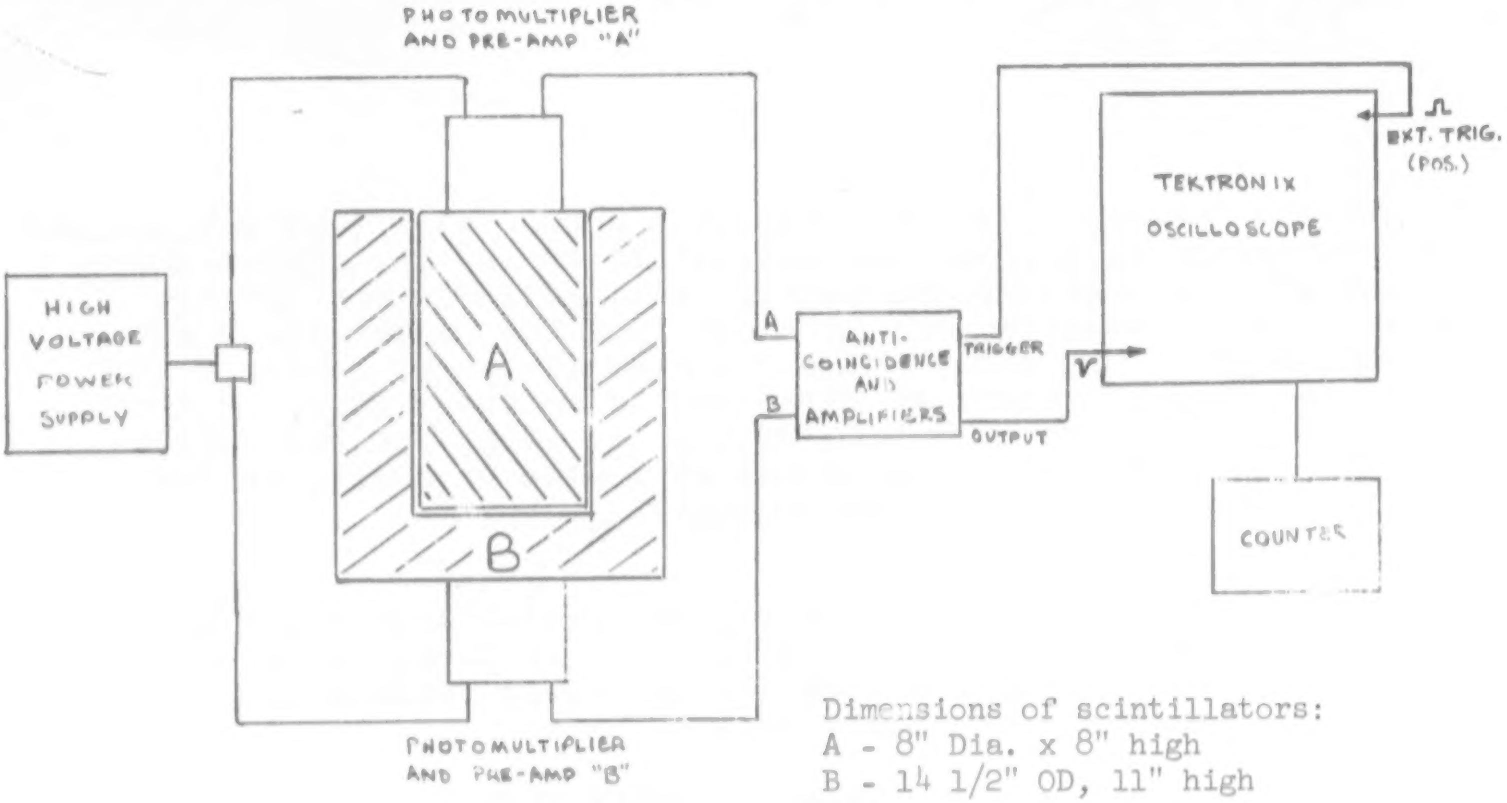
References:

1. Principles of Modern Physics by Leighton pp. 633-635 required reading.
2. High Energy Particles by Rossi pp. 151-172, 179-192.
3. Rossi and Nereson, Phys. Rev. Vol. 62, Nos. 9 & 10 pp. 417-422, Nov. 1942. This is one of the original papers in which this experiment is discussed. A copy is available in the Junior Lab Office.
4. Notes on Statistics for Physicists by Orear UCRL-8417. Maximum likelihood method.

At sea level about half of the cosmic radiation observed consists of mu mesons. These particles are radioactive and decay according to

$$\mu^{\pm} \rightarrow e^{\pm} + \nu$$

in about 2.2 X 10^{-6} seconds. These energetic particles are created in the upper atmosphere by decaying pi mesons. We wish to measure their mean lifetime and will use an apparatus set up as follows:


PHOTOMULTIPLIER AND PRE-AMP "A"
HIGH VOLTAGE POWER SUPPLY
A
B
A
B
ANTI-COINCIDENCE AND AMPLIFIERS
TRIGGER
OUTPUT
V
TEKTRONIX OSCILLOSCOPE
EXT. TRIG. (POS.)
COUNTER
PHOTOMULTIPLIER AND PRE-AMP "B"

Dimensions of scintillators:
A - 8" Dia. x 8" high
B - 14 1/2" OD, 11" high

A large quantity (A) of plastic scintillator* is surrounded by a cup (B) of similar material. We wish to observe mu mesons which stop in A and their subsequent decay electrons. This is accomplished by connecting the vertical deflection input of a Tektronix scope to the amplified output of photomultiplier A and then triggering the scope from the output of an anticoincidence circuit. Thus when a particle is stopped in A, an output appears from the A detector only, and the anticoincidence circuit provides a positive triggering pulse. The anticoincidence circuit is such that if it senses a simultaneous pulse from both A and B, as would happen if a particle traversed both detectors, no trigger pulse is produced.

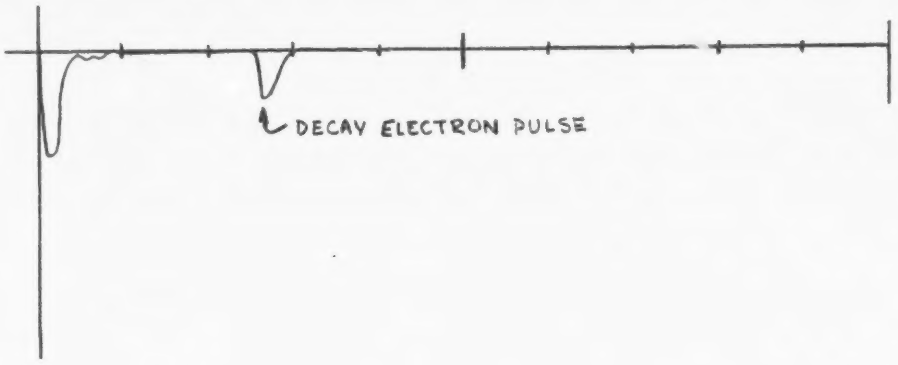
In the notation we are using, the logic for this experimental apparatus may be expressed as:

$A\bar{B}$ = trigger **

A = output

You should understand how this logic eliminates triggers from particles which do not stop in A.

When the scope triggers on an anticoincidence pulse we see:



By setting the oscilloscope sweep at a known value (one microsecond per centimeter is a good starting value) we may photograph these pulses and the superimposed oscilloscope reticule and get the decay time by measuring the distance from the trace origin to the leading edge of the decay pulse. The oscilloscope reticule is graduated in centimeters so times can be measured directly from the photograph. The amplitude of the "A" output pulses will be about 1 volt and the trigger pulse will also be small, so the oscilloscope should be set accordingly.

---

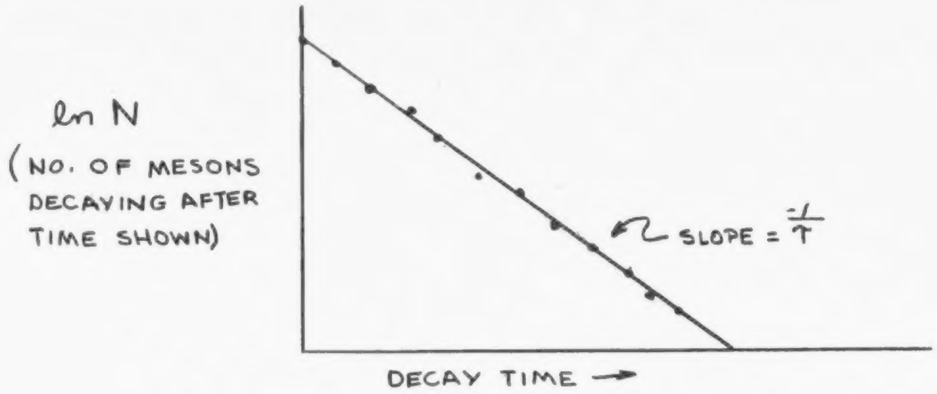
*This is a material consisting of special scintillation materials dissolved in clear plastic. It is inexpensive to manufacture and can be machined easily, so it finds many applications in experiments of this nature.

**$\bar{B}$ read "not B"; indicates no pulse from B.

The photomultipliers used in this experiment are Dumont type 6364 and operate from 900-1500 volts. 1300 volts is the recommended high voltage.

Except for the high voltage supply, scope, and counter, all electrical circuits used with this experiment are transistorized and require no warm up time. Diagrams of these transistor circuits are available in the Junior Lab Office. Since the transistor circuitry has no pilot light, be sure to turn it off when not in use. In addition: the first stages of the photomultiplier pre-amplifiers are very sensitive to surge pulses created when the high voltage is switched on and off. Power should not be applied to the transistor circuitry when switching the high voltage on and off or adjusting it. This means that high voltage power must be applied before switching power on and transistor power must be removed before switching the high voltage off.

Since the mu meson decay pulses are relatively infrequent and since you will find that only about 1/4 of the triggers will actually be mu mesons decaying, the best procedure is to open the camera shutter after setting the oscilloscope, then leave it open until about 20 triggers are indicated on the counter connected to the scope. After this the photo may be examined for mu decays. Take photos until you have a representative number of decay times and then plot them integrally as shown below.



The theoretical equation for the above integral plot is

$$N = N_0 \tau e^{-t/\tau}$$

where τ is the lifetime. Thus the lifetime can be obtained directly from the slope of the semi-log plot. A maximum likelihood error analysis yields

$$\sigma_\tau = \frac{\tau}{\sqrt{n}} \qquad n = \text{total \# of decays.}$$

as the standard statistical error in the lifetime. This statistical error may be added to your ordinary measurement error as follows:

$$\sigma^2 = \sigma_\tau^2 + \sigma_m^2$$

to give the total error estimate. Remember that an error limit of plus or minus one standard deviation represents a confidence interval of about 68%.

Write Up:

In your research you should try to answer the questions below. Your write-up should be concise and informative. Be sure to discuss the error in your value of the lifetime. Remember that the results of an experiment is always accountable, if enough is known about the experiment to estimate error, both systematic and statistical, correctly. If your mean value does not agree with the best published measurement an estimate of your error should be able to account for the difference. In making this comparison you should realize that the best published value also has an error associated with it.

Questions:

1. What are the properties of μ mesons?
2. How are they created? How do they disappear in matter?
3. Why is the layer of lead absorber useful?
4. How do negative mu mesons affect your results?
5. What is the mean lifetime?
6. How do mu mesons created at the top of the atmosphere "live" long enough to reach sea level?

MESON DECAY: ANALYSIS OF BUBBLE CHAMBER EVENTS

Columbia University

References: Wehr and Richards, Sections 9-7, 5-12, and Chapter 12.

Thorndike, Mesons, McGraw-Hill.

*Object: To measure the approximate masses of the pion and muon (π-and μ-mesons) from analysis of photographs of the bubble chamber in which they decayed.

GENERAL DISCUSSION

Introduction

When nuclei are bombarded by high energy particles which are found as cosmic rays or are produced by large accelerators, a variety of unstable 'elementary' particles may be produced. Most of the experiments performed in the past twenty years with beams from the growing number of cyclotrons and synchrotrons have involved the discovery of new particles and the determination of their properties, e.g. mass, charge, mean life until decay, decay products, interaction with other particles, etc.

Visual Detection Techniques

Electronic detectors such as Geiger counters or scintillation counters, are particularly useful for recording large fluxes of charged particles (as in Experiment 24) or for accurate timing measurements. Where more accurate spacial measurements are needed, however, 'visual' detection techniques, such as photographic emulsions, cloud chambers, or bubble chambers, or the recently developed spark chambers may be used. All of the visual devices record the trail of ionization left in the wake of a charged particle. In a bubble chamber, small bubbles, which form along the trails of ionized molecules, are illuminated and stereoscopically photographed milliseconds after the passage of the particle. The detailed trajectory of the particle can be examined on the photograph. If the chamber was in a strong magnetic field, the momentum of the particle can be determined from the curvature of its trajectory in the plane normal to the field. If the particle comes to rest in the liquid, its kinetic energy can be determined from its range. Since slower particles ionize more heavily, an estimate of the velocity can be made from the density of bubbles along the track.

* Arrangements have been made to supply copies of original bubble chamber photographs (on 35 mm film strips) and of transparent templates for measurement of track curvature. For information on obtaining this material, write to Professor A. M. Sachs, Pupin Laboratory, Columbia University, New York 27, N. Y.

Relativistic Kinematics

For particles travelling near the speed of light (c), the relations among rest mass (m_o), velocity ($v = \beta c$) momentum ($p = mv$), kinetic energy (T), and total energy ($E = mc^2$) must be derived from the special theory of relativity. (See Wehr and Richards, Chapter 5.)

The most useful of the derived results are:

$$E^2 = (pc)^2 + (m_o c^2)^2 , \qquad (1)$$

$$E = T + (m_o c^2), \qquad (2)$$

$$(pc)^2 = T^2 + 2T(m_o c^2), \qquad (3)$$

and

$$\beta = v/c = pc/E. \qquad (4)$$

Note that E, T, pc, and $m_o c^2$ have the dimensions of energy, and it is customary in particle physics to express each of them in units of MeV, (10^6 electron volts).

Momentum Measurements

If the momentum, $p_\perp$, of the particle with charge e, is in a plane perpendicular to the applied magnetic field, $\vec{B}$, the radius of curvature, R, is given by an expression identical to the classical result:

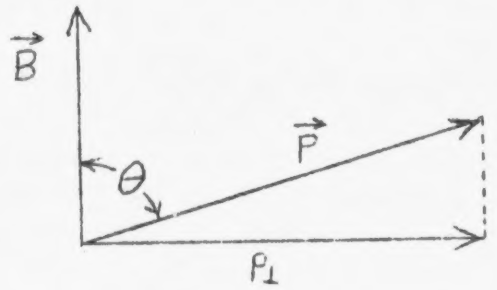
$$R = \frac{mv}{eB} = \frac{p_\perp}{eB} \qquad (5)$$

The quantities in equation (5) must, of course, be expressed in a consistent set of units. Converting from mks or cgs units to a system where $p_\perp$ is expressed in Mev/c, it follows that

$$p_\perp = 3 \times 10^{-4} \text{ BR MeV/c} \qquad (6)$$

where B is expressed in gauss and R in cm. (You are expected to prove this in your report.)

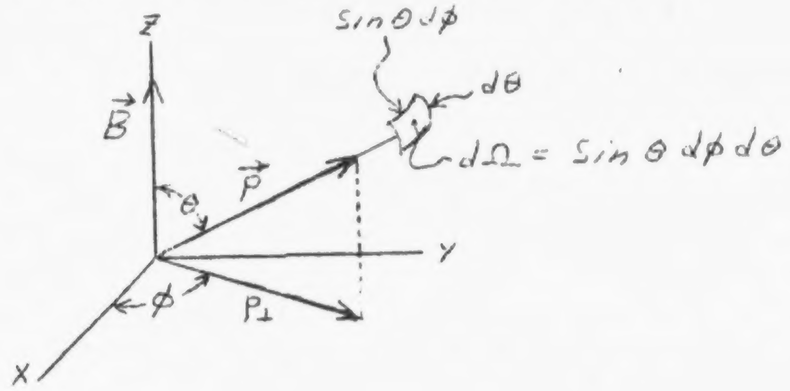
In the more general case where the angle between the initial momentum, $\vec{p}$ and magnetic field direction, $\vec{B}$, is θ (rather than 90°), the particle travels in a helix and



$$p = \frac{p_\perp}{\sin\theta} \qquad (7)$$

In a complete research experiment, stereoscopic photography (using two, or sometimes three cameras), permits reconstruction of a track in space, and thus gives a value of θ for each track. You will measure only one view in the present experiment, giving only the projection of the track in the $\theta = 90^\circ$ plane. For a single track, you will be able to find only $p_\perp$, not p. If a large number of tracks are measured, however, the average momentum will be the average value of $p_\perp$ divided by the average value of sin θ.

Assuming that all directions of momenta are equally probable (as they are from the 'isotropic' decay of a π-meson at rest), the averaging of sin θ is performed over area elements of a unit sphere; i.e. elements of solid angle



$$\sin\theta = \frac{\int \sin\theta \, d\Omega}{\int d\Omega}$$

(Integrations are over the unit sphere.)

$$= \frac{\int_0^{2\pi} d\phi \int_0^{\pi} \sin^2\theta \, d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta} = \frac{\pi}{4} = .785 \qquad (8)$$

Energy Measurement for Stopped Particles

A charged particle loses energy in ionizing the material through which it passes. It can be shown that, to a good approximation, the energy lost per cm. by a charged particle traversing liquid hydrogen in a bubble chamber is

$$\frac{dE}{dx} \approx \frac{2.1\rho}{\beta^2} \text{ MeV/cm} \qquad (9)$$

where ρ is the density of the liquid in gm/cm^3 and $\beta = v/c$. This energy loss is directly related to the number of bubbles per cm produced by the particle. Thus a particle traveling at almost the speed of light will produce a relatively sparse track, while a slow particle will produce a heavy, almost solid track. This, combined with knowledge about the momentum = mv obtained from the radius of curvature, makes it easy to tell mesons from electrons and often mesons from protons. For example, a 10 MeV/c electron has a speed very close to c, while a 10 MeV/c meson has a speed about 0.1 c, so that its track would appear much heavier.

Consider a particle which is created and also loses all its energy inside the bubble chamber so that its measured track length, L, corresponds to its range in liquid hydrogen. Then

$$L = \int_0^T \frac{dx}{dE} dT = \int_0^T \frac{\beta^2}{2.1} dT \qquad (10)$$

from equation (9). For a non-relativistic particle

$$T = \frac{1}{2} m_o v^2 = \frac{1}{2} m_o c^2 \beta^2 \qquad (11)$$

if $m_o c^2$ is expressed in MeV. Therefore:

$$L = \frac{2}{2.1 m_o c^2 \rho} \int_0^T T dT$$

and

$$T = \sqrt{2.1\, m \rho L} = \sqrt{2.1\, m_o c^2 \rho L} \qquad (12)$$

Again the measured track length, $L_\perp$, is related to the actual track length, L, by

$$L_\perp = L \sin \theta \qquad (13)$$

If a large number of tracks are measured, L is related to the average of the measured track lengths, $\bar{L}_\perp$ by

$$L = \bar{L}_\perp / 0.785 \qquad (14)$$

using the same argument that was used to derive equation (8).

π-μ-e DECAYS

Introduction

The π^+ meson (pion) decays, with a mean lifetime of 2.5×10^{-8} sec., into a μ^+ meson and a neutrino:

$$\pi^+ \longrightarrow \mu^+ + \nu \qquad (15)$$

The emission of a single neutrino (a neutral particle with zero mass) is assumed to balance the constant value of momentum of the muon observed in all decays of pions at rest. (Interactions of these neutrinos have been recently observed in giant spark chambers by Lederman, Schwartz, Steinberger and co-workers at Brookhaven.)

The μ^+ meson (muon) then decays, with a mean lifetime of 2.2×10^{-6} sec., into a positron (positive electron) and two neutrinos:

$$\mu^+ \longrightarrow e^+ + \nu + \nu' \qquad (16)$$

The emission of two neutrinos is assumed, since positrons are emitted with a continuous spectrum of energies from decays of muons at rest, indicating that energy and momentum is shared among 3 decay products. (The neutrinos have been shown to be of two different kinds in the recent experiment referred to above.)

Measurement of $m_{o\mu}$

The rest mass of the muon (approximately one hundred MeV) is much larger than that of the positron (0.5 MeV), so that $m_{o\mu}c^2$ goes almost entirely into the kinetic energy of the decay products:

$$\begin{aligned} E_\mu = m_{o\mu}c^2 &= E_e + E_\nu + E_{\nu'} \\ &\approx p_e c + p_\nu c + p_{\nu'} c \end{aligned} \qquad (17)$$

The three 'light' decay products are thus on an almost equal footing, and if no special law were acting in the reaction, each might be expected to receive, on the average, 1/3 of the total energy. (The interaction among the decay products is 'weak', and detailed experimental results indicate that this expectation is valid to a good approximation.) In any one decay event, the visible positron track can receive any energy from zero up to $E_\mu/2$ (prove this), but averaged over a large number of events:

$$\bar{p}_e c \approx \bar{p}_\nu c \approx \bar{p}_{\nu'} c \approx E_\mu/3 = m_{o\mu}c^2/3 \qquad (18)$$

Equation (18) serves as the basis for the measurement of $m_{o\mu}$ in this experiment.

Measurement of $m_{o\pi}$

In the decay (15) of a pion at rest, conservation of total energy yields:

$$m_{o\pi}c^2 = \sqrt{(p_\mu c)^2 + (m_{o\mu}c^2)^2} + p_\nu c \qquad (19)$$

But, by conservation of momentum in the two-body decay:

$$p_\mu = p_\nu \qquad (20)$$

Therefore

$$m_{o\pi}c^2 = \sqrt{(p_\mu c)^2 + (m_{o\mu}c^2)^2} + p_\mu c. \qquad (21)$$

The unique value of T_μ, the kinetic energy of the decay muon, can be determined by equations (12) and (14) from the measurement of $\bar{L}_\perp$ - the average of the projected range of the muon tracks from a large number of decays.

From T_μ, and a knowledge of $m_{o\mu}$, a value for p_μ can be obtained; substituting p_μ and $m_{o\mu}$ in equation (21) yields a value for $m_{o\pi}$.

PROCEDURE

Experimental Arrangement

The bubble chamber photographs to be scanned were taken using the 385 MeV proton synchrocyclotron at Columbia's Nevis Laboratory. Pions were produced by collisions of accelerated protons with a copper target inside the vacuum chamber of the cyclotron. The pion beam emerged through a thin window, passed through a hole in the surrounding shielding, and was allowed to enter a liquid hydrogen bubble chamber.

The experimental arrangement is indicated in Fig. 1 . An absorber was placed in the pion beam, so that a sizeable fraction of the incident pions come to rest in the liquid hydrogen. The emerging muon travels a short distance before coming to rest, and the final positron makes a long, lightly ionizing track, passing through the hydrogen until it leaves the chamber. If the positron leaves through a side of the chamber which is parallel to the picture plane, the track will appear to stop at an arbitrary point in the projected view.

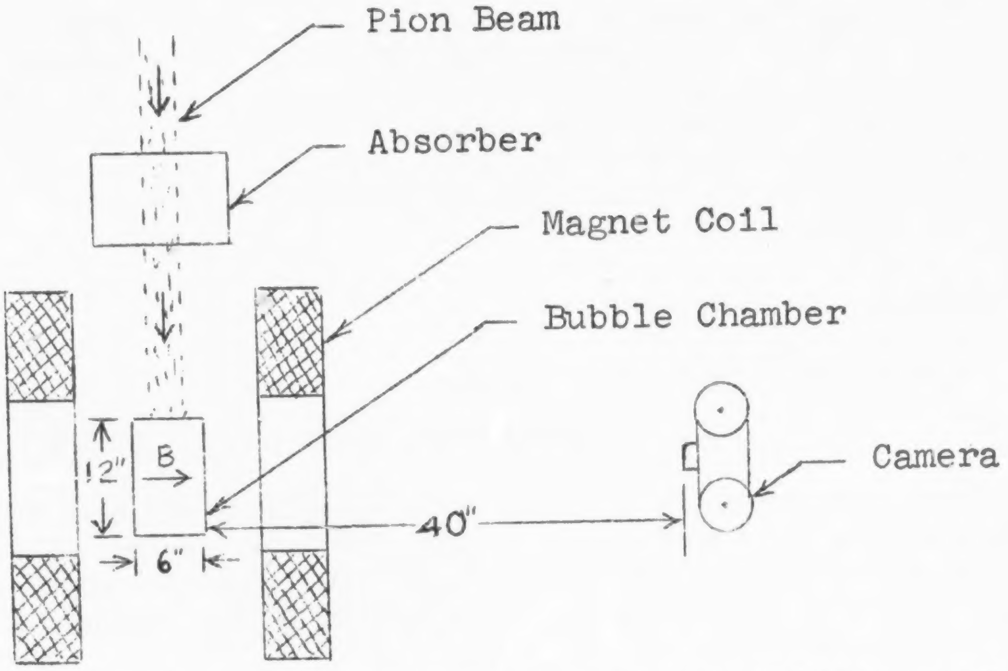


Fig. 1. Arrangement of bubble chamber and auxilliary equipment.

Measurements

You are to scan the actual 35-mm film projected on a ground glass screen. For a large number of identifiable π-μ-e events, you are to measure:

(1) The radius of curvature of the positrons, from which the average positron momentum and, finally, the mass of the muon are derived.

(2) The length of the muon tracks, from which the muon momentum and, finally, the mass of the pion are derived.

The radius of curvature of a track is measured on the screen by the use of a template, a transparent sheet on which there are drawn a series of circular arcs of different radii. The length of a track is measured with a plastic ruler. In order to convert measurements on the screen to actual distances in the bubble chamber, a scale factor must be determined from three crosses scratched on the front glass plate of the chamber. The crosses are at the vertices of an equilateral triangle, 22.4 cm on a side. To compensate for a possible distortion in the projection system an average should be taken of measurements made between the three pairs of crosses.

A correction to the scale factor of 43/40 is necessary since the average track is 43 inches from the camera compared to the 40-inch distance from the camera to the glass plate.

In using the average value of sin θ for the average of the large number of events measured, it is assumed that there is no scanning bias - i.e. that you are as likely to identify and measure one event as another. Although this goal is unlikely to be achieved, try not to favor events in the picture plane by failing to measure events in which the projection of the track is short.

Calculations

Constants: $B = 8800$ gauss

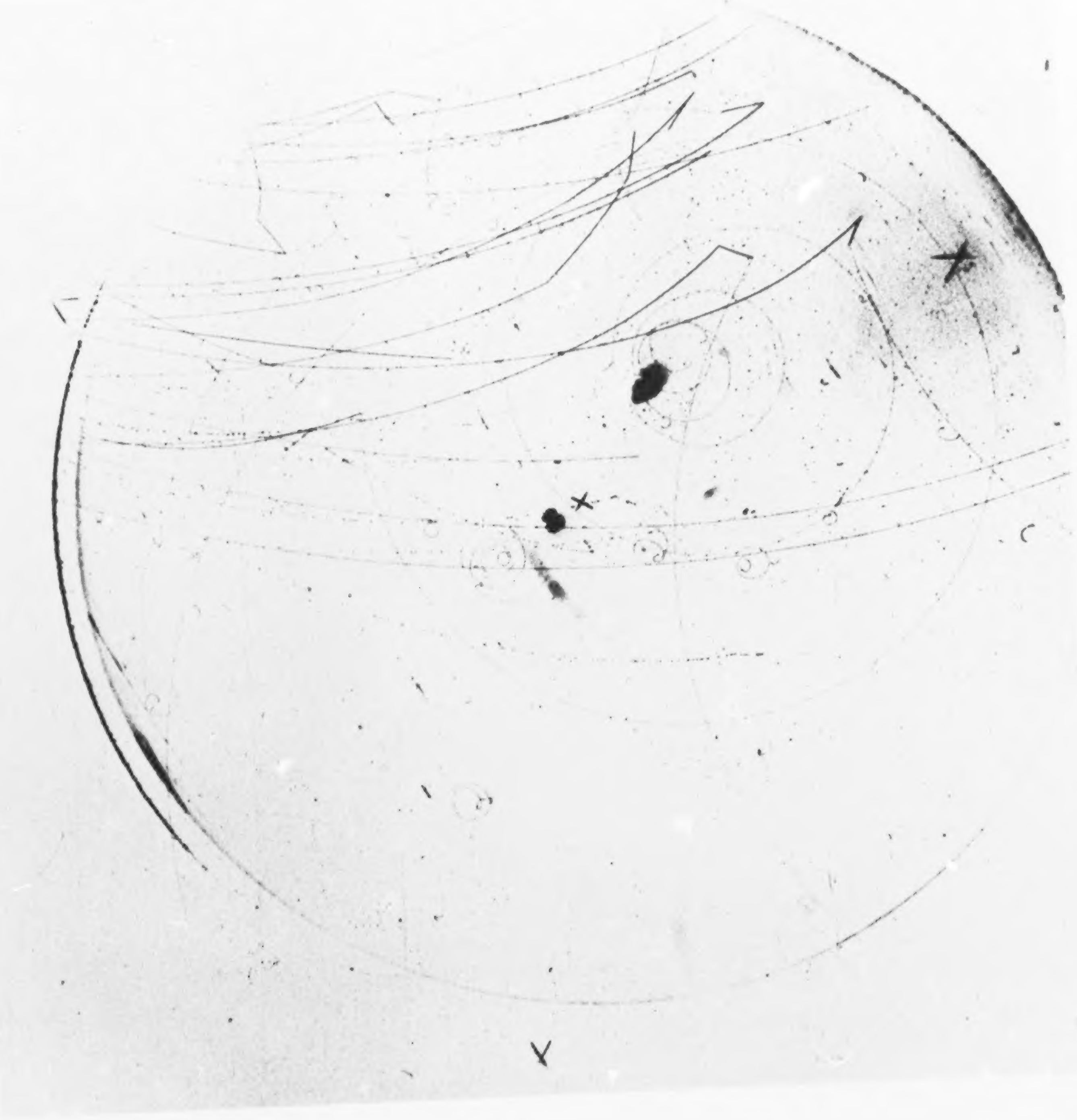
ρ of liquid hydrogen = .07 gm/cm^3

Find the average radius of curvature of the positrons, corrected for the measured scale factor. From this, calculate the average momentum of the positrons and the mass of the muon. Compare your result to the accepted value of $m_{0\mu}c^2 = 106$ MeV.

Find the average projected length of the muon track, and, from this, calculate the muon momentum and the pion mass. Compare your result to the accepted value of $m_{0\pi}c^2 = 140$ MeV.

Question

Why does the radius of curvature of each positron decrease along the track?



Positive pions enter from the upper left. Decays of pions at rest show a short muon track and a less dense (positive) electron track. Note that the long electron spiral originates from a heavy track that is much longer than the usual muon track - probably a more energetic muon that has decayed from a pion in flight. The heavy track above it with two "kinks" is a pion that scatters in the liquid hydrogen before coming to rest and decaying into a muon.

ANALYSIS OF A K+ MESON DECAY IN NUCLEAR EMULSION

University of California, Berkeley

References:

1. Brown, Camerini, Fowler, Muirhead, Powell and Ritson, "Observations with Electron-Sensitive Plates Exposed to Cosmic Radiation," Nature (London), 163, 82 (1949).
2. Gell-Mann and Rosenfeld, "Hyperons and Heavy Mesons: Systematics and Decay", U.C.R.L. Report No. 3799.
3. Ritson, Pevsner, Fung, Widgoff, Zorn, Goldhaber and Goldhaber, "Characteristics of K^+ Particles," Phys. Rev. 101, 1085 (1956).
4. Atkinson and Willis, "High Energy Particle Data," Volume II, U.C.R.L. Report No. 2426 (revised).
5. Fretter, "Introduction to Experimental Physics," Chapter 14.

Reference 1 describes the discovery of the τ meson. Reference 2 is an up-to-date summary of our knowledge of K mesons. The article is quite technical but parts of it may help the student. Reference 3 contains a description of the set-up by which the emulsion stack used in this experiment was exposed at the Bevatron. Reference 4 contains range-energy and range-momentum curves and tables of masses. It will be indispensable in preparing the report. Copies of references 2 and 4 have been placed on 2-hour reserve in the Physics Library. Reference 5, in the textbook, should be read in full, especially the sections concerning range-energy relations.

Purpose: To become familiar with some of the types of measurement which can be made in nuclear emulsions and to use these techniques to analyze the decay of a K^+-meson.

Cautions: Since the emulsion stack used in this experiment is irreplaceable, the glass-backed plates must be handled with the utmost care. Students must observe the following rules:

(a) Handle the plates carefully.
(b) Never have more than four plates out of the boxes at any one time.
(c) Always wipe off the immersion oil with Kleenex before replacing the plates in the box.
(d) Put all plates back in the humidified boxes when finished for the day.

A student will not be penalized just because a plate gets broken- accidents will happen. But students who are observed to violate any of the above rules may be barred from completing the experiment.

When working near the top edge of the plates, it becomes necessary to remove from the turret the objective lenses not in use, since they will not clear the microscope stage. Fastening and unfastening these lenses is a hazardous operation in which it is very easy to drop and smash a $150 lens. However, a simple rule will avoid accident: Always use both hands and never release the lens unless it is completely screwed in.

Introduction

At the time of exposure, the emulsion sheets - without glass - were compressed into a block or "stack." The stack was exposed to a beam of positively charged particles emerging from a Bevatron target. The beam had been magnetically analyzed and was roughly homogeneous as to momentum (about 400 Mev/c) but contained protons, K mesons and π mesons (plus a few μ mesons). Viewing a plate with the brass "L" at the upper right hand corner, the particles entered at the right. The protons stopped at a distance of one inch from entry where a very noticeable change in blackening can be observed with the naked eye. The range of the π-mesons was such that they did not end in the stack (unless they interacted in flight). The relatively rare K-mesons could be picked up by scanning the emulsion just beyond the proton endings where the grain density of the K tracks was appreciably higher than the background π tracks (because of the lower velocity of the K's). K's found in this way could then be followed until they interacted-in-flight or until they came to rest. A K^+ meson at rest is prevented from entering the nucleus by coulomb repulsion and thus decays according to one of a variety of schemes. In about six percent of the cases, the K^+ decays into three π mesons - two π^+'s and one π^-. K mesons decaying by this mode were originally called τ (tau) mesons and the nomenclature has survived. The τ decay is most distinctive since the three π mesons are coplanar, carry off a unique amount of kinetic energy among them, and have themselves very distinctive endings. (A π^+ meson usually stops and then decays according to the scheme $\pi^+ \rightarrow \mu^+ + \nu$. In this two-body decay, the μ^+ is emitted with a unique energy and has a unique range. The μ^+ decays to a positron at the end of its range. π^- mesons usually produce a nuclear disintegration or "star" at the end of their range since they are attracted to atomic nuclei.) The experiment will consist in checking a τ decay for coplanarity, in following 2 or more of the π-mesons to the end of their range, and in determining the energy release or "Q" value of the disintegration from which the mass of the K^+ meson can be determined.

Apparatus

1. The Microscope

The illumination system has been lined up and adjusted to give clear, bright images under all three oil-immersion objective lenses. Don't adjust the illumination system other than to move the condenser up and down (large knurled knob beneath the stage at left) for optimum illumination of the field of view. If you suspect that the microscope has become mis-aligned get one of the instructors to adjust it. Familiarize yourself with the controls which move the stage, with the coarse and fine focus adjustments, with the left ocular focus, with the binocular spacing adjustment and with the goniometer in the right ocular. Note that all of the Leitz objectives require immersion oil. The X20 objective requires rather a lot of immersion oil to function clearly since the working distance is long for an oil-immersion lens. Use only the non-hardening oil from the polyethylene squeeze bottle! When leaving the microscope for the day, always wipe off the immersion oil from lenses and stage using Kleenex, turn off the lamp, and cover the instrument against dust.

2. The Plates

At the time of exposure, the stack was a solid block of emulsion--- composed of slices 600 microns thick. After exposure, the slices were separated and mounted on glass for processing. The plates are numbered and are arranged in the boxes in the same relative positions they originally occupied in the stack. The plates were positioned in the stack by rods passing through holes punched in the emulsion. At the time these holes were punched, a 1 mm grid was photographed on the bottom of the emulsion slices. Corresponding points on adjacent emulsion slices can now be located by observation of the numbered grid squares. Under the microscope a grid square appears thus:

049
+
126

The upper or "x" coordinate increases as you move to the right in the field of view. The lower or "y" coordinate increases as you move down in the field of view. The plates are positioned in the plate holder by the brass "L" at the upper right corner. When positioned by these, the same grid square on all plates in the stack should be under the microscope lens.

Techniques

1. "Following through" -- If a track is to be followed for any distance it must be be pursued from one plate to the next. The grid can be used for this purpose if a map is made showing the exits and entrances of the tracks from the plates.

Plate 43	
S(urface)	G(lass)
034	036
+	+
152	154

Plate 42	
S	G
036	038
+	+
154	154

For example the track mapped here is going down in the stack (from "surface" to "glass" in each plate). In plate 43 it moves from square 034/152 to 036/154 being scattered somewhat in the process (note the change of direction). To follow it into plate 42, the observer would align the eyepiece reticle parallel to the track, insert the new plate, focus on the grid, center the reticle on the anticipated spot, then focus on the surface and look for a track parallel to the reticle near the center of the field. In the example, the track was found very near where it was expected. "Steep" tracks should not be displaced in going from one plate to the next by more than the "grid error." "Flat" tracks will be displaced somewhat along the line of motion due to the motion through the slight air gap between plates. Whenever following tracks, keep maps of this type. The "idle" lab partner can prepare the blocks in the notebook in which the observer enters the maps. As a rule the X20 objective is recommended for following the π's in this experiment.

2. Range measurements

The range a particle can be computed if x, y, z coordinates are recorded at occasional points along the track -- especially at places where the track undergoes marked scattering. The grid maps discussed above are suitable for x and y coordinates since the grid lines are accurately (within a few percent) 1 mm apart. Note that numerals and crosses within the grid lines are not accurately located. To make distance measurements within a field of view or when special accuracy is desired, the reticle scale is used. The scale must be calibrated against one of the screws which propel the stage (a separate calibration being required for each objective lens.) These screws have backlash but are accurate when advanced consistently in the same direction. "z" coordinates can be determined from the scale on the fine focus screw (there is backlash here, too). The 1 micron per division scale on this screw refers to mechanical motion and does not equal depth in the emulsion since the refractive index of various media are involved. Thus the scale divisions should be taken as arbitrary and the scale calibrated in terms of the original 600 micron emulsion thickness (the emulsion shrinks by about a factor of two during the development process). When reading depth measurements it is best to fasten the plate to the stage with modeling clay. Otherwise the plate may be dragged up and down with the oil immersion objective.

3. Angle measurements

Projected angles (i.e. the angle in the x-y plane) are easily measured with the goniometer provided. Dip angle for a track is measured by measuring the change in "z" corresponding to a convenient number of reticle divisions -- from which the tangent of the dip angle can be computed. Work fairly close to to the star always (since tracks scatter in dip as well as in projected angle) under the X100 objective lens. Again, use modeling clay.

4. Other measurements possible in nuclear emulsions are ionization measurements (by grain counting) -- a measure of particle velocity -- and multiple scattering measurements -- principally a measure of particle momentum. These types of measurement are not required for this experiment.

Procedure

Obtain from the instructor the coordinates of a decaying τ meson together with any special instructions for studying the event. Follow the π tracks to their end, making observations during the process from which the range can be computed. Sketch the endings of the π's, showing any π-μ-e events found at the end of π^+ tracks and showing any disintegration stars found at the end of π^- tracks. Measure the range of any μ's found. Measure carefully the projected angles and dip angles of the outgoing π's. From these angles, calculate the space angles between each two π's. From these results, determine the degree of coplanarity of the event. Use the tables in Reference 4 to determine the energy and momentum of the π's followed to rest. If three π's were followed to rest, check to see if the momentum triangle closes. If only two π's were followed, use the momentum triangle to infer the momentum of the third. Calculate the mass of the K^+ meson and compare with the value given in Reference 4. Also compute from Reference 4 (assume zero rest mass for the neutrino) the expected range of the μ's in π decay and compare with your measured values.

Questions

1. If the tables in Reference 4 have been used to infer both momentum and kinetic energy from the range of π's, verify by direct calculation that these quantities do correspond.
2. Would the event you have analyzed be a good one to include in a precision determinatin of K^+ mass? Explain.